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Leverage and Interest Rates

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On the Heterogeneous Response of Leverage to Interest Rates*

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Abstract

According to the trade-off theory, optimal leverage falls in the level of the risk-free interest rate. We show that the opposite result emerges in two connected units, where a Sponsor may help backing the other unit's debt. With lower interest rates, the Sponsor is more likely to display zero-leverage due to its own lower tax shield. This frees up cash flows to bail-out the Backed company when it is insolvent but profitable. The latter is thus able to increase its debt at an increasing spread, containing the reduction in its own tax shield despite the fall in the risk-free rate. Overall debt in both units may thus increase. Our results, which apply to both fund-LBO target and (multinational) groups, imply that we should observe a divergent response of debt and expected default costs to changes in the risk-free rate for different borrowers.

Keywords: capital structure, tax-bankruptcy trade-off, default, LBOs, multinationals, securitization, structured finance

JEL Classification: G32, H32, L32

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1. Introduction

In response to a reduction in the level of the risk-free rate, optimal firm debt falls according to the trade-off theory of optimal capital structure- holding fixed investment opportunities. This paper shows that this result holds for the Stand-Alone unit in Leland (2007). The opposite result instead obtains when we consider a complex organization where one unit, that we will call the Sponsor, may back the other unit's debt in given contingencies. Such complex organizations, ranging from the fund-Leveraged Buyout target in private equity deals, to the parent-subsidiary structure in multinationals and the Sponsor-Special Purpose Vehicle in a securitization, increased their leverage in the decades of falling interest rates.

To understand such heterogeneous response, consider that tax savings of the Stand-Alone firm fall together with the risk-free rate, prompting an increase in its default costs. Indeed, the lower tax shield leads to lower after-tax profits available to repay debt, thereby increasing default probability. Therefore both tax savings reductions and default costs increases provide the Stand-Alone firms an incentive to reduce the face value of debt, when interest rates fall. Clearly, the tax shield reduction is the driving force behind both effects.

We find a similar, but more extreme, pattern in the bankruptcy-remote Sponsor of a complex organization consisting of two separate legal entities. Specifically, there is a cut-off level of the risk-free rate below which the Sponsor will optimally raise zero debt. On the contrary, a Stand-Alone unit has positive debt (Luciano and Nicodano, 2014). The Sponsor's incentive to become zero-leverage rests with the possibility of using its cash-flows to bailout its connected unit - should it become insolvent while profitable. In turn, such conditional support allows the Backed unit to raise more debt, thereby increasing the spread component of interest expenses. This will help contain the fall in its own tax shield. Hence, the standard result that holds in the Stand-Alone unit reverses in the Backed unit. This occurs to such an extent that the total debt of the complex organization may increase when interest rates fall.

To fully understand the logic, consider a case with equal parameters in connected units whose cash-flows are also perfectly correlated. Then the Sponsor is optimally zero-leverage. In turn, the Backed company pays a much higher credit spread than the Stand-Alone unit, due to not only its higher optimal leverage (see Luciano and Nicodano (2014)) but also the lower recovery rate of its lenders. Indeed, the bailout transfer from the Sponsor in positive cash-flow states reduces lenders' recovery upon default in the Backed company relative to the Stand-Alone case, thereby increasing their required spread - the more so the higher is its debt. At the initial debt level, when the risk-free rate falls, the proportional change in the tax shield is thus lower in the Backed unit than in the Stand Alone unit. Moreover, the higher losses upon default expected by lenders of the Backed unit translate in higher spread and tax shield.

This paper contributes to capital structure theory. First, our results for the Stand-Alone company in Leland (2007) remind the ones of dynamic trade-off models (Fischer, Heinkel, and Zechner (1989), Ju and Ou-Yang (2006) and Duarte, Öztekin, and Saporito (2022)). Optimal debt is increasing in the level of interest rates, due to a rising tax benefit of debt accompanied by a reduction in default costs (We also prove that the market value of the Stand-Alone company falls when the risk-free interest rate increases, despite the increase in its tax shield and the reduction in its default costs. This is a valuation effect, since the future payoff of both its debt and its equity are discounted at a higher rate. This effect is especially strong in our model where debt is discount, only.

Second, we improve our understanding of capital structure in complex organizations, where optimal leverage trades off the tax benefits of debt with default costs in both units.[2] In Regis and Nicodano (2019) a Sponsor is more likely to raise debt when its trade-off ratio, that is the ratio between the tax and the bankruptcy cost rates, is higher than the one in the Backed unit - at a given level of the risk-free rate. There is indeed an incentive to leverage the Sponsor relative to the opposite incentive to increase support. Starting from a situation with debt outstanding in both units, we observe significant debt transfers to the Backed unit as the risk-free rate falls. This occurs because a lower interest rate shifts the incentive balance in favour of providing support towards the latter. Such debt and default-risk transfer from the Sponsor to the Backed unit is stronger when the cash-flows of the two units are positively correlated, since the tax motive for pooling cash-flows is stronger

[2] Bianco and Nicodano (2006) focus on asymmetric information with lenders rather than trade-off parameters, as we do. Models in Bolton and Oehmke (2018), Cestone and Fumagalli (2005), Segura (2018), Loranth Segura and Zeng (2022) address other aspects of complex organizations.

than the diversification motive. The total optimal face value of debt of the two units will be higher at the lower level of the risk-free rate than at the initial one, provided that cash flow correlation, and hence the tax savings motive, is sufficiently high. Thus, we show that the traditional insight holds in the special case of Stand-Alone firms but not in the case of the Backed units. More generally, we uncover the heterogeneous response of optimal debt to changes in the level of interest rates both across complex and Stand-Alone organizations and within the Complex organization.

This new insight may be relevant beyond the boundaries of capital structure theory. The relationship between debt and interest rates is a building block of much macro banking literature investigating the effects of monetary policy shocks. Observing that traditional corporate finance models cannot explain a leverage increase when interest rates fall, Fahri and Tirole (2009) argue that lower interest rates increase (aggregate) loan demand when they generate the expectation of further accommodative monetary policy or bailouts. They also show that highly leveraged companies benefit most from such policies. In our corporate finance model, the risk-free rate is (expected to be) invariant for the relevant company horizon, once it has fallen. What leads to higher debt demand is the tax-shield preservation due to conditional support increasing the credit spread associated with lenders' losses-upon-default. Our focus on the tax-bankruptcy trade-off also distinguishes our mechanism from the one in Bräuning and Wang (2020), where a lower level of interest rates makes a larger service of debt sustainable with the same income flow. Other models point to lenders' incentives to take on more risk when the level of interest rates falls. For instance, the increase in fund supply to low-rated firms is explained by the search for higher yield by investors (Martinez Miera and Repullo, 2017). Our model provides a demand explanation for the debt increase, assuming an infinitely elastic supply at fair prices.

The new insight of this paper sheds light on leverage responses to interest rates in Leveraged Buyouts (LBOs) deals. Our model applies well to LBOs, as private equity managers consider the trade-off between the tax shield and the bankruptcy costs when deciding the capital structure of their deals and cite interest rates as one of the most important determinants of leverage (see Gompers et al., 2016). Moreover, private equity funds inject capital in the distressed companies they back (Bernstein et al., 2019; Hotchkiss et al., 2021 and Haque et al., 2023). The Backed unit of our model displays a high tax shield, consistent with that observed in LBO targets (see Kaplan, 1989, Acharya et al., 2013, Renneboog et al., 2007) and is also characterized by higher leverage and default rates than other companies, in line with existing evidence (Hotchkiss et al., 2021). Our results indicate that the optimal debt in the LBO target bears an inverse relationship with the level of interest rates, in contrast to Stand-Alone companies. This insight provides a theoretical backing for the opposite relationship between leverage and interest rates observed in public firms on the one hand and in LBO targets on the other (Axelson et al., 2013). Our results also indicate that the Sponsor remains zero-leverage after the reduction in interest rates when it had zero-leverage at the initial, higher risk-free rate. This situation occurs when the Backed company has trade-off parameters that are at least as favourable to debt as those of the Sponsor, so that it pays the to provide maximum cash-flows injections to its Backed Target.

Our insight also helps understand the reason why other highly leveraged units with a bankruptcy-remote Sponsor increased leverage when interest rates were falling, as in the case of securitization arrangements (see for instance Powell (2019) and Rosengren (2019)). Securitization sponsors may indeed support their SPVs (Gorton and Souleles, 2007) and are bankruptcy remote (Ayotte and Gaon (2011)). Similarly, the implications of our model may shed light on multinationals' capital structure, since parent companies provide contingent guarantees to their subsidiaries (Bodie and Merton, 1992) while being protected by limited liability.[3]

The paper unfolds as follows. Section 2 presents the model for Stand-Alone units and for the complex organization. In Sections 3 and 4 we provide insights into leverage, default probability and lenders' losses-upon-default adjustments following a change in interest rates. While Section 3 studies the case of a zero-leverage Sponsor, Section 4 shifts attention to a leveraged Sponsor. Section 5 concludes.

[3]Moreover, empirical studies support the trade-off theory for both multinationals and business groups (see Hanlon and Heitzman (2022) and Brok (2022)).

2. The Model

This section describes our set-up, that follows Leland (2007) in modeling Stand-Alone companies.

At time 0, a controlling entity owns two units, $i = S, B$. Each unit has a random exogenous operating cash flow X_i that is realized at time T. We denote with $G(\cdot)$, the cumulative distribution function and with $f(\cdot)$ the density of X_i, identical for the two units; $g(\cdot, \cdot)$ is the joint distribution of X_S and X_B and ρ their correlation. At time 0, the controlling entity selects how to finance the risky cash flows, either through a face value F_i zero-coupon debt with maturity T or equity. She does so to maximize the total arbitrage-free value (ν_{SB}) of equity, E_i, and debt, D_i in the two units:

$$\nu_{SB} = \max_{F_S, F_B} \sum_{i=S,B} \left(E_i + D_i\right). \tag{1}$$

Each unit pays a flat proportional income tax at an effective rate $0 < \tau_i < 1$ and suffers proportional default costs $0 < \alpha_i < 1$.[4] Interest on debts are entirely deductible from taxable income. The tax advantage for debt generates a trade-off. On the one hand, increasing leverage generates tax benefits, while on the other it increases expected default costs because – everything else being equal – higher leverage increases default likelihood.

At time T, cash flows are realized and distributed to claim-holders. First, corporate income taxes are paid. Then, debt obligations are fulfilled, if possible. When a unit cannot meet its debt obligations, its income, net of taxes and the dead-weight costs of default, is distributed to the lenders. Once debt is fully repaid, equity-holders receive the net residual income.

Maximizing the value of debt and equity for the owner is equivalent to minimizing the expected cash flows not to be redistributed to claim-holders, namely expected taxes (T_i) and default costs (C_i):

$$\nu_{SB} = \min_{F_S, F_B} \sum_{i=S,B} T_i + C_i. \tag{2}$$

[4] No tax credits or carry-forwards are permitted.

The expected tax burden of each unit is proportional to the expected operational cash flow X_i, net of the tax shield X_i^Z, defined as the interest deductions, which are equal to the difference between the nominal value of debt F_i, and its market value D_i: $X_i^Z = F_i - D_i$. Default costs are proportional to income.

Units can be owned as two separate unconnected units, or they can be connected through a conditional bailout guarantee.

2.1. The Stand-Alone Companies

It is useful to start with the benchmark case of unconnected, Stand-Alone units. The expected tax burden in each Stand-Alone (SA) unit is equal to:

$$T_{SA}^i(F_i) = \tau_i \phi \mathbb{E}[(X_i - X_i^Z)^+], \tag{3}$$

where the expectation is computed under the risk neutral probability and $\phi = \frac{1}{(1+r)^T}$ is the discount factor for the time-T horizon at which the cash flows are realized. The superscripts and subscripts, i, indicate whether the Stand-Alone unit is endowed with the Sponsor ($i = S$) or Backed unit ($i = B$) parameters.

Each Stand-Alone unit defaults when its realized net cash flow is lower than the face value of debt; in other words, default occurs when cash flows are lower than the default threshold, $X_i^d = F_i + \frac{\tau_i}{1-\tau_i} D_i$. Expected default costs, that are a dead-weight loss, are equal to:

$$C_{SA}^i(F_i) = \alpha_i \phi \mathbb{E}\left[X_i 1_{\{0<X_i<X_i^d\}}\right]. \tag{4}$$

They are proportional to the default cost parameter, α_i, and they increase in realized cash flows, when the unit goes bankrupt. A rise in the nominal value of debt, F_i, increases the default threshold, X_i^d, thereby increasing the expected default costs.

When units are owned separately, the value of the objective function (2) is simply the sum of the values of the taxes and default costs in each unconnected unit. Notice that the value of each unit can be written as:

$$V_i(F_i^*(\phi)) = V_i(0;\phi) + TS_i(F_i^*(\phi);\phi) - C_i(F_i^*(\phi);\phi),$$

where $V_i(0;\phi)$ is the value of the unlevered firm, and $TS_i(F_i;\phi) = T^i_{SA}(0;\phi) - T^i_{SA}(F_i;\phi)$ is the present value of the tax savings from leverage, equal to the difference between the taxes paid by an unleveraged firm and a firm which issues debt F_i. It is possible to show that the tax shield of a Stand-Alone unit is a convex function of F_i. Increasing the nominal value of debt increases the tax shield, thereby reducing the tax burden because the market value of debt, D_i, increases with F_i at a decreasing rate (reflecting a higher risk). On the contrary, the default threshold X_i^d is concave in the face value of debt, F_i. Luciano and Nicodano (2014) prove that a Stand-Alone unit has positive optimal debt if the sum of tax burden and default costs is convex in the face value of debt. Nicodano and Regis (2019) show that it raises positive debt even if the risk-free rate is zero, because of the endogenous spread.

We can now analyze the optimal response to a reduction in the risk-free rate, that increases the discount factor, ϕ. Throughout the analysis, while we let ϕ vary, we keep the cash flow distribution fixed. Indeed, we assume that valuation is always performed under the risk neutral probability and let the expected present value of cash flows vary with ϕ. At a given debt level F_i, the interest rate influences (3) and (4) through two channels. First, they are both discounted expected values, and depend on ϕ directly. Second, they depend on the thresholds X_i^Z and X_i^d, which are influenced by the market value of debt D_i, which in turn depends on ϕ. Hence, when the level of interest rate changes, expected taxes and default costs change. The following lemma describes how they change.

Lemma 1. *In a Stand-Alone company, the expected values of both taxes and default costs increase with the discount factor ϕ (i.e. decrease with the interest rate), for a fixed face value of debt.*

Proof. See the Appendix. ■

The first part of the Lemma concerning taxes is straightforward, as the tax shield, X^Z, falls together with the risk-free interest rate. The result concerning the increase in default costs is instead not obvious. Indeed, the no-default threshold increases as the interest rate lowers. It stems from a reduction in net after-tax income available to repay debt, due to the reduction in the tax shield, which increases the probability of default. It turns out that, for reasonable values of the tax rate ($\tau < \frac{1}{2}$), the tax shield decreases faster, as the risk-free rate falls, than the increase in the no-default threshold. The loss of benefits from leverage is therefore first order relative to the increase in default costs.

It is possible to prove also the following:

Lemma 2. *The spread y, i.e. the part of the rate of return on debt due to its riskiness, $y = \left(\frac{F}{D}\right)^{\frac{1}{T}} - \left(\frac{1}{\phi}\right)^{\frac{1}{T}}$, increases in the interest rate (decreases in ϕ) for fixed F.*

Proof. See the Appendix. ■

The previous lemma clarifies that the incentives to leverage due to interest tax deductions increase with the interest rate.

The Proposition below, finally, explores the associated effects on the market value of the Stand-Alone company. Due to the increase in the default probability, the present value of debt falls. However, the discount factor effect, which pushes debt value up as the interest rate falls, prevails. For fixed face value, the market value of debt increases as interest rates fall. Moreover, also the market value of equity increase in the discount factor, leading to the increase in value for any F_i and at the optimum, following a drop in interest rates:

Proposition 1. *In a Stand-Alone company, the market values of both debt and equity increase with the discount factor ϕ, for fixed face value of debt, F_i. As a consequence, the market value of the firm increases with ϕ (decreases with the interest rate) for any face value of debt (and, thus, at the optimum).*

Proof. See the Appendix. ■

The results above highlight that, for stand alone companies, the incentives to raise debt in our classical tax-bankruptcy trade-off setting lower as the risk-less interest rate lowers.

2.2. The Unit Backed by a Sponsor

We allow the owner to set up units, which are connected through a conditional guarantee, as in Luciano and Nicodano (2014). We let the Sponsor transfer part of its net profits to an insolvent, but profitable, Backed unit if such transfer is able to prevent its insolvency. Formally, the Sponsor transfers an amount $F_B - X_B^n$ to the Backed unit, provided its net profits are large enough ($X_S^n - F_S \geq F_B - X_B^n$) to repay its claimholders first, so that both units become solvent. Importantly, the Sponsor enjoys limited liability relative to the debt of its Backed company.

The presence of the bailout modifies the tax/bankruptcy trade-off, for fixed capital structure (F_S,F_B), as follows. The transfer never increases the default costs in the Backed unit, C_B, and decreases them as soon as the $F_B > 0, F_S < +\infty$. It increases its tax burden, because debt becomes more valuable and less risky interests can be deducted. On the contrary, the default costs of the Sponsor and its tax burden are unaffected.

Indeed, the value of the guarantee, i.e. the reduction in expected default costs (Γ) due to the rescue mechanism, is equal to:

$$\Gamma = \alpha_B \phi \mathbb{E}\left[X_B 1_{\{0<X_B<X_B^d, X_S \geq h(X_B)\}}\right] \geq 0, \quad (5)$$

where the indicator function $1_{\{\cdot\}}$ defines the event of a rescue, which occurs when the Backed unit would default without transfers (first term) and the Sponsor cash flows are sufficient for rescue (second term). As discussed in Luciano and Nicodano (2014), this mechanism is thus always value enhancing if the sponsor commits to honouring its guarantee. The rescue by the Sponsor is likelier the smaller the Sponsor debt, F_S, as the function $h(X_B)$, defined in the Appendix, clearly shows. On the contrary, increasing the Sponsor debt reduces support to the Backed unit through the bailout guarantee. Thus, it is immediate to identify a trade-off between tax shield maximization in the sponsor and default cost savings in the backed unit. Also, the rescue mechanism allows the Backed unit to raise additional leverage and increase its tax shield. Indeed, it may be optimal to foregone the tax gains in the sponsor and exploit the guarantee maximally, by – contextually – increasing it in the backed unit, in which default costs are curbed by the rescue mechanism. The trade-off described above may lead, differently from the stand-alone case, to higher incentives to leverage even when the risk-free rate is decreasing.

We can prove the following proposition:

Proposition 2. *There exists an interest rate level $\bar{r}(\tau_S, \tau_B, \alpha_B, G, g)$ below which the Sponsor is zero-leverage.*

Proof. See Appendix. ∎

Table 1: Base-case parameters		
Symbol	**Parameter**	**Value**
τ	Tax Rate	20%
α	Default Costs rate	23%
r	Interest rate	5%
ϕ	Discount Factor	0.78
$X(0)$	Cash flow present value	100
V_U	Unleveraged firm value	80.05
T	Time Horizon	5
σ	Cash flow volatility	$22 * \sqrt{5}$
ρ	Cash flow correlation	0.2

Table 1: This table displays the base-case parameters, following Leland (2007).

The proposition shows that there is a low enough level of the interest rate such that the Sponsor becomes zero-leverage. This happens because, as the risk-free rate lowers, exploiting the guarantee becomes more valuable. While the sponsor becomes zero-leverage it is possible to raise additional debt in the backed unit, increasing its tax shield, while containing default costs thanks to the support from the parent. As a consequence, the leverage of the backed unit and of the organization as a whole may increase when the interest rate falls, as we show numerically in the following section.

3. Optimal leverage and credit risk sensitivity to the risk-free rate

In this section, we numerically analyse the changes in the optimal capital structure following a drop in interest rates. We compare Stand-Alone units with connected units displaying the same parameters. We analyze the changes in the endogenous default probability, spread and loss given default as we vary the level of the risk-free rate.

Table 1 displays the base-case calibration parameters, which we borrow from Leland (2007) and refer to a typical BBB company.

We set the tax rate and the proportional bankruptcy costs to $\tau_i = 20\%$ and $\alpha_i = 23\%$, $i = S, B$ respectively, and then proceed to parametric changes. We fix the marginal distributions of cash flows at maturity (5 years) to a normal distribution with mean $100 * (1.05)^5$ and Standard Deviation $\sigma = 22 * \sqrt{5}$ and we maintain a joint normality assumption for connected

Table 2: Optimal Stand-Alone		
Parameter	Interest Rate	
	5%	1%
Principal (F^*)	57.1	34.3
Value (V^*)	81.47	97.45
Debt (D^*)	42.21	31.84
Equity (E^*)	39.26	65.61
Tax Shield (X_Z^*)	14.89	2.46
No-default threshold (X_d^*)	67.65	42.26
UnLeveraged Firm Value (V_U)	80.05	97.20
Value of Leverage ($V^* - V_U$)	1.42	0.25
Debt Yield(y^*) (spread,s^*)	6.23% (1.23%)	1.50% (0.50%)
Taxes (T^*)	17.70	23.84
Tax Savings(TS^*)	2.32	0.47
Default Costs (C^*)	0.89	0.22
5- Year Default Probability (DP^*)	11.14%	4.13%
Loss Given Default (LGD^*)	28.96	20.16

Table 2: This table displays the optimal figures of a Stand-Alone unit with parameters as in Table 1, for two levels of interest rates, 5% and 1%. The yield is computed as $(F^*/D^*)^{\frac{1}{5}} - 1$, the loss given default as $\frac{F^* - D^* \frac{1}{\phi}}{DP^*}$. Yields and spreads are annualized, while the default probability is over the 5-year horizon. Cash flow distribution is fixed: $X \sim N(127.63, 49.19)$.

units, letting correlation vary. We compare changes in the capital structure of the Stand-Alone and of the connected units, when the risk-free rate falls from 5% to 1%.

3.1. The Stand-Alone Company

Our first observation is that, in the Stand-Alone case, the decrease in interest rates reduces the incentives towards leverage (see Table 2).

Indeed, following the interest rate drop, the optimal face value of debt for a Stand-Alone company decreases by almost 40%, from 57.1 to 34.3. Also the total market value of debt drops by almost 25%, from 42.2 to 31.8. While the value of a hypothetical zero-leverage company increases sharply (from 80.05 to 97.20) when the interest rate drops, due to the higher discount factor, the value of leverage, i.e. the difference between the optimally leveraged and the zero-leverage firm value, drops dramatically, from 1.42 to 0.25. The reduction in the value of leverage is explained by the lower relevance of the tax shield, which falls from 14.89 to 2.46. As a consequence, the tax savings from leverage fall, from 2.32 to 0.47. Symmetrically, taxes increase from 17.70 to 23.84.

Since the optimal debt is smaller, the default threshold shrinks, albeit less sharply than the tax shield, from 67.65 to 42.26. Default costs reduce accordingly, from 0.89 to 0.21, reflecting the lower risk for lenders.[5] Interestingly, default costs drop not only in absolute terms, but also relative to both the optimal value and the present value of expected cash flow. The lower riskiness of the optimal Stand-Alone company as interest rates fall is mirrored in a much smaller default probability. In the base-case, it is 11.14% at the 5-year horizon[6]. This is largely due to leverage, as the probability of default for the zero-leverage firm is 0.47%, only. When interest rate drops to 1%, leverage decreases and the default probability decreases accordingly, down to 4.13%. The endogenous (annualized) spread[7] reflects such change, decreasing from 1.23% to 0.50%.

Lenders' losses upon default instead reduce in absolute terms as the interest rate decreases, from 28.96 to 20.16. However, this is due to the reduction in the optimal debt principal value. Indeed, as a percentage of the principal, the loss given default worsens, because only about 41% of the principal is recovered by lenders in default when the interest rate is 1% vs. 50% when it is 5%.[8] Summarizing, while defaults are less frequent, and this drives the drop in expected default costs, they have more severe consequences, as a proportion of outstanding debt. This happens because the default threshold is closer to zero, and defaults occur in most cases when after-tax profits are negative or very small, leading to zero or little recovery for debt-holders.

The decrease in optimal leverage when interest rate decreases is a consistent pattern across parametric changes. It occurs when cash flow volatility is higher (44%) or lower (15%), when the tax rate is higher (24%) or lower (16%) and when the proportional default cost parameter is higher (26%) or lower (20%), consistent with the insight deriving from Proposition 1. The decrease in leverage due to a fall in interest rates is milder the higher

[5] In a traditional trade-off model with exogenous bankruptcy probability, we would obtain an even sharper reduction in optimal leverage, with a falling tax benefit of debt together with fixed bankruptcy probability.

[6] The default probability is defined as the probability that the firm is not able to repay its debtholders after $T = 5$ years, when the cash flows are realized, i.e. $DP = \int_{-\infty}^{X^d} f(x)dx$.

[7] The spread s is the difference between the yield, defined as $y = (F/D)^{\frac{1}{5}}$ and the interest rate r.

[8] Such increase in losses upon default, which are defined as $LGD = \frac{F - D\frac{1}{\phi}}{DP}$, derives from the increase in the discount factor in the calculation of expected discounted losses (at the numerator) and by the decrease in the default probability (at the denominator).

the incentive toward leverage, i.e. the higher the volatility and the tax rate and the lower the default cost rate.

We can summarize these results as follows, assuming that everything else is unchanged including the distribution of future cash flows:

Observation 1. *In a Stand-Alone company, the optimal leverage falls when the risk-free interest rate decreases. Both expected default costs and default probability decrease, while losses upon default increase.*

3.2. A Zero-Leverage Sponsor: the Private Equity Case

In this section we turn to the case where a Sponsor supports the service of debt of a Backed company. We start by addressing the case where both are endowed with the base-case parameters presented in Table 1. Table 3 reports the optimal capital structure and relevant figures for different levels of correlation between cash flows between the connected units. However, we will focus our discussion on the case of a weak (0.2) positive correlation between unit cash flows. This is also the correlation maintained in Figure 1 and Figure 2, that display the implied changes in connected units vs. two equivalent Stand-Alone units when the risk-free rate varies in the interval [1%,5%].

When the risk-free rate is 5%, the Sponsor has optimal zero leverage as in Luciano and Nicodano (2014). On the contrary, the bankruptcy-remote Backed company has an optimal face value of debt which is almost four times that of the Stand-Alone (220 versus 57.1). Such polarized capital structure is typical of both Sponsor/SPV arrangements (Gorton and Souleles, 2006) and private equity fund-LBO target firm (Cohn et al., 2014). Thanks to zero leverage, the Sponsor maximizes the bailout support provided to the highly leveraged company, which is in turn able to maximally exploit its tax shield. The tax shield indeed reaches 103 in the subsidiary unit, up from 14.9 in the Stand-Alone company. As a consequence, tax savings from leverage in the Backed unit are far higher than in the Stand-Alone (14.62 vs. 2.32). Expected default costs increase as well, and are almost ten times larger than in a Stand-Alone, reaching 8.13 (vs. 0.89). Such "extreme" exploitation of the tax-bankruptcy trade-off is allowed for by the conditional guarantee provided by the Sponsor, that limits the rise in default costs relative to tax savings. Indeed, the Sponsor/Backed unit organization

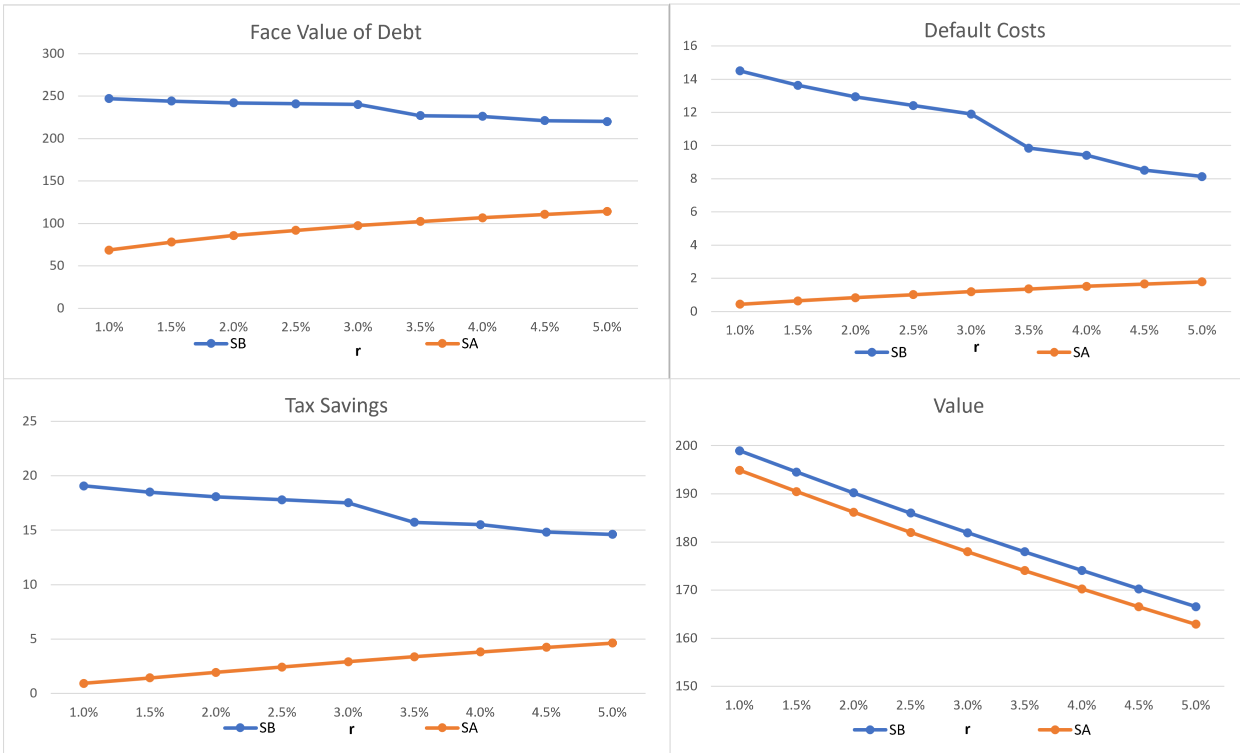

Figure 1: This figure contrasts the optimal face value of debt, default costs, taxes and total value of a company with its bankruptcy-remote Sponsor (in blue) and the equivalent Stand-Alone arrangements (in orange) for different interest rate levels, ranging from 1% to 5%. The parameters are collected in Table 1. The cash flows of the units are jointly normally distributed, with marginal distributions as in Table 1 and correlation parameter 0.2.

is more valuable than two equivalent Stand-Alone (166.59 vs. 162.94). As evident also from Figure 2, despite the Sponsor, the Backed unit is optimally a very risky entity. Indeed, its default threshold (i.e. the cash flow level below which default occurs) grows to a startling 242, up from 67.7 of the Stand-Alone case. Its default probability is much higher and the losses upon default are far larger than those of a Stand-Alone unit (47% vs. 11% and 148.98 vs. 28.96, respectively). Lenders' losses upon default are larger because the Sponsor supports the Backed company when the latter has positive cash-flows (and the Sponsor has enough funds). As a consequence, the endogenous spread, which reflects the credit risk compensation demanded by the lenders rises to 8.45%, up from 1.23% in the Stand-Alone. It is the large spread that leads to the high tax savings we just illustrated.

Let us now turn to changes in response to a drop in the risk-free rate. In the supported unit the face value of debt increases as the interest rate decreases, reaching 247 when the risk-free rate is 1%. For any interest rate level, it remains optimal for the Sponsor to have zero leverage. Indeed, while the incentive towards leverage in the Stand-Alone company decreases due to the lower interest rate, in the Sponsor/Backed unit organization the drop in interest rates results in a more extreme exploitation of the tax-bankruptcy trade-off. We now study how this result derives from endogenous debt pricing and costly default.

When debt increases, several things happen. First, tax savings increase. Second, the default probability increases, driving up the spread. As a consequence, the dead-weight costs of default increase as well. However, in the Sponsor/Backed unit arrangement, they are mitigated by the bailout transfer occurring when the Backed unit has positive cash flows. More precisely, the tax savings increase as the interest rate falls, from 14.6 to 19.1 and the expected default costs (concentrated in the Backed unit) almost double, rising from 8.1 to 14.5. The ratio of expected default costs to total group value rises from 4.88% to 7.29%, outpacing the growth of expected tax savings to group value, which instead rises from 8.78% to 9.59%. While the value of leverage, i.e. the difference between the optimal and the zero-leverage value, falls from 6.49 to 4.54, the Sponsor/Backed unit remain 2.3% more valuable than the Stand-Alone (198.96 vs. 194.42). While debt market value – concentrated in the Backed company – increases by 14%, from 117.06 to 133.43, the equity value (concentrated in the Sponsor) increases much more, by almost 33%. This is why the market value of leverage does not increase. Such increase in equity value, due to the discounting effect of lower interest rates, provides the capital buffer needed to enhance the support provision to the Backed unit.

The riskiness of the Backed unit increases, as the risk-free rate decreases. Indeed, as portrayed in Figure 2, the default probability increases as the rate decreases, hitting 63.3% when the risk-free interest rate is 1%, up from 47.38% when $r = 5\%$. The spread of the Backed unit increases accordingly, topping 12.11%, as does the loss given default, from 148.98 to 168.68 (or from 67.72% to 68.29% in percentage terms). Again, the Backed unit behaves very differently relative to the Stand-Alone, whose default probability and spread decrease when the interest rate decreases. On the contrary, the zero-leverage Sponsor is insolvent very rarely in all interest rate scenarios. This happens only when both its cash flow realization is negative and the Backed unit payout is not large enough to cover such losses. This is why the joint default probability is unaltered when the interest rate changes.

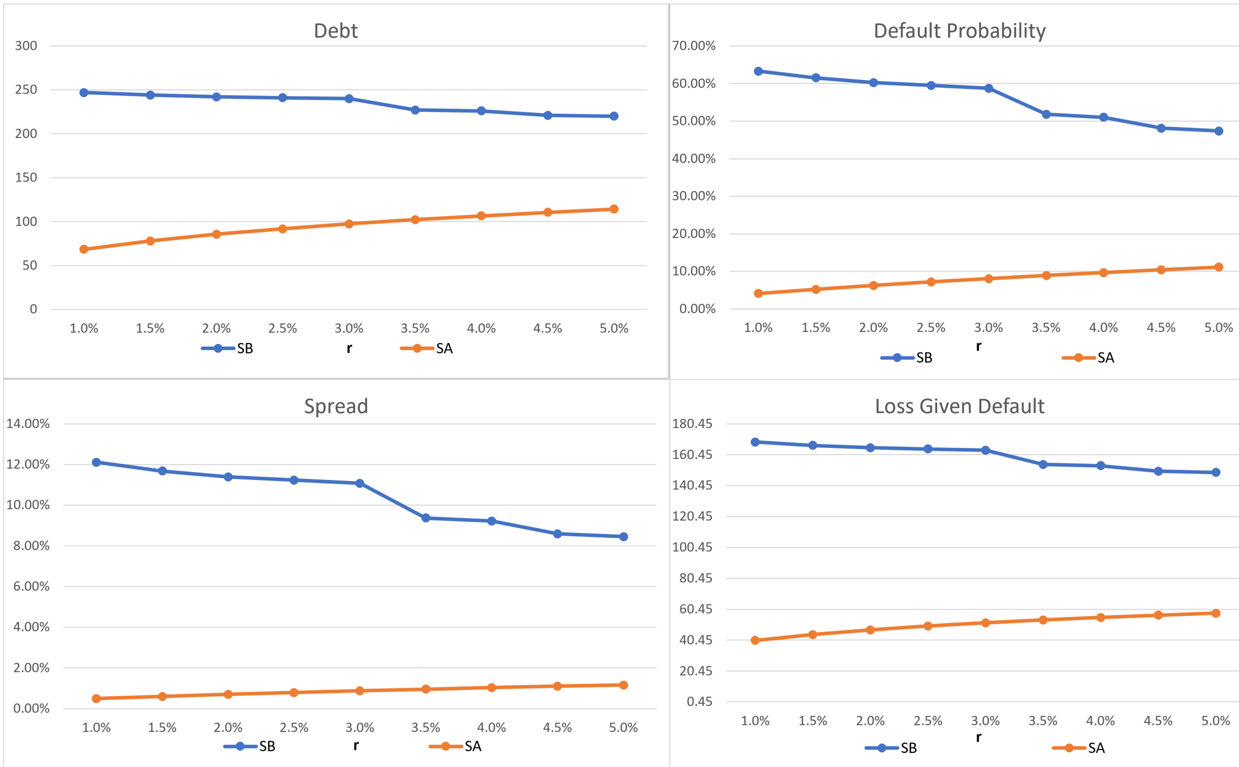

Figure 2: This figure contrasts the optimal face value of debt, 5-year default probability, annualized credit spread and loss given default of a company Backed by a bankruptcy-remote Sponsor (in blue) and the equivalent Stand-Alone (in orange) for different interest rate levels, ranging from 1% to 5%. The parameters are collected in Table 1. The cash flows of the units are jointly normally distributed, with marginal distributions as in Table 1 and correlation parameter 0.2.

Figure 1 and Figure 2 show that the patterns we just discussed hold uniformly as the level of the risk free rate falls from 5% to 1%. Furthermore, the above results qualitatively hold true for different correlation levels, as reported in Table 3.

In particular, higher cash flow correlation makes support more valuable because it allows for higher tax savings. To obtain such tax savings, the Sponsor has to be able to provide funds when the supported unit has positive cash flows. Both the optimal face value of debt of the Backed unit and its riskiness, as captured by the spread, increase with correlation and top 257 and 12.99%, respectively, when cash flow correlation is equal to 0.8 and the interest rate is 1%.

The statement below summarizes these patterns, assuming that only the risk-free rate varies:

Observation 2. *In a company Backed by a zero-leverage Sponsor, the optimal debt increases*

Table 3: Optimal Value and Debt: Sponsor/Backed Unit						
	Correlation					
	-0.8		0.2		0.8	
Parameter	Interest Rate		Interest Rate		Interest Rate	
	5%	1%	5%	1%	5%	1%
Face Value of Debt	183 (0;183)	201 (0;201)	220 (0;220)	247 (0;247)	227 (0;227)	257 (0;257)
Market Debt Value	133.58 (0;133.58)	153.38 (0;153.38)	117.06 (0;117.06)	133.43 (0;133.43)	115.53 (0;115.53)	133.55 (0;133.55)
Equity Value	32.74 (32.74;0)	42.88 (42.88;0)	49.52 (49.52;0)	65.53 (65.53;0)	51.84 (51.84;0)	66.70 (66.70;0)
Total Value	166.32 (32.74;133.58)	196.26 (42.88;153.38)	166.59 (49.52;117.06)	198.96 (65.53;133.43)	167.36 (51.83; 115.53)	200.24 (66.70;133.54)
Value of Leverage	6.23	1.86	6.50	4.56	7.27	5.86
Tax Savings	7.57 (0; 7.57)	8.87 (0; 8.87)	14.62 (0; 14.62)	19.07 (0; 19.07)	15.50 (0; 15.50)	20.16 (0; 20.16)
Taxes	32.45 (20.01;12.44)	39.73 (24.30;15.43)	25.40 (20.01;5.39)	29.53 (24.30;5.23)	24.52 (20.01;4.51)	28.44 (24.30;4.14)
Default Costs	1.91 (0;1.91)	7.29 (0;7.29)	8.13 (0;8.13)	14.50 (0;14.50)	8.24 (0;8.24)	14.29 (0;14.29)
Yield Sponsor (Spread)	N/A (N/A)	N/A (N/A)	N/A (N/A)	N/A (N/A)	N/A (N/A)	N/A (N/A)
Yield Backed Unit (Spread)	6.50% (1.50%)	5.56% (4.56%)	13.45% (8.45%)	13.11% (12.11%)	14.46% (9.46%)	13.99% (12.99%)
Default Probability Sponsor	0%	0%	0%	0%	0%	0%
Default Probability Backed Unit	11.05%	30.76%	47.38%	63.30%	50.43%	64.73%
Joint Default Probability	0.01%	0.22%	0.47%	0.47%	0.47%	0.47%
Loss Given Default Sponsor	0	0	0	0	0	0
Loss Given Default Backed Unit	113.28	129.38	148.98	168.68	157.77	180.21

Table 3: This table displays the optimal figures of a Backed unit with its bankruptcy remote Sponsor when both units displaying the parameters in Table 1, for two levels of interest rates, 5% and 1%. Cash flows are jointly normally distributed, with marginal distributions as in Table 1 and correlation parameter ranging from -0.8 to 0.8. Yields and spreads are annualized, the default probabilities are the probabilities that debtholders are not repaid in full when cash flows are realized at $T = 5$ years. Sponsor and Backed Unit figures are reported in brackets, respectively.

when the risk-free interest rate decreases. Both the spread and tax savings, along with default probabilities and losses upon default increase. These changes are larger the higher is the cash-flow correlation between the Sponsor and its Backed unit.

These findings are broadly consistent with the divergent response, by public companies and comparable LBO targets, to lower interest rates which has been observed. Axelson et al. (2013) find that the ratio of debt to EBITDA is higher in LBO targets but not in matched public companies when interest rates fall. The spreads they find depend on the type of debt. The median is equal to 262bp and 937bp for senior and junior bank loans respectively, reaching up to 916bp and 1048bp for senior and subordinated bonds respectively. The spread implied by our model, when 1% (5%) is the level of interest rates, is equal to 441bp (130bp) when cash-flow correlation between the fund and the LBO target is -0.8, reaching 1130bp (762bp) when cash-flow correlation is 0.2. Our numerical exercise also shows that these spreads allow to reduce the tax burden of the LBO target to up to one fifth of the taxes paid by a similar zero-leverage company.

These results can also provide a rationale for the disproportionate increase in lending through securitization vehicles such as CLOs as interest rates were falling in the first two decades of this century (Powell, 2019). Rosengren (2019) raises concerns of potential financial instability observing the rise – from below 4 to above 5 - in the multiple of average total debt to EBITDA for leveraged transactions priced at or above LIBOR + 225bp. Our results indicate that, when the leverage in structured finance increases in association with lower interest rates, so do both default probabilities and, to a lesser extent, lenders' losses upon default. However, our numerical exercise allows for neither increases in mean cash flows triggered by a lower risk-free rate nor for productivity increases relative to Stand-Alone companies that have been observed in some private equity research. Furthermore, our model reveals that the default rates of Stand-Alone and Backed units move in opposite directions. This suggests that an economy composed by both complex organizations and Stand-Alone units displays smoother changes in aggregate default costs as interest rates vary relative to one composed just of the latter.

4. A Leveraged Sponsor: the Parent-Subsidiary Case

The previous section shows that the optimal debt of a Backed company may increase in response to a decrease in interest rates, in contrast to the case of a Stand-Alone company. This pattern holds when the Sponsor is optimally zero-leverage before the interest rate reduction and remains optimally zero-leverage afterwards. In turn, this occurs when the tax/bankruptcy-cost ratio is equal across the Sponsor and Backed company, or when it is lower for the Sponsor than for the Backed company.

A change in the level of the risk-free interest rate may however prompt a restructuring of the Sponsor's capital structure, as well. This section analyzes one such case for robustness purposes. The Sponsor initially displays positive optimal leverage at the base-case interest rate level because of a higher tax-bankruptcy-cost ratio than its Backed company. This case is empirically relevant since we often see a leveraged parent in multinationals and other corporate groups, which also acts as Sponsor by backing its subsidiaries' debt (Bianco and Nicodano, 2006; Brok, 2022; Anantavrasilp et al., 2020). This case is theoretically interesting because it involves a trade-off between leveraging the Backed subsidiary, in order to exploit the Sponsor's limited liability, and leveraging the parent, which enjoys higher marginal tax rates and/or a lower proportional bankruptcy cost parameter.

Our numerical exercise shows that there are conditions under which the optimal response to a reduction in interest rates is the creation of a LBO-like structure with debt concentrated in the Backed company and a zero-leverage Sponsor. In other words, when interest rates drop, a parent may find it profitable to sell its subsidiary to a zero-leverage private equity fund that will lever up the LBO target, rather than entertaining itself this financial restructuring.

Let us consider the case when the tax rate of the Sponsor company ($\tau_S = 24\%$) exceeds the one of the Backed unit ($\tau_B = 16\%$). Since the incentive to raise debt also in the Sponsor is in general stronger the higher is cash flow volatility, we set $\sigma_S = \sigma_B = \sigma = 44 * \sqrt{5}$, similarly to Nicodano and Regis (2019). We first focus on the base-case correlation ($\rho = 0.2$). Figure 3 reports the optimal debt, market leverage, tax savings and default costs of the Sponsor/Backed unit organization and the equivalent two Stand-Alone units for interest rate levels ranging between 1% an 5%. Figure 4 displays the optimal debt, default probability, spread and loss given default of the Sponsor and the Backed unit, comparing them with their equivalent Stand-Alone values. Table 4 reports the numerical values of the optimal characteristics.

When $r = 5\%$, our choice of parameters leads to optimal positive leverage in both units. The total face value of optimal debt exceeds the one in two equivalent Stand-Alone units (199 vs. 169), and the Sponsor raises more debt than its Backed unit (124 vs. 75). This may seem counter-intuitive, because the parent has a much higher tax rate, and the debt tax shield is therefore more valuable in that unit. However, to preserve its ability to provide support, the Sponsor raises less debt than the equivalent Stand-Alone (75 vs. 103), while the Backed unit raises more (124 vs. 68). The Sponsor bears much lower default costs than the Stand-Alone peer as it receives the payout from the Backed unit, a mechanism that emerges in Anantavrasilp et al. (2020). In turn, to endow the Sponsor with a positive payoff, the subsidiary is not as leveraged as in the private equity case depicted in the previous Section 3.2, but still raises higher debt than the Stand-Alone. Overall, tax savings are higher relative to the Stand-Alone case (10.58 vs. 10.32), and default costs are mitigated (5.67 vs. 6.09), leading to higher total value (170.59 vs. 169.88). The Sponsor and the Backed unit appear to be similarly risky, with a (5-year ahead) default probability of around 35% and 37%, respectively, but while the Sponsor displays a lower (5.86% vs. 7.25%) (annualized) spread than its Stand-Alone equivalent, the opposite happens for the Backed unit (5.86%

vs. 4.74%).

As the interest rate decreases to 1%, we highlight two main effects. First, the total face value of debt raised by the organization has a non-monotone behaviour. It first decreases, decreasing from 5% to 3% and then increases relative to its 5% level. As Figure 4 captures, this is in sharp contrast with what happens to the two equivalent Stand-Alone units, whose combined optimal debt falls monotonically from 170 to 121 as interest rates drop. Second, debt is entirely raised in the Backed unit only when the interest rate is small enough. Indeed, when the interest rate falls below a certain level, the Sponsor optimally specializes in providing support as in the private equity-like structure described in Section 3.2. This occurs because a lower interest rate shifts the balance between two opposing incentives, increasing the Sponsor tax shield versus providing additional support to the Backed unit, toward the latter. The drop in interest rates reduces the incentive to leverage up the Sponsor, since it should bear higher default costs to reach the same tax savings level. The combination of connected units is however able to exploit the tax shield, while shielding the Sponsor from bankruptcy, by leveraging up the Backed unit. While the Sponsor becomes zero leverage, the Backed unit maximally exploits the tax shield, allowing the organization to become more valuable at the cost of increasing its riskiness.

The default costs of the organization increase sharply as the interest rate falls. In particular, those of the Backed unit are more than 11 times the default costs of an equivalent Stand-Alone unit (9.08 vs. 0.79) and the (5-year) default probability reaches 54% when the interest rate is 1%. Losses upon default deteriorate as well, rising to 175.42 from 88.42 (75.61% vs. 71.31% in percentage terms) relative to the 5% interest rate case. These changes affect the endogenous (yearly) spread, which rises from 5.86% to 12.20%. The default probabilities of the two units move in two opposite directions. In the Sponsor, as debt decreases the default probability drops when the interest rate moves from 5% to 1%. Below 3%, when the Sponsor becomes a zero-leverage entity, it defaults only when its realized cash flows are negative (9.73% probability), bearing no losses due to its limited liability.[9]

[9] As the risk-free rate approaches 3% from above, lenders' losses upon default get closer to 100% since the default threshold approaches zero, implying that there is hardly any recovery due to negligible or negative after-tax profits.

For our selected parameters, we observe a transformation to a zero-leverage structure composed of a Sponsor and a Backed unit for high enough correlation, leading us to the following observation, under the usual *coeteris paribus* assumption:

Observation 3. *When the risk-free rate falls, a parent-subsidiary structure with balanced debt may transform into a zero-leverage Sponsor and a highly leveraged company, even if the tax rate (proportional bankruptcy cost) of the parent exceeds (is lower than) the subsidiary's. This is more likely to happen the higher the correlation level.*

This observation is in line with the result in Proposition 2. In Table 4, we let cash flow correlation vary for two risk-free interest rate value: 5% and 1%. The table allows us to observe that the level of cash flow correlation has an impact on whether the transformation occurs. In particular, the zero-leverage Sponsor is optimal when correlation is high enough. This happens because the value of the conditional bailout guarantee is more effective in saving default costs the higher the correlation. With negative cash flow, the Sponsor would be able to provide support when the Backed company would not suffer from default, or viceversa the Sponsor would not be able to provide support when the Backed company faces positive default costs. Under both interest rate scenarios, as a consequence, total default costs increase with correlation.

When moving from a 5% to a 1% level of the risk-free rate, we find that total debt of the connected units increases for high enough correlation ($|\rho| > -0.2$). In those cases, the support mechanism is valuable enough to mitigate the increased marginal default cost associated with higher leverage. The same happens for default costs.

When the interest rate is 5% default costs in the connected units are always smaller than the costs in the two equivalent Stand-Alone units (6.09), as in the case presented in Figure 3, unless correlation is very high (0.8). On the contrary, when the interest rate is 1%, this happens only for negative correlation levels (-0.8 and -0.2), because the capital structure of the connected units is balanced. Indeed, when transformation to a Sponsor/Backed unit following a rate drop is privately optimal, the organization is not welfare-optimal, as Stand-Alone units display lower default costs. In the 5% interest rate scenario, instead, the privately optimal and socially optimal firm combination is the same for almost all correlation levels.

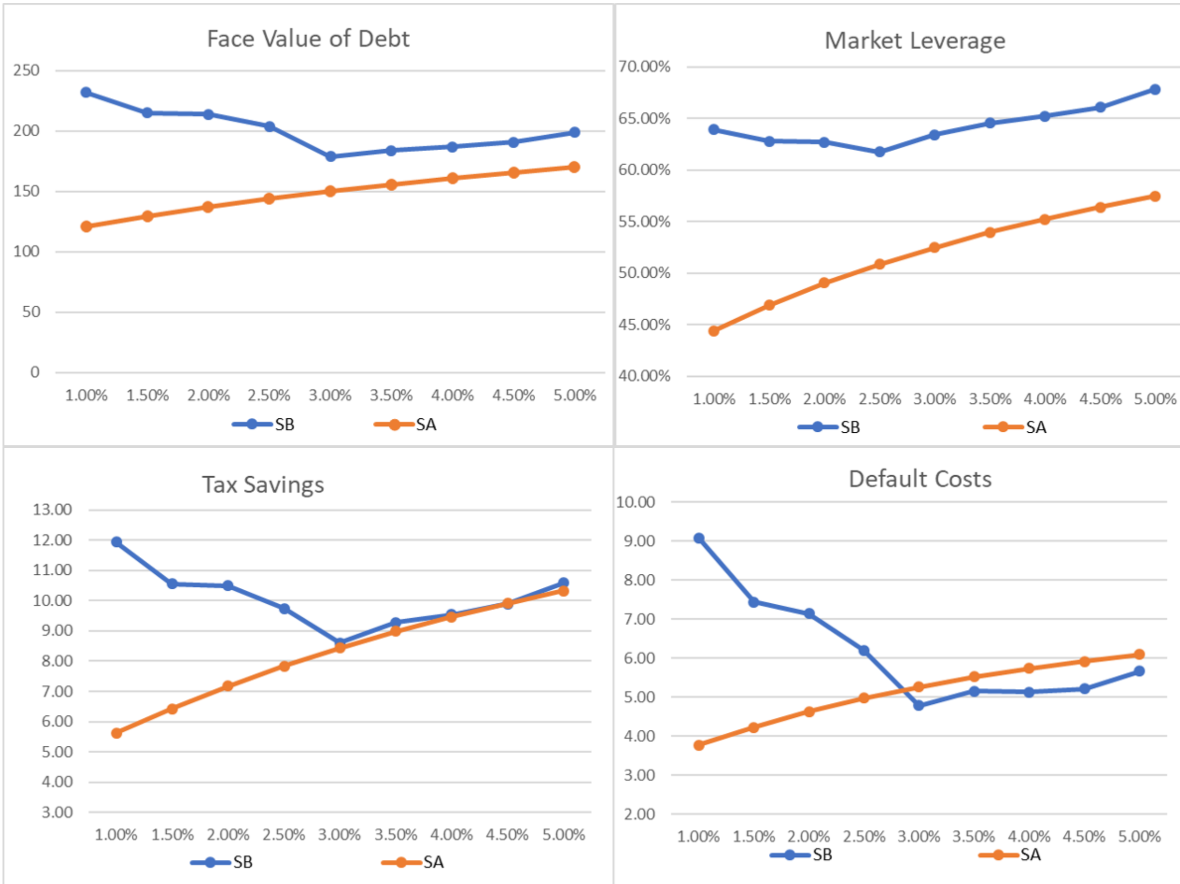

Figure 3: This figure portrays the optimal debt (face value and market value), default costs and tax savings of the Sponsor/Backed unit arrangement (in blue) when interest rate ranges from 1% to 5% and compares the figures with those of equivalent Stand-Alone units (orange). In the upper left panel, the red line depicts the optimal debt of the Sponsor.

Finally, default probabilities and spreads increase in the subsidiary for the correlation levels for which, following a drop in interest rates, the transformation occurs. The 5-year-ahead default probability tops an impressive 57.44% when $\rho = 0.8$, and the annualized spread consequently reaches 12.85%.

Two further effects are worth noticing. Firstly, the probability of a joint default decreases for all analyzed correlation levels when interest rate drops to 1%. This happens because of the limited liability of the Sponsor, which, being zero-leverage, defaults only when its cash flows are negative. Secondly losses upon default in the Backed unit (Sponsor) increase (decrease) for all correlation levels when interest rates drop from 5% to 1%.

Again, some model-based insights are broadly consistent with observation.

The first observation derives from a study of Small Medium Enterprises (SMEs), covering

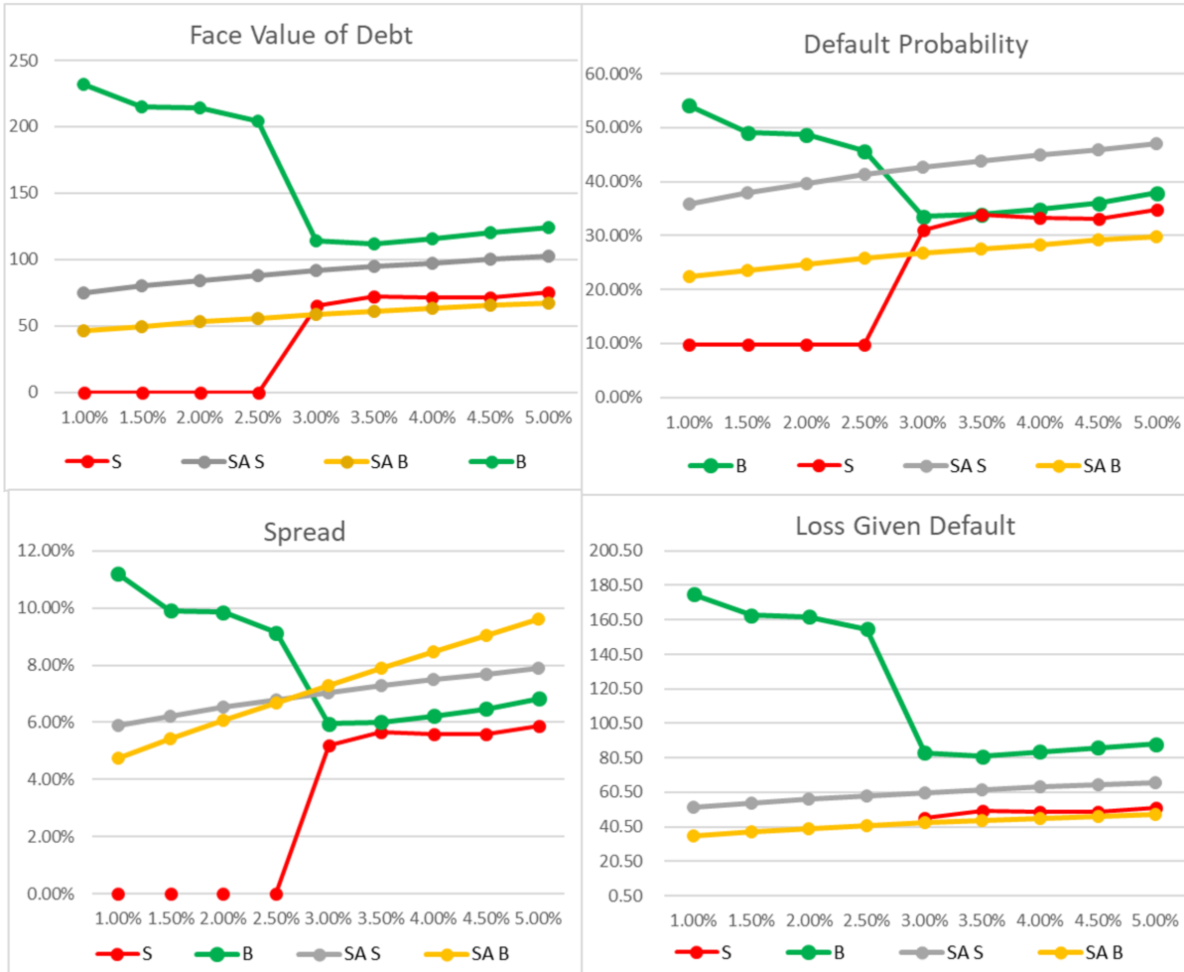

Figure 4: This figure portrays the debt (face value), default probabilities, spreads and loss upon defaults of the Sponsor (red), the Backed unit (green) and compares their figures with those of equivalent Stand-Alone units (in grey and yellow, respectively). Spreads are annualized, the default probabilities are the probabilities that debtholders are not repaid in full when the cash flows are realized at $T = 5$ years.

approximately 70 percent of total corporate and industrial loans made to U.S. firms from 2012 to 2019 (Caglio et. al (2021)). The impact of monetary policy in the full sample is driven by private companies, which are mostly Small and Medium Enterprises (SMEs). SMEs with higher leverage borrow more, at a higher cost, during monetary expansions. This result is driven by their higher demand for credit, while their lenders do not increase risk-taking. The higher cost of borrowing these leveraged SMEs pay relative to others is a result of their higher credit demand. On the contrary, highly leveraged public firms obtain less credit and pay higher spreads during monetary expansions. While Caglio et al.(2021) explain their findings with a relaxation of credit constraints for SMEs, we can read their evidence through the lenses of our model, provided Sponsors are more likely to be present in the SME than in the public company samples. Our results suggest that Backed units increase their borrowing relative to the Stand Alone firms when interest rates fall. Furthermore, they are also more leveraged at the initial interest rate level.

The second observation relates to the model prediction of increased LBO activity (that is, zero-leverage Sponsors) when interest rates fall, and a consequent increase in default risk. Lower interest rates have indeed accompanied higher LBO activity, although there may be other factors behind such association (Ivashina, 2022) beyond the demand-side factors we stress.

Table 4: Optimal Value and Debt: Transformation of a Parent Subsidiary Structure

Parameter	Correlation -0.8, Interest Rate 5%	-0.8, 1%	-0.2, 5%	-0.2, 1%	0, 5%	0, 1%	0.2, 5%	0.2, 1%	0.8, 5%	0.8, 1%
Face Value of Debt	187 (73;114)	171 (54;117)	186 (82;104)	158 (58;100)	190 (80;110)	192 (0;192)	199 (75;124)	232 (0;232)	239 (29;210)	251 (0;251)
Market Value of Debt	116.79 (43.83;72.96)	133.82 (41.21;92.61)	111.76 (47.92;63.85)	120.42 (43.79;76.62)	112.86 (47.04; 65.82)	124.74 (0;124.74)	115.70 (44.79;70.91)	130.40 (0;130.49)	116.64 (19.48; 97.16)	131.22 (0;131.22)
Equity Value	54.33 (28.58;25.75)	70.27 (41.62;28.65)	58.94 (29.14;29.80)	83.20 (46.33;36.87)	57.79 (30.24;27.56)	79.01 (71.05;7.96)	54.89 (32.16;22.72)	73.60 (70.12;3.48)	54.54 (49.39;5.14)	74.53 (74.53;0)
Total Value	171.12 (72.41;98.71)	204.09 (82.83;121.26)	170.70 (77.06;93.64)	203.62 (90.13;113.49)	170.66 (77.28;93.38)	203.75 (71.05;132.70)	170.59 (76.95;93.64)	204.10 (70.12;133.97)	171.18 (68.88;102.30)	205.75 (74.53;131.22)
Value of Leverage	5.47	2.94	5.05	2.47	5.01	2.60	4.96	2.98	5.53	4.66
Taxes	32.20 (20.06;12.14)	44.42 (27.57;16.85)	31.51 (19.28;12.23)	44.27 (27.29;16.98)	31.28 (19.46;11.82)	41.83 (30.17;11.66)	30.83 (19.89;10.94)	38.35 (30.17;8.17)	29.15 (23.25;5.90)	36.77 (30.17;6.59)
Tax Savings	9.22 (4.79;4.42)	5.86 (2.60;3.26)	9.90 (5.66;4.33)	6.02 (2.89;3.13)	10.13 (5.39;4.74)	8.45 (0;8.45)	10.58 (4.96;5.62)	11.94 (0;11.94)	12.26 (1.60;10.66)	13.52 (0;13.52)
Default Costs	3.78 (2.12;1.66)	3.10 (1.39;1.71)	4.86 (2.77;2.09)	3.58 (1.64;1.94)	5.15 (2.62;2.53)	6.01 (0;6.01)	5.67 (2.26;3.41)	9.08 (0;9.08)	6.76 (0.26;6.50)	8.97 (0;8.97)
Yield Sponsor (Spread)	10.74% (5.74%)	5.55% (4.55%)	11.34% (6.34%)	5.78% (4.78%)	11.20% (6.20%)	N/A	10.86% (5.86%)	N/A (N/A)	8.28% (3.28%)	N/A (N/A)
Yield Backed Unit (Spread)	9.34% (4.34%)	4.79% (3.79%)	10.25% (5.25%)	5.47% (4.47%)	10.82% (5.82%)	9.01% (8.01%)	11.83% (6.83%)	12.20% (11.20%)	16.67% (11.67%)	13.85% (12.85%)
DP Sponsor	33.91%	26.88%	37.82%	28.52%	36.93%	9.73%	34.77%	9.73%	17.37%	9.68%
DP Backed Unit	23.98%	21.37%	29.44%	25.38%	32.62%	41.90%	37.88%	54.07%	53.97%	57.44%
Joint Default Probability	3.94%	2.63%	14.14%	10.08%	17.71%	7.93%	21.72%	9.30%	17.36%	9.74%
LGD Sponsor	50.30	39.76	55.12	41.97	54.04	0	51.29	0	23.80	0
LGD Backed Unit	87.11	92.05	76.46	76.71	79.69	145.34	88.42	175.42	159.34	196.88

Table 4: This table displays the optimal figures of a Parent/Subsidiary structure when the parameters for the two units are those in Table 1, apart from $\tau_P = 24\%, \tau_S = 16\%, \sigma_P = \sigma_S = \sigma = 44 = \%$ for two levels of interest rates, 5% and 1%. Cash flows are jointly normally distributed, with marginal distributions as in Table 1 and correlation parameter ranging from -0.8 to 0.8. Sponsor and Backed Unit figures are reported in brackets, respectively.

5. Concluding Remarks

This paper contributes to capital structure theory, uncovering the heterogeneous response of optimal debt to changes in the level of interest rates. Our results for the Sponsor-Backed company depart from previous knowledge in corporate finance, that is focused on the Stand-Alone unit. To get them, we rely on the trade-off theory of structured finance, where leverage, credit spreads, bailout transfers and ownership links of two units are endogenous. Our results thus shed light on leverage and default of complex structures, such as private equity, securitization and multinationals, in different interest rate scenarios. Importantly, they provide a theoretical backing for the puzzling opposite relationship between leverage and interest rates in public firms and Leveraged Buyouts targets.

The implications of our analysis concerning optimal leverage and spreads appear to be broadly consistent with the observed concentration of leverage increases among high-risk companies in the years of falling interest rates. The trade-off theory of structured finance may thus complement existing explanations that stress supply considerations - such as investors' search for yield and time-inconsistent monetary policy. More generally, a key take-away of our analysis is that the overall structure of the company is essential to understand both leverage choices and default predictions, as their response to interest rates is opposite for a Stand-Alone and a Backed company with the same characteristics. This observation adds another strand for future empirical research on leverage to the suggestions in Frank and Goyal (2022).

Finally, our analysis relieves default concerns regarding highly leveraged entities when interest rates increase, as their optimal debt falls. Conversely, it supports the financial stability concerns arising from the increasing leverage of riskier entities, that appeared in association with lower interest rates. In fact, Backed companies in our model display higher default probabilities and default costs in comparison to Stand-Alone counterparts, both of which further increase when interest rates fall. This result is however conditional on the Backed companies having the same cash-flow distribution and the same horizon as their Stand-Alone counterparts, while in practice they have shorter horizons and operate in defensive industries when belonging to Private Equity funds. Furthermore, these concerns do not consider that bankruptcy-remote Sponsors hardly ever default contrary to their Stand-Alone counterparts.

Last but not least, the default probability of Backed units increases in those interest rates scenarios when the one of Stand-Alone activities falls, suggesting that heterogeneous company types smooth variation of aggregate defaults across interest rate scenarios. A thorough assessment of financial stability implications of highly leveraged units in alternative interest rate scenarios therefore deserves a much closer scrutiny, which we leave for future work.

References

Acharya, V. V., Gottschalg, O. F., Hahn, M., Kehoe, C.,2013. Corporate Governance and Value Creation: Evidence from Private Equity. The Review of Financial Studies 26(2), 368–402.

Ayotte, K., and Gaon, S., 2011. Asset-Backed securities: costs and benefits of "bankruptcy remoteness". The Review of Financial Studies, 24(4), 1299-1335.

Axelson, U., Jenkinson, T., Strömberg, P., Weisbach, M. S., 2013. Borrow cheap, buy high? The determinants of leverage and pricing in buyouts. The journal of finance, 68(6), 2223-2267.

Beaver, W. H., Cascino, S., Correia, M., McNichols, M. F., 2019. Group affiliation and default prediction. Management Science, 65(8), 3559-3584.

Beck, T., Perotti, Peltonen,T., E., Sánchez Serrano, A. and J. Suarez, forthcoming. Corporate credit in the EU: recent evolution, main drivers and financial stability implications, ESRB Report of the Advisory Scientific Committee

Bianco, M., and G. Nicodano, 2006. Pyramidal groups and debt, European Economic Review, 50(4), 937-961.

Brok, P., 2022, Debt and Taxes: The Role of Corporate Group Structures. ssrn.com

Caglio, C.R., Darst, R.M., and S.Kalemli-Özcan, 2021. Risk-Taking and Monetary Policy Transmission: Evidence from Loans to SMEs and Large Firms, NBER Working Paper Series 28685

Cohn, J., Mills, L., and E. Towery, 2014. The evolution of capital structure and operating performance after leveraged buyouts: Evidence from U.S. corporate tax returns. Journal of

Financial Economics 111, 469–494.

Darmouni, O., and M. Papoutsi, 2022. The Rise of Bond Financing in Europe, ssrn.com.

Duarte, D., Öztekin, Ö., and Y.F. Saporito, 2022. Capital Structure and the Yield Curve, The Review of Corporate Finance Studies, forthcoming.

Farhi, E., and Tirole, J., 2009. Leverage and the central banker's put. American Economic Review, 99(2), 589-93.

Fischer, E. O., Heinkel, R., and J. Zechner, 1989. Dynamic Capital Structure Choice: Theory and Tests. The Journal of Finance, 44(1), 19–40.

Frank, M.Z., and V.K. Goyal, 2022. Empirical Corporate Capital Structure, HKUST Business School Research Paper No. 2022-091.

Gompers, P., Kaplan, S. N., and Mukharlyamov, V, 2016. What do private equity firms say they do?. Journal of Financial Economics, 121(3), 449-476.

Gorton, G., and N. Souleles, 2006. Special purpose vehicles and securitization. In: Stulz, R., Carey, M. (Eds.), The Risks of Financial Institutions. University of Chicago Press, Chicago, IL.

Han, J., Park, K., and G. Pennacchi, G., 2015. Corporate taxes and securitization. The Journal of Finance, 70(3), 1287-1321.

Hanlon, M. and S. Heitzman, 2022. Corporate Debt and Taxes. Annual Review of Financial Economics 14:1

Haque, S., Jang, Y.S., and S. Mayer, 2023. Private Equity and Corporate Borrowing

Constraints: Evidence from Loan Level Data. ssrn.com

Hotchkiss, E. S., Smith, D. C., Strömberg, P., 2021. Private equity and the resolution of financial distress. The Review of Corporate Finance Studies, 10(4), 694-747.

Ivashina, V., 2022. When the Tailwind Stops: The Private Equity Industry in the New Interest Rate Environment, CEPR Press, London.

Ju, N., and Ou-Yang, H., 2006. Capital Structure, Debt Maturity, and Stochastic Interest Rates. The Journal of Business, 79(5), 2469–2502.

Kaplan, Steven. 1989. Management buyouts: Evidence on taxes as a source of value. The journal of finance, 44 (3), 611-632.

Kaplan, Steven, and Per Stromberg. 2009. Leveraged Buyouts and Private Equity. Journal of Economic Perspectives, 23 (1), 121-46.

Leland, H.E., 1994. Corporate Debt Value, Bond Covenants, and Optimal Capital Structure. The Journal of Finance, 49, 1213-1252.

Leland, H., 2007. Purely financial synergies and the optimal scope of the firm: implications for mergers, spin offs, and structured finance. Journal of Finance 62, 765-807.

Luciano, E., Nicodano, G., 2014. Guarantees, leverage, and taxes. Review of Financial Studies 27, 2736-2772.

Martinez-Miera, D., and R. Repullo, 2017. Search for Yield, Econometrica, 85, 351-378.

Moody's, 2006. Default and Migration Rates for Private Equity-Sponsored Issuers.

Nicodano, G. and L. Regis, 2019. A trade-off theory of optimal ownership and capital structure, Journal of Financial Economics, 131 (3), 715-735.

Powell, J., 2019, Mapping the Financial Frontier: What Does the Next Decade Hold? Speech 20190520, 24th Annual Financial Markets Conference, Federal Reserve Bank of Atlanta.

Rosengren, E.S., 2019, Assessing Economic Conditions and Risks to Financial Stability, Speech, Bostonfed.

Renneboog, L., T. Simons, and M. Wright, 2007. Why do firms go private in the UK?, Journal of Corporate Finance, 13, 591-628.

Schularick, M., 2021. Corporate indebtedness and macroeconomic stabilisation from a long-term perspective, ECB Forum on Central Banking Sintra.

6. Appendix

6.1. Definition of the $h(\cdot)$ and $k(\cdot)$ functions

The function $h(X_B)$ defines the set of states of the world in which the Sponsor has enough funds to intervene and save its affiliate from default while at the same time remaining solvent. The rescue happens if the cash flows of the Sponsor X_S are enough to cover both its own debt obligations and the remaining part of those of the subsidiary. The function $h(X_B)$, which defines the level of parent cash flows above which the rescue occurs, is defined as:

$$h(X_B) = \begin{cases} X_S^d + \frac{F_B}{1-\tau_B} - \frac{X_B}{1-\tau_B} & X_B < X_B^Z, \\ X_S^d + X_B^d - X_B & X_B \geq X_B^Z. \end{cases}$$

Similarly, the function $k(X_S)$ describes the level of dividends required to rescue the

Sponsor from default. It is defined as

$$k(X_S) = \begin{cases} X_B^d + \frac{F_S - X_S}{(1-\tau_B)} & X_S < X_S^Z, \\ X_B^d + \frac{F_S - \tau_S X_S^Z - (1-\tau_S) X_S}{(1-\tau_B)} & X_S \geq X_S^Z. \end{cases}$$

When $X_B < X_B^Z$ ($X_S < X_S^Z$) the cash flow X_B (X_S) of the subsidiary does not give rise to any tax payment, as it is below the tax shield generated in that unit.

6.2. Proof of Lemma 1

The derivatives of the expected discounted values of taxes and default costs are respectively (we suppress dependence on F_i and the subscript i for notational convenience):

$$\begin{aligned} \frac{\partial T}{d\phi} &= \frac{T}{\phi} - \frac{\partial X^Z}{d\phi}(1 - F(X^Z))\phi\tau \\ \frac{\partial C}{d\phi} &= \alpha\phi\frac{\partial X^d}{d\phi}X^d f(X^d)\phi\alpha + \frac{C}{\phi}. \end{aligned}$$

Recalling that $X_Z = F - D$, $X^d = F + \frac{\tau}{1-\tau}D$, indeed we have: $\frac{\partial X^Z}{d\phi} = -\frac{\partial D}{d\phi}$ and $\frac{\partial X^d}{d\phi} = \frac{\tau}{1-\tau}\frac{\partial D}{d\phi}$.

Hence, we need to focus on the derivative of market debt value with respect to ϕ, for fixed F (dependence of D on F is suppressed for notational convenience):

$$\begin{aligned} \frac{\partial D}{d\phi} &= \frac{D}{\phi} + \phi\left[(1-\alpha)\frac{\partial X^d}{d\phi}X^d f(X^d) - \tau\frac{\partial X^d}{d\phi}\left(X^d - X^Z\right)f(X^d) + \tau\frac{\partial X^Z}{d\phi}\left[F(X^d) - F(X^Z)\right] + \right. \\ &- \left. F\frac{\partial X^d}{d\phi}f(X^d)\right] \\ \frac{\partial D}{d\phi} &= \frac{D}{\phi} + \phi\left[(1-\alpha)\frac{\partial X^d}{d\phi}\left(\frac{\tau}{1-\tau}D\right)f(X^d) - \alpha F\frac{\partial X^d}{d\phi}f(X^d) - \tau\frac{\partial X^d}{d\phi}\left(X^d - X^Z\right)f(X^d) + \right. \\ &+ \tau\frac{\partial X^Z}{d\phi}\left[F(X^d) - F(X^Z)\right]. \\ \frac{D}{\phi} &= \frac{\partial D}{d\phi}\left[1 - \phi(1-\alpha)\left(\frac{\tau}{1-\tau}\right)\left(\frac{\tau}{1-\tau}D\right)f(X^d) + \phi\alpha F\frac{\tau}{1-\tau}f(X^d) + \right. \\ &+ \left. \phi\tau\frac{\tau}{1-\tau}\left(X^d - X^Z\right)f(X^d) + \phi\tau\left[F(X^d) - F(X^Z)\right]\right]. \end{aligned}$$

The derivative of D is positive if the term multiplying it is positive. Rearranging it, it becomes:

$$
\begin{aligned}
1 & \quad -\phi(1-\alpha)\left(\frac{\tau}{1-\tau}\right)\left(\frac{\tau}{1-\tau}D\right)f(X^d)+\phi\alpha F\frac{\tau}{1-\tau}f(X^d)+ \\
+ & \quad \phi\tau\frac{\tau}{1-\tau}\frac{D}{1-\tau}f(X^d)+\phi\tau\left[F(X^d)-F(X^Z)\right].
\end{aligned}
$$

Finally, we get:

$$
1+\phi\alpha\left(\frac{\tau}{1-\tau}\right)\left(\frac{\tau}{1-\tau}D\right)f(X^d)+\phi\alpha F\frac{\tau}{1-\tau}f(X^d)+\phi\tau\left[F(X^d)-F(X^Z)\right]>0.
$$

As a consequence,we have that $\frac{\partial T}{d\phi}>0$ and $\frac{\partial C}{d\phi}>0$, which proves the lemma.

6.3. Proof of Lemma 2

The derivative of the spread with respect to ϕ is

$$
\frac{\partial y}{d\phi}=(-F^{\frac{1}{T}}\frac{1}{T}D^{-\frac{1}{T}-1}\frac{\partial D}{d\phi}+\frac{1}{T}\phi^{-\frac{1}{T}-1})=\frac{1}{T}(\phi^{-\frac{1}{T}-1}-\left(\frac{F}{D}\right)^{\frac{1}{T}}\frac{1}{D}\frac{\partial D}{d\phi}).
$$

This derivative is increasing whenever

$$
phi^{-\frac{1}{T}-1}\geq F^{\frac{1}{T}}D^{-\frac{1}{T}-1}\frac{\partial D}{d\phi}
$$

and, since $\frac{\partial D}{d\phi}\leq\frac{D}{\phi}$, a sufficient condition for $\frac{\partial y}{d\phi}\geq 0$ is

$$
\phi^{-\frac{1}{T}-1}\geq F^{\frac{1}{T}}D^{-\frac{1}{T}-1}\frac{D}{\phi}.
$$

This is true iff

$$
\left(\frac{1}{\phi}\right)^{\frac{1}{T}}\geq\left(\frac{F}{D}\right)^{\frac{1}{T}},
$$

which implies $\phi\leq\frac{D}{F}$. This is never the case unless $F=0$ because $D\leq\phi F, D<F$when $F>0$.

6.4. Proof of Proposition 1

When proving Lemma 1, we proved that $\frac{\partial D}{d\phi} > 0$ for fixed F. We want to prove now that the equity value is increasing in ϕ as well, for fixed F:

$$\begin{aligned}
\frac{\partial E}{d\phi} &= \frac{E}{\phi} + \phi\left[-(1-\tau)\frac{\partial X^d}{d\phi}X^d f(X^d) + F\frac{\partial X^d}{d\phi}X^d f(X^d)\right] = \\
&= \frac{E}{\phi} + \phi\frac{\partial X^d}{d\phi}f(X^d)\left[\tau X^d - \frac{\tau}{1-\tau}D\right] = \frac{E}{\phi} + \phi\frac{\partial X^d}{d\phi}f(X^d)\left[\tau F + \tau\frac{\tau}{1-\tau}D - \frac{\tau}{1-\tau}D\right] = \\
&= \frac{E}{\phi} + \phi\frac{\partial X^d}{d\phi}f(X^d)\left[\tau F - \tau D\right].
\end{aligned}$$

If $\phi \leq 1$, the above expression is always strictly greater than zero. This implies that the value of the firm, which is the sum of D and E, is increasing in ϕ (decreasing in the interest rate) for any F and, a fortiori, at the optimum.

Finally, notice that, since $\frac{\partial X^Z}{d\phi} = -\frac{\partial D}{d\phi}$ and $\frac{\partial X^d}{d\phi} = \frac{\tau}{1-\tau}\frac{\partial D}{d\phi}$, it follows that

$$\begin{aligned}
|\frac{\partial X^Z}{d\phi}| > |\frac{\partial X^d}{d\phi}| \implies 1 &> \frac{\tau}{1-\tau} \\
i.e.\ \tau &< \frac{1}{2}.
\end{aligned}$$

6.5. Proof of Proposition 2

Following Nicodano and Regis (2019), a sufficient condition for the Sponsor to be zero-leverage is:

$$\frac{\tau_S(1-\tau_S)G(0)(1-G(0))}{\alpha_B[1-\tau_S G(0)]} \leq \int_0^{X_{SA}^{Z,B*}} xg\left(x, \frac{F_{SA}^{B*}}{1-\tau_B} - \frac{x}{1-\tau_B}\right)dx + \int_{X_{SA}^{Z,B*}}^{X_{SA}^{d,*}} xg\left(x, X_{SA}^{d,B*} - x\right)dx$$

The right hand side of the above inequality is increasing in ϕ, because its derivative relative to ϕ is

$$\frac{\partial X_{SA}^d(F_{SA}^*)}{d\phi}X_{SA}^d g(X_{SA}^d) > 0.$$

It has a lower bound when $\phi \to 0$ equal to

$$L = \alpha_B \int_0^{F_{SA}^{B*}} xg(x, \frac{F_B}{1-\tau_B} - \frac{x}{1-\tau_B})dx,$$

while it diverges to $+\infty$ when $\phi \to \infty$. For given τ_S, α_B and $G(0)$ the condition is more likely to be satisfied the higher is ϕ, i.e. the lower the interest rate. If $\frac{\tau_S(1-\tau_S)G(0)(1-G(0))}{\alpha_B[1-\tau_S G(0)]} > L$, then there exists $\bar{\phi}$ (or, equivalently, $\bar{r}$) such that (6) is satisfied as an equality.