



Research
Education
Outreach

CCA

Allievi Program, Master in Economics, and Ph.D. in Economics

MEASURE THEORY

January-February 2025

Instructor: Bertrand Lods

Contact Information

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Course Description

The course introduces the theory of Lebesgue integration. After defining appropriate structures of sets such as algebras, semi-algebras and sigma-algebras, we provide the notion of Lebesgue measure, and its construction from semi-algebras to sigma-algebras. We then constructively define integrals with respect to a Lebesgue measure and state their most relevant properties. The relationship with classical Riemann integration and the connection with probability theory are also examined.

Exam

Written exam at the end of the course.

Course Outline

- Classes of subsets: algebras, semi-algebras, sigma-algebras, monotone classes
- Measures: definition and properties; finite-additivity and sigma-additivity; construction of measures on sigma-algebras; completions of measures; Lebesgue-Stieltjes measures
- Measurable functions
- Lebesgue-Stieltjes integrals: construction and properties
- Convergence theorems
- Null-measure sets and properties holding almost everywhere
- Comparison with Riemann integration
- Tonelli-Fubini theorems
- Radon-Nikodym's theorem

Textbooks

Detailed lecture notes will be provided. Some reference books are:

- BILLINGSLEY, P. (1986) — *Probability and measure*. Wiley.
- DUDLEY, R.M. (2004) — *Real analysis and probability*. CUP.
- FOLLAND, G.B. (1999) — *Real analysis*. Wiley.
- ROYDEN, H.L. (1988) — *Real analysis*. Prentice-Hall.
- RUDIN, W. (1976) — *Principles of mathematical analysis*. McGraw-Hill.