



# Probability Theory and Stochastic Calculus

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## COURSE OBJECTIVES

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The module aims to offer a comprehensive yet accessible introduction to the tools of stochastic calculus used in mathematical finance for pricing and hedging derivative contracts.

## COURSE CONTENTS

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- Mathematical models for option pricing
  - from ODEs to SDEs
- Probability theory
  - from random variables to stochastic processes
- Information and conditioning
  - Conditional expectation
  - Martingales
- Brownian motion
  - Scaled symmetric random walk
  - Brownian motion and its properties
  - Quadratic variation
  - Markov property
- Stochastic calculus
  - Ito integral and its properties
  - Ito process
  - Ito formula
- Stochastic differential equations
  - Definition of SDEs
  - Markov property of solutions
  - Geometric Brownian motion

- Vasicek mean-reverting process
- Connection with PDEs
  - Feynman-Kac theorem
- Representation theorems
  - Martingale representation theorem
  - Stochastic integrals as time changed Brownian motions
- Change of measure
  - Radon-Nikodym theorem
  - Bayes rule
  - Girsanov theorem
- Option pricing models
  - Black-Scholes model

## TEXTBOOKS

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- Ballotta, Laura and Fusai, Gianluca, Tools from Stochastic Analysis for Mathematical Finance: A Gentle Introduction (2018). Available at SSRN: <https://ssrn.com/abstract=3183712>
- Shreve (2004). Stochastic calculus for finance II, Springer