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ESG asset demand with information costs

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Abstract

We study a market with non-iid returns linked to an ESG (Environmental, Social and Governance) and a market factor. Motivated by empirical evidence, we assume that the investor does not know which part of the return is due to the ESG component, unless he pays a cost. We provide conditions on the persistence, risk premium and observability of the ESG factor, relative to the market one, to invest in ESG assets. Information should be acquired when its costs are below a threshold that we find explicitly.
We calibrate the model to the German twin bonds, separate the ESG from the market risk factor, compute their risk premia and simulate optimal asset allocation.

Keywords: ESG assets, Information costs, Optimal filtering, Greenium, ESG risk premium, Unobservable ESG-factor returns.

JEL Classification: G11, G14.

The empirical evidence on whether ESG assets provide extra returns with respect to non-ESG assets is somewhat mixed: some papers, such as Hong and Kacperczyk (2009), show that stocks who do not have ESG features, such as sin stocks (tobacco, oil & gas) deserve an extra-return for market participants, while some others show that social features, like employees' satisfaction, or good governance ones, generate higher ex post returns (see respectively Edmans (2011) and Gompers et al. (2003) for seminal contributions). More recent equilibrium theory shows that it is possible to reconcile this evidence. Pastor et al. (2021), in a model with ESG-motivated or ESG-aware investors, suggest that in the long run ESG assets should not deserve an extra-return, but can have it in the convergence to the equilibrium, or when new shocks to ESG features arrive. Avramov et al. (2021) show that the extra-return can disappear, as uncertainty on ESG

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ratings, which characterizes the past and current phase of ESG investing, disappears.

It is a matter of fact that, for the time being, it is hard and costly to isolate the part of the risk and return of ESG assets that depends exclusively from sustainability. Our interest is then in studying investor demand for ESG assets when the separation of the two sources of returns does not come for free, because information is costly.

To do so, we build a multiperiod model, in a realistic, non-iid return setting. We depart from the iid assumption, because both Avramov et al. (2023) and Giglio et al. (2021) find significant parameters for a VaR model of green asset returns.

Our investors live in a world with autoregressive returns, optimally filter information and maximize the expected utility of final wealth, as in Avramov et al. (2021), Pastor et al. (2021), Pedersen et al. (2021), without having explicit ESG preferences. We assume that they are green-neutral, to focus on the unobservability feature. We assume however that they are green-aware: they know that ESG returns are related to an additional risk factor, the ESG one. They can therefore diversify by inserting a product with ESG features in their portfolio, and do so without getting informed about its contribution to returns, or getting the appropriate information about the ESG risk-return profile, at a cost. We examine their cost-of-information versus diversification benefit trade-off assuming Bayesian learning.

We provide conditions on the persistency, importance in determining returns and observability of the ESG factor, under which the investor should better buy an ESG product than stick to his "old", market portfolio. If information costs are low, he will also buy information on it, and the opposite if information costs become too high. As a result, *ESG-related assets can provide both a risk premium and a discount with respect to the market-based asset, even in expectation, and not only ex post.*

We show how this applies to the German Govies market, where both conventional and green bonds are traded - the so-called twin German bonds studied in Pastor et al. (2022) - and parametrize their greenness from market data. This allows us to uncover whether in that market there is an expected extra or lower return on green assets (greenium), what the optimal policy of a green neutral investor should be and which welfare increase - measured by the rate of return on the certainty equivalent of his utility - the optimal policy should provide.

The outline of the paper is as follows. In Section 1 we add to the portfolio choice of a green-neutral investor ESG assets, characterize the optimal filtering of information in case he does not pay a cost to separate different sources of returns, determine the maximum cost or fee that he is willing to pay to acquire ESG information and make optimal asset allocation and welfare explicit. In Section 2 we study the German Govies market through the lens of our model. In Section 3 we summarize and conclude.

1 Investor's preferences and asset allocation

Portfolio models in the presence of assets with ESG-related returns can be divided into several strands: those in which some of the investors have a preference for greenness per se, or get some non-pecuniary benefits from green stocks, those in which they do not, but are still aware of the ESG properties of asset returns, and those in which they ignore the greenness features completely. Pedersen et al. (2021) call the former investors ESG-motivated, the second ESG-aware, the third ESG-unaware. We focus on ESG-aware investors, to isolate the unobservability effect, show how and when they can integrate ESG assets in their portfolio, and what drives their choice. In a static portfolio setting, with non-perfectly correlated returns on the ESG and the market factor, an ESG-aware investor should invest in both, to increase diversification. This is not the case any more with non-iid returns.

Because of unobservability, our model resembles the one in Guasoni et al. (2019), whose investors either know the returns on single commodities or on a fund made out of them. Guasoni et al. assume that investors have power utility, of which log utility, that we are going to use below, is a special case, in which there is no hedging demand. In Guasoni et al. (2019) investors are simply either informed on the separate returns of different commodities or informed about the fund returns only. We add to their model the possibility to collect information to switch from one to the other type of information, at a cost.

To increase the clarity of the effects, we assume first that there are only assets M, whose risk premium is market-related, then introduce green assets N, whose returns depend on both factors.

1.1 Without ESG-diversifying assets

Suppose first that the representative investor can invest (going long or short) in a riskless asset and buy a risky asset subject to a market-wide risk source or factor. For the sake of simplicity we normalize the riskless rate to zero, so that the riskless asset is worth $B_0 \in \mathcal{R}^+$ at all times.

The market, non-ESG risk factor follows an Ornstein-Uhlenbeck process,

$$dJ_1(t) = -\lambda_1 J_1(t)dt + dW_1(t)$$

where $\lambda_1 > 0$ and $W_1(t)$ is a standard Brownian motion. Call $\mathcal{F}(t)$ the augmented filtration generated by $W_1(t)$.

Let σ be the instantaneous standard deviation of the log returns on the risky asset, μ its drift, and let both of them be constant. On $(\Omega, \mathcal{F}, P, \mathcal{F}(t))$

the risky asset has the price dynamics

$$\begin{aligned}\frac{dM(t)}{M(t)} &= \mu dt + \sigma dJ_1(t) \\ &= (\mu - \sigma\lambda_1 J_1(t))\, dt + \sigma dW_1(t).\end{aligned}$$

with μ, $\sigma \in \mathcal{R}, \sigma > 0$.

Note that returns are not iid because the shock dJ_1 has a mean-reverting component: the higher is the mean-reverting parameter λ_1, the less persistent will be the shock. The iid return is nested as a subcase when $\lambda_1 = 0$. In the case of interest to us, $\lambda_1 > 0$, the mean reverting factor J_1 has long term variance $\frac{1}{2\lambda_1}$: the higher is the parameter of the mean reversion, the less noise will be accumulated over time and the lower will be the long term variance of non-ESG assets.

We assume that investors are log and solve their expected utility maximization problem

$$\mathbb{E}(\mathbb{U}; x, T) = \sup_{\pi(t)} \left(\mathbb{E} \frac{\ln(W(T))}{T}; X(0) = x \right) \tag{1}$$

where $W(T) = X(T) + B_0$, $X(t)$ is the total amount invested in M at time t, $x > 0$, the utility function is log and $\pi(t)$ is the fraction of wealth invested in M at t. It follows that:

$$X(t) = x \exp\left[\int_0^t \left[\pi(s)\mu - \pi(s)\sigma\lambda_1 J_1(s) - \frac{1}{2}\pi(s)^2\sigma^2\right] ds + \int_0^t \pi(s)\sigma dW_1(s)\right]. \tag{2}$$

The budget constraint or self-financing condition is:

$$\frac{dX(t)}{X(t)} = \pi(t)\frac{dM(t)}{M(t)}$$

We then compute the limit, when the horizon of the investor tends to infinity, of the derived utility $\mathbb{E}(\mathbb{U}; x, T)$ per unit of time. Because of the certainty equivalent definition with log utility, this represents the rate of growth of the certainty equivalent itself over time. This is the measure that we use to evaluate the welfare of optimal policies

$$u(x) = \lim_{T\to\infty} \frac{\mathbb{E}(\mathbb{U}; x, T)}{T}$$

It is easy to prove - see Guasoni et al. (2019) - that:

Theorem 1 *(Guasoni et al. (2019)) The optimal strategy in non-ESG assets $\pi^*(t)$, and the corresponding rate of growth of expected utility $u(x)$,* *which solve problem* (1) *subject to* (2) *are*

$$\pi^*(t) = \frac{\mu - \sigma\lambda_1 J_1(t)}{\sigma^2}, \tag{3}$$

$$u(x) = \frac{\mu^2}{2\sigma^2} + \frac{\lambda_1}{4}. \tag{4}$$

The portfolio allocation differs from the standard, myopic one with iid returns ($\lambda_1 = 0$), which coincides with its first addendum, and represents now a strategic component. The second addendum represents a tactical component and is time dependent, because of the mean reversion in the risk process. The higher is the current value of the mean reverting part of returns, if positive, the lower is the expected return, and therefore the lower is the position in the risky asset. Also, the tactical allocation is inversely proportional to both the short and long run variance. As a result, the utility is greater, by a factor proportional to mean reversion - and inversely proportional to the long-run variance - with respect to the one which would obtain with iid return.

1.2 With ESG-diversifying assets

Suppose now that the investor can buy/sell either the same riskless asset as above or a risky asset that, on top of the dependence on a market-wide risk factor, depends also on an ESG risk factor. Call the new asset N. Since its returns depend on J_1 and on another risk source, the ESG factor J_2, we say that an investor preferring N to M diversifies.

Also J_2 follows a mean-reverting Ornstein-Uhlenbeck process

$$dJ_2(t) = -\lambda_2 J_2(t)dt + dW_2(t) \tag{5}$$

with $\lambda_2 > 0, \lambda_2 \neq \lambda_1, W_1(t), W_2(t)$ independent Brownian motions. Let N be exposed by $p_1 > 0$ to the market risk factor, by $p_2 > 0$ to the ESG one, with $p_1^2 + p_2^2 = 1$. Note that the market-only case is nested as a subcase for $p_2 \to 0$. Define as $Y(t)$ the $p-$weighted sum of J_1 and J_2:

$$Y(t) = p_1 J_1(t) + p_2 J_2(t) \tag{6}$$

and let N satisfy the following dynamics

$$\frac{dN(t)}{N(t)} = \mu dt + \sigma dY(t) \tag{7}$$

where μ, σ are the same as for asset M.

The long-term variance of Y is $p_1^2/(2\lambda_1) + p_2^2/(2\lambda_2)$, which depends on both mean reversions.

When he invests in the ESG asset, the investor observes its price, returns, but cannot observe separately the returns on the first and second risk factor, unless he pays for distinguishing the two signals. He just observes the innovations in the "pooled signal" or risk factor Y. While he knows μ and σ, he does not observe separately the realizations of the inoovations in the processes J_i, namely how the dependence on the market and the ESG features of the asset contribute to its return. He is *uninformed*. As an alternative, the investor can decide to get full information on both risk factors, by paying a fee ϕ per unit of time, and observe separately the innovations in J_1 and J_2. He becomes *informed*.

It follows from (7) that - all others equal - the additional source of risk J_2 has a stronger effect on the uninformed investor, the higher is the mean reversion parameter λ_2 associated to it. The greater is the difference between λ_2 and λ_1, with $\lambda_2 > \lambda_1$, the greater is the "gap" in terms of information between an *informed* and an *uninformed* investor. If the investor decides to stay uninformed, his filtration is $(\mathcal{F}_U(t))_{t\in[0,+\infty)}$. The filtration for the informed investor is $(\mathcal{F}_I(t))_{t\in[0,+\infty)}$. $\mathcal{F}_U(t)$ is the augmented filtration generated by $Y(t)$ alone while $\mathcal{F}_I(t)$ denotes the augmented filtration generated by $W_1(t)$ and $W_2(t)$. Further, $\mathcal{F}_U(t)\subset\mathcal{F}_I(t)$.

From standard filtration theory, we get that the dynamics for the *uninformed* investor are

$$\frac{dN_U(t)}{N_U(t)} = \left[\mu - \sigma\left(p_1\lambda_1\hat{J}_1(t) + p_2\lambda_2\hat{J}_2(t)\right)\right]dt + \sigma d\hat{W}t$$

where $\hat{W}$ is an $\mathcal{F}_U(t)$-Brownian motion and represents the innovation process obtained from the filtering procedure:

$$\begin{aligned}\hat{W}(t) \quad &= \quad \int_0^t \left[\lambda_1 p_1(\hat{J}_1(s) - J_1(s)) + \lambda_2 p_2\left(\hat{J}_2(s) - J_2(s)\right)\right] ds \\ &\quad + \int_0^t (p_1 dW_1(s) + p_2 dW_2(s))\end{aligned}$$

The estimates of the OU processes, $\hat{J}_j(s), j = 1, 2$, solve the SDE

$$d\hat{J}_j(t) = -\lambda_j\hat{J}_j(t)dt + \alpha_j(t)d\hat{W}_j(t) \tag{8}$$

The vector $\alpha(t)$ is defined as

$$\alpha(t) = p' - \gamma(t)b',$$

where $p = [p_1\ p_2], b = [\lambda_1 p_1\ \lambda_2 p_2]$ and γ is the variance covariance matrix of the errors in the learning procedure of the return processes, namely

$$\gamma(t) = \mathbb{E}[\left(J_1(t) - \hat{J}_1(t)\right)\left(J_2(t) - \hat{J}_2(t)\right)]$$

so that

$$\alpha(t) = \left[\begin{array}{c} p_1 - \lambda_1 p_1\gamma_{11}(t) - \lambda_2 p_2\gamma_{12}(t) \\ p_2 - \lambda_1 p_1\gamma_{21}(t) - \lambda_2 p_2\gamma_{22}(t)\end{array}\right]$$

For the *informed* investor the dynamics can be written substituting (6) in (7) as follows

$$\frac{dN_I(t)}{N_I(t)} = (\mu - p_1\sigma\lambda_1 J_1(t) - p_2\sigma\lambda_2 J_2(t))\, dt + \sigma dW_I(t)$$

where $W_I(t) = p_1 W_1(t) + p_2 W_2(t)$ is an $\mathcal{F}_I(t)$-Brownian motion.

The risk premium on the market or green-neutral asset M was

$$k(t) = \mu - \sigma\lambda_1 J_1(t) - \sigma^2/2$$

The risk premium on the green asset N, under the informed filtration, is

$$k_I(t) = \mu - \sigma p_1\lambda_1 J_1(t) - \sigma p_2\lambda_2 J_2(t) - \sigma^2/2 - \phi$$

while under the uninformed is

$$k_U(t) = \mu - \sigma p_1\lambda_1 \hat{J}_1(t) - \sigma p_2\lambda_2 \hat{J}_2(t)) - \sigma^2/2$$

Both can be lower or higher than the risk premium on the non-ESG asset. The first and third coincide when $p_2 \to 0$, when the second is dominated because it is net of the (unuseful) information fees. This is of the outmost importance in our achievements: there is no positive or negative greenium implicit in the model, because *ESG-related assets can provide both a risk premium and a discount with respect to the market-based asset, even in expectation, or ex ante, and not only ex post.* The premium can be positive or negative, and it changes sign over time, because of the evolution of the factors. Also, unless the weight of the ESG factor is null, it is differently perceived by uninformed and informed investors.

We are ready to spell out the conditions under which the investor participates in the ESG-market as an *uninformed* (U) or as an *informed* (I) investor. For this we solve the logarithmic utility maximization problem in the two cases, namely (1) with $W = W_\kappa = X_\kappa + B_0$, $\kappa = U, I$, and $z = \pi_\kappa$, $\kappa = U, I$. The self-financing condition for the *uninformed* investor is

$$\frac{dX_U(t)}{X_U(t)} = \pi_U(t)\frac{dN_U(t)}{N_U(t)} \tag{9}$$

while the self-financing condition for the *informed* investor is

$$\frac{dX_I(t)}{X_I(t)} = \pi_I(t)\frac{dN_I(t)}{N_I(t)} - \phi dt. \tag{10}$$

Let us introduce the vector $\alpha = p' - \gamma b'$, where γ is the stationary value of the variance covariance matrix of the errors in the learning procedure of the return processes, namely

$$\gamma = \lim_{t\to\infty} \mathbb{E}[\left(J_1(t) - \hat{J}_1(t)\right)\left(J_2(t) - \hat{J}_2(t)\right)]$$

The matrix γ exists unique, if $\gamma(0)$ is positive definite, and satisfies the Riccati equation (see Guasoni et al. (2019))

$$-\lambda\gamma - \gamma\lambda + I - \alpha\alpha' = 0$$

where λ is the diagonal matrix with the parameters λ_i on the main diagonal and I is the 2 x 2 identity matrix. This is a CARE matrix equation, whose solution obtains from the eigenvalues and vectors of the following matrix

$$Z = \begin{pmatrix} b'p - \lambda & -BB' \\ -I + p'p & -p'b + \lambda \end{pmatrix}$$

and

$$B = \begin{pmatrix} b_1 & 0 \\ b_2 & 0 \end{pmatrix}$$

Indeed, if we select the two eigenvectors of Z with negative real parts, z_1 and z_2, split them into the first two and second two components, define by so doing the submatrices U_1 and U_2, the γ matrix is the product

$$\gamma = U_1^{-1} U_2.$$

It can be proved, as a straightforward extension of Guasoni et al. (2019), that the optimal portfolio allocation in

Theorem 2 *The optimal strategies in ESG assets π^*_κ, $\kappa = U, I$ and the asymptotic rate of growth of the certainty equivalent u_i, $i = U, I$, which solve problem* (1) *subject to* (9) *for the uninformed investor,* (1) *subject to* (10) *for the informed one, exist. They are*

$$\pi^*_U(t) = \frac{\mu - \lambda_1 \sigma p_1 \hat{J}_1(t) - \lambda_2 \sigma p_2 \hat{J}_2(t)}{\sigma^2} \tag{11}$$

$$u_U(x) = \frac{\mu^2}{2\sigma^2} + \frac{p_1^2 \lambda_1 \alpha_1^2 + p_2^2 \lambda_2 \alpha_2^2}{4} + \frac{p_1 p_2 (\lambda_1 + \lambda_2) \alpha_1 \alpha_2}{4}. \tag{12}$$

$$\pi^*_I(t) = \frac{\mu - \lambda_1 \sigma p_1 J_1(t) - \lambda_2 \sigma p_2 J_2(t)}{\sigma^2} \tag{13}$$

$$u_I(x, \phi) = \frac{\mu^2}{2\sigma^2} + \frac{\lambda_1 p_1^2 + \lambda_2 p_2^2}{4} - \phi \tag{14}$$

The strategic allocation components are the usual, myopic ones. With perfect information, the tactical one is a straightforward extension of the one with the market factor only. With imperfect information, the tactical component incorporates also the estimates of the variance-covariances of the forecast errors. Note that the utility of the informed investor u_I is higher than the one of the uninformed agent u_U if and only if the information cost is low enough, $\phi < \phi^*$, where

$$\phi^* = \left[p_1^2 \lambda_1 (1 - \alpha_1^2) + p_2^2 \lambda_2 (1 - \alpha_2^2) - p_1 p_2 \alpha_1 \alpha_2 (\lambda_1 + \lambda_2)\right]/4. \tag{15}$$

Also, for the informed investor, utility u_I is higher than with the market factor only u if and only if the information cost is low enough, $\phi < \phi^{**}$, where:

$$\phi^{**} = \frac{(\lambda_2 + \lambda_1)\, p_2^2}{4} \tag{16}$$

while for the uninformed one the comparison with the market case depends on the parameters driving his estimates of the role of the two factors, namely the p, λ and the covariances in α. It follows that, when the returns on green assets are relatively less persistent than the ones on the market-related asset, it is optimal to invest in the ESG assets. The investor acquires information when fees are low and remains uninformed when fees are too high. If they are relatively more persistent, the investor remains invested only in the market factor, and therefore does not acquire information, if fees are high, or diversifies in the ESG one acquiring information on it, if fees are low.

Theorem 3 **a)** *If $\lambda_2(p_2^2\alpha_2^2 + p_1p_2\alpha_1\alpha_2) > \lambda_1(1 - p_1^2\alpha_1^2 - p_1p_2\alpha_1\alpha_2)$, the investor should invest in ESG assets, and acquire information only if it is not too costly, since*

$$u(x) < u_U(x) < u_I(x, \phi) \qquad if\ \phi < \phi^*, \tag{17}$$

$$u_I(x, \phi) < u_U(x) \qquad if\ \phi > \phi^* \tag{18}$$

$$u(x) < u_U(x) \tag{19}$$

b) *If the opposite inequality holds, the investor should neither invest in ESG assets nor acquire information if $\phi > \phi^*$, since*

$$u_I(x, \phi) < u_U(x) < u(x) \qquad if\ \phi > \phi^* \tag{20}$$

or if $\max(\phi^{**}, \phi^*) = \phi^*, \phi^{**} < \phi < \phi^*$*, since*

$$u_I(x, \phi) \quad < u_{,U}(x) < u(x) \quad if \max(\phi^{**}, \phi^*) \quad = \phi^*, \phi^{**} < \phi < \phi^*$$

He should invest and get information, if $\phi < min(\phi^*, \phi^{**})$, since

$$u_U(x) < u(x) < u_I(x, \phi) \qquad if\ \phi < min(\phi^*, \phi^{**}) \tag{21}$$

Proof *If the inequality sub a) holds, then* $u_U > u$. *If it is also true that* $\phi < \phi^*$, *then* $u_I > u_U$ *and consequently* $u < u_U < u_I$. *If instead it is true that* $\phi > \phi^*$, *then* $u_I < u_U$ *and consequently* $u < u_U$, $u_I < u_U$. *This proves the statement sub a).*

If the inequality sub b) holds, then $u_U < u$. *If it is also true that* $\phi < \phi^*$, *then* $u_I > u$ *and consequently* $u_U < u, u_U < u_I$. *We need to compare u and* u_I. *If* $\phi^* > \phi^{**}$, *then* $u > u_I$ *if* $\phi^{**} < \phi < \phi^*$ *and* $u < u_I$ *if* $\phi < \phi^{**}$. *If* $\phi^* < \phi^{**}$, *then* $u < u_I$. *Still under sub b), if it is true that* $\phi > \phi^*$, *then* $u_I < u_U$ *and consequently* $u_I < u_U < u$. *This proves the statements sub b).*

The main reason for these results is that higher mean reversion implies lower long-run variance of the factor and enhances the value of information.

Note that, in the special case of no costs (ϕ =0), the investor should invest in ESG assets and get information, independently of the inequality that distinguishes the cases a) and b) of the theorem. This happens because of diversification benefits, and would be even more pronounced if we had ESG-motivated investors.

At the opposite, if information is very costly or cannot be bought ($\phi \to \infty$), the investor should invest in ESG asset without getting information in the case sub a), and do not diversify in the ESG risk factor sub b). The decision would be based on the persistence properties and long-run variance of returns, given that imperfect information cannot be overcome..

1.3 Fee-threshold for information acquisitio

The previous theorem allow us to spell out under which conditions an investor participates in the market as informed or uninformed, as a function of the cost ϕ. Among the thresholds in the theorem, we focus on ϕ^*, the level of indifference fees which matches $u_U(x)$ in (12) and $u_I(x, \phi)$ in (14). It represents the welfare gain from information on the role of the ESG versus market factor.

Note that ϕ^* does not depend on μ and σ, which are common knowledge, but only on the weights p_1, p_2 and the mean reversion parameters λ_1, λ_2, which enter the risk factors J_1, J_2 and their weighted sum Y. Both in the first and second subcase of the previous theorem, if the level of ϕ remains below the value ϕ^* it is better to be informed, because

$$\phi < \phi^*_N \Rightarrow u_I(x, \phi) > u_U(x).$$

If the level of ϕ is greater than the value ϕ^* it is better to be uninformed

$$\phi > \phi_N^* \Rightarrow u_I(x,\phi) < u_U(x).$$

Note that

$$\begin{aligned}
\lim_{\substack{p_1\to 0\\ p_2\to 1}} \phi^* &= \lambda_2(\gamma_{22}\lambda_2^2 + 2\gamma_{22}\lambda_2)\\
\lim_{\substack{p_1\to 1\\ p_2\to 0}} \phi^* &= \lambda_1(\gamma_{11}\lambda_1^2 + 2\gamma_{11}\lambda_1)
\end{aligned}$$

This means that, when either the ESG represents the overall riskiness of the asset ($p_2 \to 1$) or gives no contribution to it ($p_2 \to 0$), the value of getting to know it in isolation is positive, since $\gamma_{11} > 0, \gamma_{22} > 0$.

When $p_2 \to 1$, the higher the mean reversion of the ESG factor λ_2, as well as the variance of the error in its estimate γ_{22}, the higher the value of the information fee. This is because an higher mean reversion provides more information and less noise, that allows an investor who has access to all the information to reap his welfare gain, even though he does not like greenness per se. Also an higher volatility of the error from limited information increases the maximum acceptable fees, as intuition would suggest.

When $p_2 \to 0$ instead the limit value of the fee is independent of λ_2, as intuition suggests, because only the market factor remains. Considerations symmetric to the ones for λ_2 and the variance of the corresponding estimate now apply to the market factor.

In the second case studied in the theorem, also the comparison with the level of fees which equates the informed and non-diversified investor, ϕ^{**}, enters, because, when it is optimal to get information, it may well be case that the cost of getting it is so high that sticking to the market factor is better.

2 Twin German bonds

We now provide a calibrated example, which allows us to extract the μ, σ, but above all λ and p parameters from market data, to specify the optimal asset allocation for the corresponding market, to comment on whether on that market the market premium for greenness is positive or negative and to simulate from it.

In the example we study the first issue of the so-called twin German bonds, which are issued by the German Federation and are a couple of bonds equal in all features but size of the issue, namely with the same maturity and coupon. The former (ISIN DE0001102507) is conventional, while the latter (ISIN DE0001030708) is green, since it is issued with a declaration of allocation of the resources raised through it to green projects and with an evaluation of the potential impact of such an allocation. The general scope is transition to a low-carbon economy.[1] The very first issue of the green bonds, for a face value of 6.5 billion euros and a duration and maturity of 10 years, took place in September 2020. The data we use are from September 8, 2020, to September 29, 2023 for both the green bond and the same-maturity conventional bond (source: Bloomberg). Together with them, we consider as riskless rate the OIS (Euro short-term rate - Volume-weighted trimmed mean rate (EST, B, EU000A2X2A25,WT), source: Refinitiv). We compute the daily log returns on both bonds and subtract the daily OIS rate, so as to enter into the model set up above. Consistently with the model itself, the part of wealth not invested in the Govies will be invested at the OIS rate.

We report in the table below the statistics of such issues over the observation period, namely the average yield, as well as its initial (September 8, 2020) and final (September 29, 2023) values, the average realized log return and its cumulative value, in basis points per day. We accompany them with the average, initial and final value of the annualized OIS, still in basis points.

	green bond	conventional
yield		
average	71.58	74.91
initial	−51.20	−49.60
final	269.7	271.6
realized log return		
average	−0.454	−0.454
cumulative	−0.842	−0.822
OIS		
average	48	
initial	−55.4	
final	388	

In our terminology, conventional bonds are $M-$type securities, green are $N-$ones. Note that on average, the green bond yield is smaller than the conventional one, by 3 bp per day approximately, while both are greater than the OIS rate.

Let us remind first that the log returns on the conventional bonds are

$$d \log M = \left(\mu - \sigma^2/2 - \sigma\lambda_1 p_1 J_1(t)\right) dt + \sigma dW_1$$

while the true - or informed - ones on green bonds are

$$d \log N_I = \left(\mu - \sigma^2/2 - \sigma\left(\lambda_1 p_1 J_1(t) + \lambda_2 p_2 J_2(t)\right)\right) dt + \sigma dW_I$$

Therefore, conventional bond returns have instantaneous drift

$$\mu - \sigma^2/2 - \sigma\lambda_1 p_1 J_1(t)$$

[1]The twin bonds have been discussed also in Pastor et al. (2022), For further details see also https://www.deutsche-finanzagentur.de/en/federal-securities/types-of-federal-securities/green-federal-securities/twin-bond-concept

and variance σ^2, while returns on the green bond have instantaneous drift

$$\mu - \sigma^2/2 - \sigma\left[\lambda_1 p_1 J_1(t) + \lambda_2 p_2 J_2(t)\right]$$

and variance σ^2. The long-run value of the drift is in both cases

$$\mu - \sigma^2/2$$

The drift variance is

$$\sigma^2 var(J_1(t))$$

for the conventional bond and

$$\sigma^2 var(Y(t))$$

for the green, with long-run values

$$\sigma^2 p_1^2/(2\lambda_1)$$

for the conventional bond and

$$\sigma^2\left[p_1^2/(2\lambda_1) + p_2^2/(2\lambda_2)\right]$$

for the green.

So, we can take any estimate of the instantaneous variance of log returns on either conventional or green bonds, including the variance of a time series, to get σ from either the conventional or green bond.

We can compute the drift from different time series of log returns on either conventional or green bonds and obtain the mean of the estimates. Because the stationary distribution of J_1 and J_2 has zero mean. That mean represents an estimate of the long run-mean of the drift, $\mu - \sigma^2/2$. Using the known value for σ, we can therefore obtain μ.[2]

Once the parameters μ and σ are fixed, we can use the observations on the returns on the two bonds to get the observations of dJ and dY, since from the expression of log returns it follows that

$$dJ_1(t) = \frac{d\log M - (\mu - \sigma^2/2)dt}{\sigma}$$

[2] The previous methodology assumes that the estimates of σ and μ are not different, whether one uses the green or non-green bonds. In case they are, a modification of the previous model with different μ and σ on the N and M asset must be used. Since in the data this was the case, we used the drift long-run values $\mu_i - \sigma_i^2/2, i = 1, 2$, with variance

$$\sigma_1^2 p_1^2/(2\lambda_1)$$

for the conventional bond and

$$\sigma_1^2 p_1^2/(2\lambda_1) + \sigma_1^2 p_2^2/(2\lambda_2)$$

for the green. We maintain the simplified version in which the drift and diffusion coefficient are the same in the text.

$$dY(t) = \frac{d \log N - (\mu - \sigma^2/2)dt}{\sigma}$$

The usual methods to estimate the mean reversion of an OU process can be used to get λ_1 from the observations on $dJ_1(t)$.

From different time series of log returns on conventional and green bonds we can obtain different drift estimates, and compute the variance of the drifts. We do that by simulation, bootstrapping from the original data smaller time series. If we do that on the conventional and green bonds, we get respectively

$$\sigma^2 p_1^2/(2\lambda_1)$$

and

$$\sigma^2 \left[p_1^2/(2\lambda_1) + (1-p_1)^2/(2\lambda_2)\right]$$

The former, together with the already fixed values of σ and λ_1, gives an estimate of p_1, while the second gives λ_2. All the needed parameters have therefore been obtained.[3]

For the case at hand, we obtain the following parameter values:[4]

μ_1	-0.00036184
μ_2	-0.000354343
σ_1	0.003019844
σ_2	0.003018847
λ_1	0.003018847
λ_2	44.77073546
p_1	0.13220658

which show the daily values of the drift and standard deviation, the mean reversion of the market and ESG factor and the former's weight in the green bond. We have therefore separated the two signals, as an informed

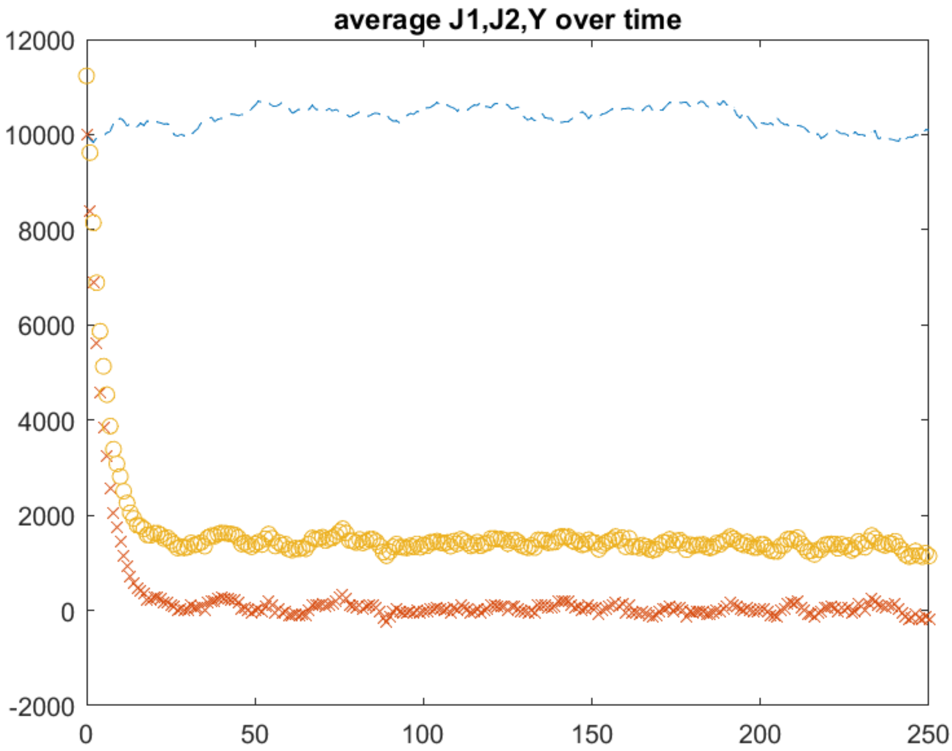
[3] To sum up, the calibration strategy consists in obtaining the variance of log returns on M or N, $\sigma^{2,}$ obtain μ from the long-run mean of the drift, compute the increments dJ_1 implicit in the log returns on M and the mean reversion of the OU market factor from the observations on its changes dJ_1, obtain p_1 and p_2 from the variance of the long-run drift of M and obtain the mean reversion of the ESG factor from the variance of the long run drift of N. In particular, after taking the variance of a time series of log returns on either conventional or green bonds as an estimate of the instantaneous variance, we bootstrap smaller ones. When we compute a mean of the drift estimates from the sub-series, we read it as a long-run mean $\mu - \sigma^2/2$.

Pastor et al. (2021) define an ESG factor, which also in their case is a second factor in ESG-asset returns, on top of the traditional CAPM one. The factor and its premium can be separately observed by agents. To build the ESG factor, they suggest running a cross-sectional regression of market-adjusted excess stock returns on the stocks' ESG characteristics, with no intercept. The ESG factor is the slope of that regression, a weighted average of market-adjusted stock returns, in which the weights are the ESG features of the stock under exam. Pastor et al. also add that, to obtain the time series of the ESG factor's actual values, one can run a series of such cross-sectional regressions. The technique cannot be applied here.

[4] See footnote 2.

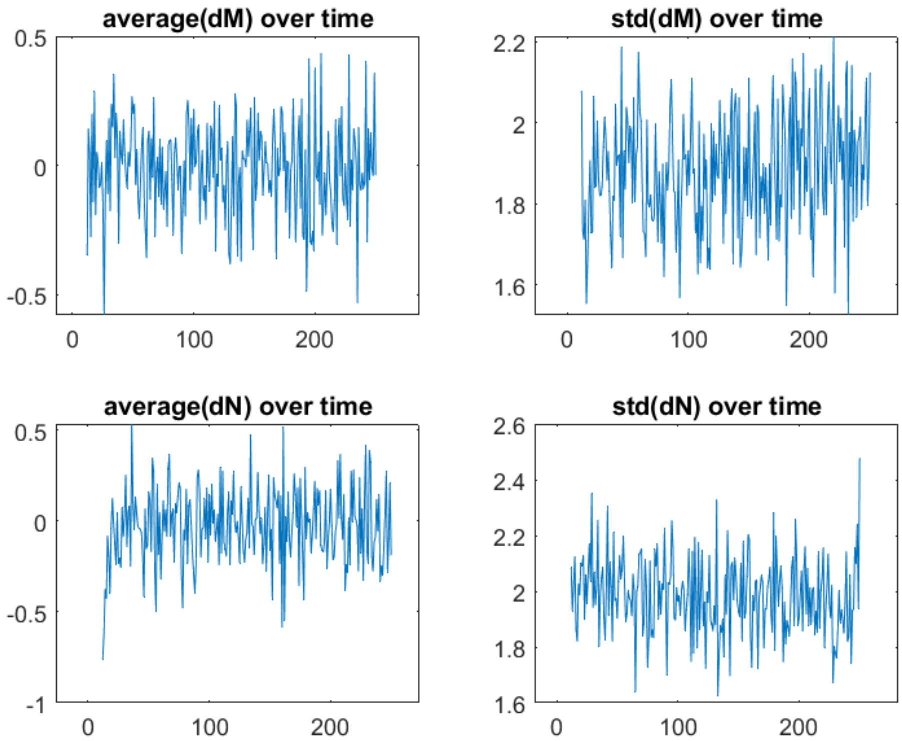
investor would do. By so doing, we discover that the drift μ of both assets is negative, consistently with the general behavior of bond returns in the observation period. We also discover that the J_1 process, which represents the market factor, has a higher persistence than the green, with a mean reversion much smaller than J_2, the ESG process. Consequently, its has a much smaller weight in determining the returns on the green bond, since $p_2^2 = 1 - p_1^2 = 98.25\%$. So, the green factor is priced and has a smaller contribution to the long-run variance.

The mean of the simulated processes J_1, J_2 and Y, over a one year horizon, using a daily time step, expressed in basis points, are as represented in the following picture. The initial value of the process J_1 is assumed to be one, and the effect of the higher mean reversion in J_2 and Y, together with the different importance of the ESG versus the market factor in determining the actual returns on N is clearly visible:



Average simulated values of the processes J_1 (dashed), J_1 (crosses), Y (dots) over 250 days. Simulations are over a one-year horizon (days on the abscissae).

The processes for returns on M and N, once simulated over a one year horizon, using a daily time step, and expressed in basis points, are as represented in the following picture, through their mean over the realizations and standard deviations. In the German market, over the horizon of interest, the changes in the two processes are quite similar, up to the second moment.



Average of the simulated changes in the conventional (top left) and green bond (bottom left), together with their standard deviations (top and bottom right, respectively), over250 days.

Also, the daily risk premium on the conventional M is

$$k(t) = -0.00036184 - 0.003019844 J_1(t) - 0.003019844^2/2$$

The daily risk premium on the green bond N, under the informed filtration, is

$$\begin{aligned} k_I(t) &= -0.000354343 - 1.2053 \times 10^{-6} J_1(t) - 0.134\ J_2(t) \\ & \quad -0.003019844^2/2 - \phi \end{aligned}$$

while under the uninformed is

$$
\begin{aligned}
k_I(t) &= -0.000354343 - 1.205\,3 \times 10^{-6} \hat{J}_1(t) - 0.134\ \hat{J}_2(t) \\
&\quad -0.003019844^2/2
\end{aligned}
$$

which evidently depend on the realizations of the noises and the fees. So, there is no dominated asset, in spite of the difference in yields. However, given that the drift is negative, and we are working on returns net of the OIS one, we can expect negative optimal positions in both bonds, with an implied positive one in the riskless contract.

In order to determine the optimal allocation policies and the corresponding rates of growth of the certainty equivalent, we need to solve for the stationary variance-covariance matrix of the error estimates, for the case at hand. By so doing, we obtain

$$
\gamma = \begin{pmatrix} 0.0233 & -0.0180 \\ -0.0180 & 0.0292 \end{pmatrix}
$$

It follows that the daily rates of growth of the certainty equivalent by investing in the bond M, computed according to Theorem 1, is

$$
0.0080
$$

That is smaller than the rate of growth by investing in the bond N, computed according to Theorem 2, while staying uninformed:

$$
0.67
$$

The informed one depends on fees:

$$
11.0043 - \phi
$$

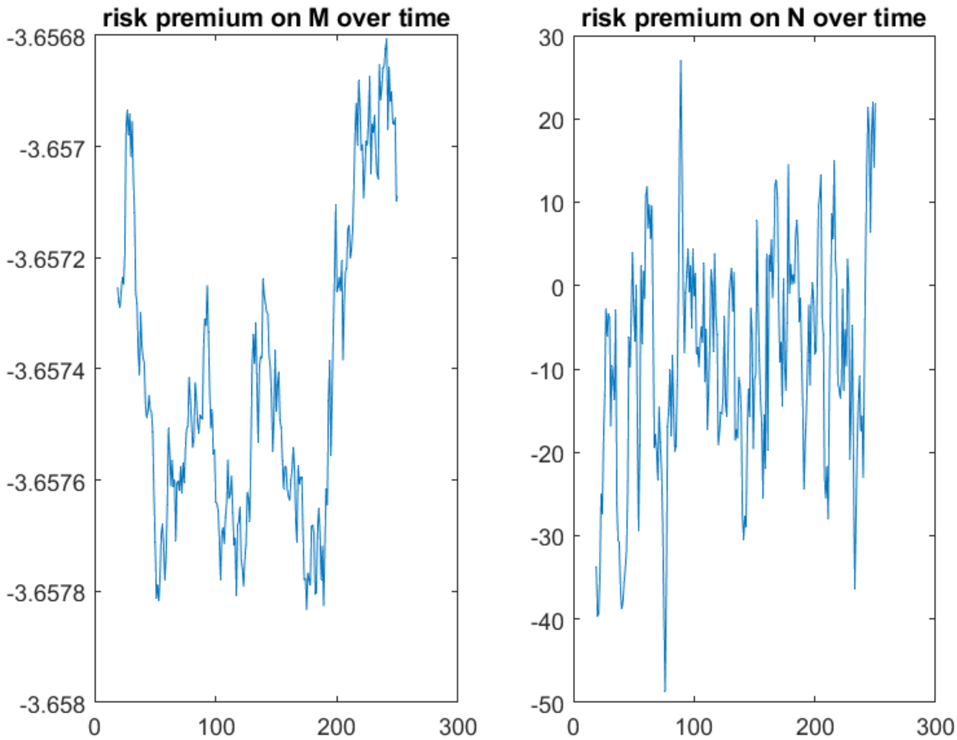
Since subcase a) of Theorem 3 applies, $\phi^* = 10.334$, $\phi^{**} = 10.998$, and the optimal policy consists in investing in the ESG asset and getting information only as long as $\phi < 10.334$ basis points per day. Investing in the ESG asset and getting no information is optimal if fees are above that threshold.

Using theorems 1 and 2, we can make the optimal policies explicit:

$$
\begin{aligned}
\pi^* &= -40 - 1.0063 J_1(t) \\
\pi_I^* &= -40 - 0.1330 J_1(t) - 1.5 \times e + 04 J_2(t) \\
\pi_U^* &= -40 - 0.1330 \hat{J}_1(t) - 1.5 \times e + 04 \hat{J}_2(t)
\end{aligned}
$$

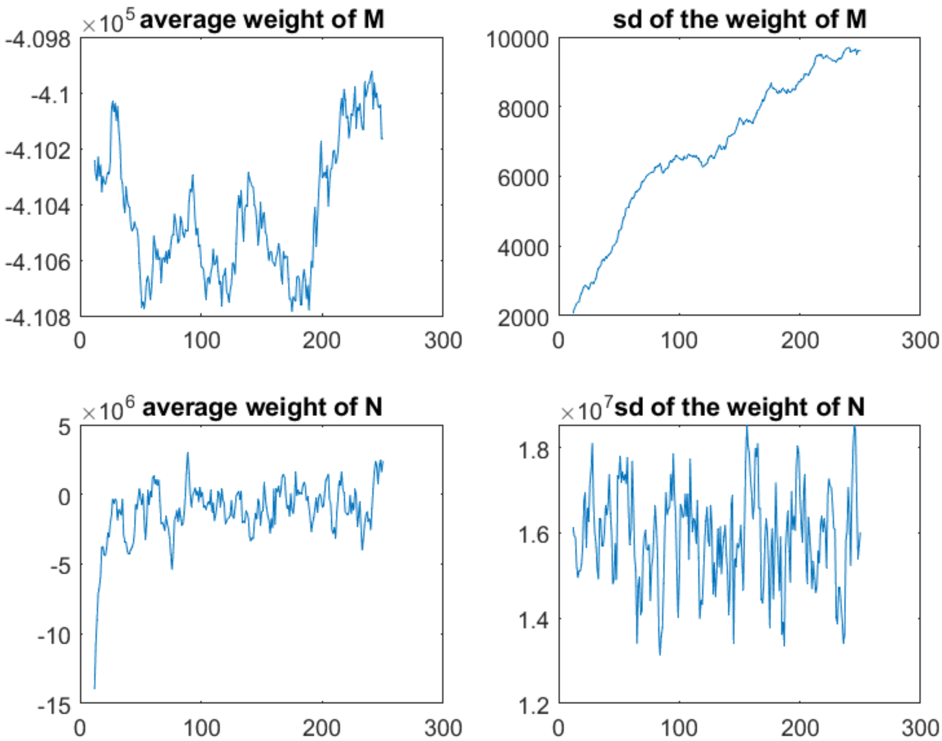
As for the risk premia, we can simulate them, using the same horizon and step as above, both for the conventional (left) and green bond (right), as seen by the informed investor. The right plot is gross of fees, and both are in basis points. Assuming an initial value of 1 for the OU processes and

100 for the bonds, the risk premium on M is on average negative, while the one on N goes from negative to positive and is much bigger in magnitude. So, on the German market exposure to the green factor pays, even if not at all times.



Average simulated values of the risk premia on the conventional (left) and green (right) bond, over 250 days.

Last, to give a sense of the asset allocation, let us simulate the weights of the two assets in the portfolio, for the cases when the optimal allocation consists in remaining invested in the riskless asset and the market one (information fees greater than 10, top), and for the case in which the optimal policy consists in investing in the ESG asset and getting information (information fees smaller than 10, bottom). The figure below represents the positions in the risky assets, obtained by simulation, using the same horizon and step as above. We compute the average and standard deviations of the positions - the weights π and π_I in the portfolio - over simulations. The bottom plots are as seen by the informed investor, gross of fees, and both are in basis points. If, as we assumed, the riskless alternative is the OIS, the optimal policies on M goes short the risky asset, while the one on N is sometimes positive, sometimes negative. The standard deviations of the optimal policies are very different, with the green bond much more erratic.



Average of the simulated portfolio weights on the conventional (top left) and green bond (bottom left), over a one-year horizon. Standard deviation of the weights on the right, respectvely top for the conventional, bottom for the green bond.

3 Summary and Conclusions

In this paper we investigate diversification benefits versus information cost effects in ESG assets, and the optimal behavior of a rational green-neutral investor facing them and choosing at the same time how much information to acquire and how to invest. When information is imperfect the investor adopts optimal filtering to process it. To get perfect information the investor has to pay a fee. Two cases are relevant: the first occurs when the shocks on the ESG factor are relatively less persistent than the market ones, the second otherwise. In the first case it is optimal to invest in the ESG assets. The investor acquires information when fees are low and remains uninformed when fees are too high. In the second case the investor remains invested only in the market factor, and therefore does not acquire information, if fees are high, or diversifies in the ESG one acquiring information on it, if fees are low. The barriers with respect to which fees are high or low are different in the two cases. The reason for this behavior is that persistency contributes to extra returns, but also to their long-term variance.

Once applied to the German bond market, taking as an alternative the OIS, the model indicates that the risk premium on green German bonds changes sign, that the optimal policy consists in taking a position in them - which depends on the current, actual realizations of the shocks - and getting informed about their returns if fees are low, while continuing to invest, in a different proportion, that depends on the current estimates of the shocks, otherwise. We quantify the welfare effect of such policies. The certainty equivalent of wealth invested like that grows at the rate 11.0043, gross of information fees, while it grows at 0.67 without information. It would grow only at .008 with no diversification.

Inserting explicit ESG preferences, as in most of the reference literature, would increase the optimal exposure to ESG assets, as if we increased diversification benefits. The same is expected to happen with a greater risk aversion.

All in all, the model and its application show that the possibility to split the effect of the ESG factor versus the market one on asset returns is important: investors should abstain from entering the ESG market based not only on returns' properties, but also on their costs. Lowering information costs is certainly beneficial, and consistent with the current regulatory approach, which aims at increasing portfolio allocation in green assets and lowering the cost of capital of their issuers.

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CRediT Author Statement

Elisa Luciano: Conceptualization; Data curation; Formal analysis; Funding acquisition; Investigation; Methodology; Project administration; Resources; Software; Supervision; Validation; Visualization; Writing – original draft; Writing – review & editing.

Antonella Tolomeo: Methodology.

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