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Corporate Fraction and the Equilibrium Term-Structure of Equity Risk*

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Abstract. The recent empirical evidence of a downward sloping term structure of equity risk is viewed as a challenge to many leading asset pricing models. This paper analytically characterizes conditions under which a continuous-time long-run risk model can accommodate the stylized facts about dividend and equity risk, when dividends are a stationary stochastic fraction of aggregate consumption. Such a cointegrating relation makes dividends riskier in the short-run than at medium horizons but also preserves the role of long-run risk: consequently, the model captures both the traditional puzzles, like the high equity premium, as well as the new evidence about the term structure of equity risk.

JEL Classification: C62, D51, D53, G12, G13

1. Introduction

Recent empirical evidence, as in van Binsbergen, Brandt, and Koijen (2012) and van Binsbergen, Hueskes, Koijen, and Vrugt (2013), questions the short-term implications of leading asset pricing models such as Bansal and Yaron (2004) and Campbell and Cochrane (1999). These models provide a long-term explanation of traditional puzzles like the equity premium and the excess volatility but are inconsistent with the highly risky returns of dividend strips in the short-term and the downward sloping term-structures of the volatility of dividends and dividend strips returns at short and medium horizons.

This paper shows that a one-channel long-run risk model can accommodate for both short-term and long-term patterns of equity returns as long as dividends are modelled as a stochastic fraction of aggregate consumption. Then, I consider an endowment economy and model fundamentals in a similar fashion to Longstaff and Piazzesi (2004) and Santos

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and Veronesi (2006) and in spirit of Bansal, Dittmar, and Lundblad (2005): the corporate fraction or dividend-share –that is dividends relative to consumption– captures the cointegrating relation between the two cash-flows streams.

The intuition of the main mechanism driving the results of the paper is as follows. When the corporate fraction is small and most of consumption is funded by labor income, financial assets constitute a claim on a small fraction of consumption which covaries little with it. The opposite holds when the corporate fraction is large. However, if consumption and dividends are cointegrated, fluctuations in the corporate fraction –even if persistent– do not alter the riskiness of long-term cash-flows. Hence, risk due to the corporate fraction dynamics concentrates in the short-run, inducing uncertainty in short-term cash-flows. Consequently, short-term equity claims, such as the dividend strips, are highly volatile and feature a downward sloping term structure of risk. Even if fluctuations in the corporate fraction would not covary with either short- or long-run risk in consumption, highly risky short-term cash-flows can significantly contribute to the equity premium. Such a short-run explanation of the premium is both alternative and complementary to the mechanism driving the premium in leading equilibrium models, that is highly volatile long-term discounted cash-flows.[1]

I analytically characterize many results about the dividend strips as well as the market asset. If dividends relative to consumption are volatile enough, the model generates, on the one hand, i) a downward sloping term-structure of volatility of dividends and dividend strips' returns and ii) the countercyclical dynamics of the equity yields and, on the other hand, iii) a low and smooth risk-free rate and iv) a high equity premium and excess volatility of the stock returns. These results obtain under standard preferences and without stochastic volatility or jumps in fundamentals. The latter can be easily introduced in the framework and can help to fit additional moments.[2]

The model is defined in continuous time by use of the differential stochastic utility of Duffie and Epstein (1992), which guarantees analytic tractability. The model has closed-form solutions with unitary elasticity of intertemporal substitution or closed form solutions up to a log-linearization of the consumption-wealth ratio around its endogenous steady-state for any value for the elasticity of intertemporal substitution. Analytic tractability allows to characterize conditions under which the model can generate simultaneously short-term and long-term patterns of equity returns and how these relate with macroeconomic fundamentals. Moreover, the model allows for a transparent comparison with the standard long-run risk model.

The model first sheds lights on the role of preferences on the equilibrium term structure of equity. When the corporate fraction is independent of consumption dynamics, the term-structure of risk is U-shaped and the term-structure of premia is monotone. The latter has positive slope if the intertemporal substitution effect dominates the wealth effect and

[1] In the habit model of Campbell and Cochrane (1999), long-term discounted cash-flows are highly volatile because variations in the aggregate relative risk aversion induce uncertainty in discount rates, which integrates with the horizon. In the long-run risk model of Bansal and Yaron (2004), the high riskiness of long-term discounted cash-flows is due to the pricing of small but persistent uncertainty in consumption dynamics, amplified by the degree of preference for the early resolution of uncertainty.

[2] The setting automatically extends to the whole class of jump-diffusion affine processes –providing an alternative framework to Eraker and Shaliastovich (2008)– and allows for the pricing of derivatives on the term structure of equity.

vice-versa. Instead, when the corporate fraction affects consumption dynamics, it becomes a priced factor under recursive utility and both the term structures of risk and premia are non-monotone. The latter can be decreasing at short and medium horizons and increasing in the long-run if the wealth effect dominates the intertemporal substitution effect. In particular, the price of risk associated to the fluctuations in the corporate fraction is non-monotone in the intertemporal elasticity of substitution. Those non-trivial effects obtain endogenously at equilibrium and enrich the standard long-run risk model. Then, the latter can accommodate the recent evidence about downward-sloping dividend and equity risk up to a minimal modification.[3]

The model calibration exploits information from the term-structure of dividend risk. This is interesting for two main reasons. First, downward-sloping dividend volatility is consistent with the predictability of dividend growth by the dividend-share that we see in the real data and, hence, supports the main model mechanism that relates the timing of dividend risk to the cointegrating relationship between consumption and dividends. Second, the term-structure of dividend risk provides information about the persistence of latent factors and, hence, about the strength of the long-run risk channel. Matching the empirical evidence about the timing of dividend risk leads to a dynamics for the long-run growth factor which is in line with most of the long-run risk literature.

Beyond the traditional asset pricing moments and the slope of equity risk, the analysis of the quantitative predictions of the model investigates a number of testable implications and compares them with the available empirical evidence. Consistently with the data, the model produces: declining volatilities of the forward equity yields; the countercylical slope of the equity yields; the variance decomposition of equity yields in which the cash-flows channel dominates the discount rate channel; conditional CAPM beta of dividend strips lower than unity and increasing with the horizon.

The paper is closely related to Ai, Croce, Diercks, and Li (2012) and Belo, Collin-Dufresne, and Goldstein (2014), which focus respectively on investment and financing decisions.[4] Namely, Belo et al. (2014) assume an exogenous financial leverage process that, in a partial equilibrium framework, makes aggregate EBIT and dividends cointegrated. Similarly to this paper, the key ingredient is modelled exogenously in such a way that the dividend-share does not affect the dynamics of aggregate consumption and, in turn, that of the state price density. However, I provide empirical evidence that the dividend-share conveys additional information, with respect to financial leverage, concerning the properties of short-run dividend risk.

An interesting challenge is to model the firm behavior, in order to generate endogenously the cash-flows dynamics. Marfè (2013) provides some economic foundation to the endowment equilibrium model of this paper. A mechanism of income insurance from shareholders to workers endogenizes the cointegrating relationship between consumption and

[3] Notice that, while downward-sloping dividend risk (and, in turn, equity risk) is a very robust feature of the data, downward-sloping equity premia is a stylized fact still under debate. See Boguth, Carlson, Fisher, and Simutin (2012), van Binsbergen and Koijen (2012b) and Muir (2014) on the empirical evidence from currently available data.
[4] Berrada, Detemple, and Rindisbacher (2013), Croce, Lettau, and Ludvigson (2014), Curatola (2015), Khapko (2014) and Marfè (2014) focus on non-standard beliefs and preferences.

dividends, its pricing at equilibrium as well as the downward sloping term-structures of dividend and equity risk.[5]

2. Empirical Support

In this section, I document some properties of aggregate dividends in the US data. These properties provide empirical support for the key features of the model and drive the main results. First, I document that various measures of dividend-share are smooth, persistent and stationary. Second, I document that these measures predict dividend growth and provide additional explanatory power with respect to aggregate financial leverage. Third, I provide support to the idea that variations in the dividend-share capture more information about the term-structure of dividend risk than aggregate financial leverage. Indeed, the effect of the latter –which can be understood through the term-structures of variance ratios of EBIT and dividends as in Belo et al. (2014)– does not capture the huge distance between the upward-sloping consumption risk and the downward-sloping dividend risk. In a nutshell, the analysis offers empirical support to the main economic mechanism of the model, that is the role of the co-integrating relationship between consumption and dividends on the timing of dividend risk.

2.1 DATA

The key variable of the analysis is an aggregate measure of shareholders' remuneration. The main data source is the National Income and Products Account (NIPA), available through the Bureau of Economic Analysis (BEA) website. Real GDP levels are from section 1.1.6, aggregate consumption (*"Nondurable goods"* plus *"Services"*) is from the same section and aggregate wages (*"Compensation of employees paid"*) are from section 1.10. Two alternative measures of shareholders' remuneration are: aggregate dividends (*"Net dividends"*) and aggregate corporate profits after tax (*"Profits after tax with inventory valuation and capital consumption adjustments"*) both from section 1.10. Data are collected at yearly frequency since 1929 to 2012. I also consider a measure of the aggregate leverage ratio, computed as in Belo et al. (2014), from the Flow of Funds Accounts of the US (Board of Governors of the Federal Reserve System) table B.102, since 1945 to 2012.

2.2 DIVIDENDS AND CORPORATE FRACTION

Both net dividends (D^1) and after tax corporate profits (D^2) are high volatile, in particular profits are extremely volatile in the pre-war sub-sample. In order to measure the fraction of total resources devoted to shareholders' remuneration, I compute the dividend-share or corporate fraction in three ways for both D^1 and D^2:

$$s^{i1} = \frac{D^i}{Y}, \quad s^{i2} = \frac{D^i}{C}, \quad s^{i3} = \frac{D^i}{W + D^i}, \qquad i = \{1, 2\}, \tag{1}$$

[5] Such an approach is in spirit of Danthine and Donaldson (1992, 2002) and is consistent with a few recent contributions relating labor frictions to finance. See Uhlig (2007), Merz and Yashiv (2007), Kuehn, Petrosky-Nadeau, and Zhang (2012) and Favilukis and Lin (2013). Marfè (2015) empirically and theoretically supports the idea that aggregate labor rigidity drives the value premium by means of its effect on the timing of dividend risk.

where Y, C and W denote real GDP, aggregate consumption and aggregate wages. All these measures of dividend-share are small in average, smooth and strongly persistent.

Panel A of Table 1 reports some summary statistics: their unconditional first and second moments vary respectively in the range (2.6%, 9.2%) and (0.9%, 1.8%). I verify for stationarity by regressing first differences on the lagged level. Panel B of Table 1 reports the estimation results. All the six measures of dividend-share are stationary and feature a similar rate of mean reversion, ranging in the interval (14%, 32.1%).

Table 1 Measures of the corporate fraction. Panel A reports yearly mean, standard deviation, min and max from time-series of the six measures of dividend-shares computed as in Eq. (1). Panel B reports the estimates with Newey-West corrected t-statistics in parentheses from univariate regressions of first differences in the six dividend-share measures on a constant and their lagged levels: $\Delta s_t^{ij} = b_0 + b_1 s_{t-1}^{ij} + \epsilon_t$. Data are yearly on the sample 1929:2012 from NIPA tables.

Panel A

	s^{11}	s^{12}	s^{13}	s^{21}	s^{22}	s^{23}
mean	0.026	0.046	0.045	0.051	0.092	0.084
std. dev.	0.009	0.012	0.015	0.009	0.018	0.016
min	0.015	0.029	0.025	0.033	0.054	0.055
max	0.054	0.082	0.094	0.071	0.135	0.118

Panel B

	Δs^{11}	Δs^{12}	Δs^{13}	Δs^{21}	Δs^{22}	Δs^{23}
b_0	0.039	0.009	0.006	0.015	0.022	0.025
	(2.63)	(3.23)	(2.62)	(3.95)	(3.41)	(4.10)
b_1	-0.149	-0.215	-.140	-0.309	-0.256	-0.321
	(-2.95)	(-3.45)	(-2.97)	(-4.14)	(-3.57)	(-4.30)
adj-R^2	0.09	0.12	0.09	0.16	0.13	0.18
N	78	78	78	78	78	78

2.3 PREDICTABILITY OF DIVIDEND GROWTH

The economic mechanism of the model implies that the current dividend-share should have predictive power for dividend growth. A stationary dividend-share implies that periods with a low fraction of total consumption (or GDP) devoted to shareholders' remuneration are followed by periods in which dividend growth is larger than in average, in order to push the dividend-share back toward its long-run mean. This leads to short-term risk of dividends and to dividend growth predictability. If fluctuations of the dividend-share are a main determinant of the dividend dynamics, then we expect that the forecasting power of the dividend-share survives over other sources of dividend predicability (e.g. time-varying expected growth in consumption or GDP) and, hence, is observable from the real data.

Similarly to Belo et al. (2014), I test the main model mechanism by verifying if the variable, responsible of the short-term risk of dividends in the model, forecasts dividend growth. Here, I consider the six measures of dividend-share commented above, whereas Belo et al. (2014) uses the aggregate leverage ratio. Then, I regress future dividend growth, computed over several horizons, on each measure of dividend-share:

$$\frac{1}{\tau}\log\frac{D^i_{t+\tau}}{D^i_t} = b_0 + b_1 s^{ij}_t + \epsilon_t, \qquad i=\{1,2\},\ j=\{1,2,3\},$$

with horizon $\tau=\{1,3,5,10,15,20\}$ years.

Table 2 Corporate fraction and net dividends growth predictability. The table reports the estimates of the regression in Eq. (2), where net dividends growth is regressed on the dividend-shares computed as in Eq. (1) and the aggregate financial leverage ratio. Panel A and B restrict the regression equation to only one independent variable. Data are yearly on the sample 1929:2012 from NIPA tables and 1945:2012 from Flow of Funds. Newey-West corrected t-statistics are reported in parentheses. The symbols *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Panel A

Horizon	1	3	5	10	15	20
s^{11}	-4.135**	-2.875***	-2.056***	-1.693***	-1.641***	-1.664***
	(-2.22)	(-2.88)	(-4.02)	(-4.35)	(-6.47)	(-5.23)
adj-R^2	0.05	0.18	0.20	0.26	0.35	0.48
N	77	75	73	68	63	58
s^{12}	-3.771***	-2.252***	-1.585***	-1.321***	-1.348***	-1.393***
	(-3.01)	(-3.14)	(-4.53)	(-3.93)	(-7.05)	(-5.60)
adj-R^2	0.09	0.23	0.25	0.30	0.44	0.59
N	77	75	73	68	63	58
s^{13}	-2.138*	-1.644***	-1.186***	-0.967***	-0.919***	-0.921***
	(-1.97)	(-2.93)	(-4.00)	(-4.61)	(-6.49)	(-5.43)
adj-R^2	0.04	0.18	0.21	0.27	0.36	0.49
N	77	75	73	68	63	58

Panel B

Horizon	1	3	5	10	15	20
lev	0.079	0.145**	0.164***	0.167***	0.160***	0.120***
	(0.88)	(2.31)	(3.13)	(5.81)	(16.65)	(8.17)
adj-R^2	-0.01	0.04	0.17	0.52	0.73	0.60
N	67	65	63	58	53	48

Panel C

Horizon	1	3	5	10	15	20
lev	-0.001	0.104	0.139**	0.165***	0.150***	0.093***
	(-0.01)	(1.40)	(2.26)	(5.26)	(15.06)	(8.73)
s^{11}	-4.650*	-2.386	-1.236	-0.110	-0.866***	-2.656***
	(-1.91)	(-1.26)	(-1.17)	(-0.17)	(-2.79)	(-10.00)
adj-R^2	0.04	0.12	0.21	0.51	0.76	0.78
N	67	65	63	58	53	48
lev	-0.038	0.087	0.131**	0.163***	0.147***	0.083***
	(-0.29)	(1.11)	(2.08)	(5.03)	(14.48)	(7.20)
s^{12}	-3.561**	-1.762	-0.899	-0.114	-0.573***	-1.725***
	(-2.18)	(-1.45)	(-1.37)	(-0.26)	(-3.34)	(-10.29)
adj-R^2	0.06	0.15	0.23	0.51	0.76	0.79
N	67	65	63	58	53	48
lev	0.006	0.105	0.139**	0.165***	0.150***	0.094***
	(0.05)	(1.39)	(2.23)	(5.24)	(15.61)	(9.75)
s^{13}	-2.521*	-1.386	-0.739	-0.087	-0.545***	-1.486***
	(-1.78)	(-1.24)	(-1.18)	(-0.23)	(-3.42)	(-11.85)
adj-R^2	0.03	0.12	0.21	0.51	0.77	0.80
N	67	65	63	58	53	48

Table 3 Corporate fraction and after tax corporate profits growth predictability. The table reports the estimates of the regression in Eq. (2), where after tax corporate profits growth is regressed on the dividend-shares computed as in Eq. (1) and the aggregate financial leverage ratio. Panel A and B restrict the regression equation to only one independent variable. Data are yearly on the sample 1929:2012 from NIPA tables and 1945:2012 from Flow of Funds. Newey-West corrected t-statistics are reported in parentheses. The symbols *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Panel A

Horizon	1	3	5	10	15	20
s^{21}	-11.800	-18.947***	-8.970***	-4.589***	-2.992***	-2.266***
	(-1.54)	(-3.70)	(-3.88)	(-3.05)	(-2.99)	(-3.17)
adj-R^2	0.06	0.53	0.66	0.58	0.57	0.60
N	77	75	73	68	63	58
s^{22}	-5.440	-8.768***	-4.750***	-2.494***	-1.667***	-1.249***
	(-1.18)	(-3.61)	(-3.79)	(-3.25)	(-3.50)	(-3.60)
adj-R^2	0.04	0.39	0.63	0.61	0.62	0.64
N	77	75	73	68	63	58
s^{23}	-7.480	-11.826***	-5.417***	-2.738***	-1.765***	-1.334***
	(-1.64)	(-3.76)	(-3.80)	(-2.90)	(-2.75)	(-2.91)
adj-R^2	0.07	0.56	0.65	0.56	0.54	0.56
N	77	75	73	68	63	58

Panel B

Horizon	1	3	5	10	15	20
lev	0.141	0.028	-0.064	0.012	0.046	0.054**
	(0.63)	(0.19)	(-0.65)	(0.22)	(1.15)	(2.25)
adj-R^2	-0.01	-0.01	-0.00	-0.02	0.04	0.14
N	67	65	63	58	53	48

Panel C

Horizon	1	3	5	10	15	20
lev	-0.090	-0.325***	-0.294***	-0.073*	-0.022	0.008
	(-0.35)	(-2.66)	(-4.41)	(-1.76)	(-0.70)	(0.31)
s^{21}	-4.796***	-7.393***	-4.908***	-1.892***	-1.356***	-0.946***
	(-2.86)	(-9.00)	(-8.06)	(-6.42)	(-3.62)	(-3.87)
adj-R^2	0.05	0.47	0.62	0.37	0.35	0.41
N	67	65	63	58	53	48
lev	-0.172	-0.343***	-0.298***	-0.077**	-0.033	0.002
	(-0.67)	(-3.02)	(-4.83)	(-2.07)	(-1.27)	(0.08)
s^{22}	-3.405***	-4.050***	-2.642***	-1.021***	-0.799***	-0.544***
	(-3.32)	(-8.44)	(-11.28)	(-7.89)	(-5.33)	(-4.88)
adj-R^2	0.10	0.51	0.64	0.42	0.44	0.48
N	67	65	63	58	53	48
lev	-0.062	-0.297**	-0.275***	-0.064	-0.011	0.016
	(-0.24)	(-2.27)	(-3.77)	(-1.46)	(-0.31)	(0.58)
s^{23}	-2.754***	-4.456***	-2.944***	-1.118***	-0.743***	-0.521***
	(-2.76)	(-8.24)	(-6.44)	(-5.23)	(-2.75)	(-3.16)
adj-R^2	0.04	0.45	0.58	0.32	0.28	0.34
N	67	65	63	58	53	48

Panel A of Table 2 and 3 show that all the six measures of dividend-share feature substantial predictive power at both short and long horizons. Coefficients are negative, as expected, and strongly statistically significant (t-statistics are computed by means of Newey-West corrected standard errors). Adjusted R^2 varies from about 5% to 60% over the one year to 20 years horizon. Results are similar when the dependent variable is measured using either net dividends or after tax corporate profits.

The stationarity of the dividend-share mechanically induces predictability in dividend growth. The result here is a quantitative one: the extent to which the dividend-share forecasts dividend growth is quite large. Therefore, the co-integrating relationship between dividends and consumption (or GDP) seems to be an important determinant of the dynamics of dividends. Hence, the dividend-share is an important building block of an equilibrium model, in addition to time-varying long-run growth, in particular if we are interested at capturing the short-run properties of cash-flows growth rates and, in turn, of equity returns.

Belo et al. (2014) argue that the short-run risk of dividends should be imputed to variation in stationary financial leverage ratios. From the perspective of an endowment economy equilibrium model I am interested into the joint dynamics of consumption and dividends. Therefore, I wonder whether the dynamics of the dividend-share essentially captures the same economic channel pointed out by Belo et al. (2014) or the dividend-share also conveys additional important information which I lose by focusing on financial leverage only. In order to answer such a question, I verify the predictability of future dividend growth by both financial leverage and dividend-share:

$$\tfrac{1}{\tau}\log\tfrac{D^i_{t+\tau}}{D^i_t} = b_0 + b_1 s^{ij}_t + b_2 \mathrm{lev}_t + \epsilon_t, \qquad i=\{1,2\},\, j=\{1,2,3\}, \tag{2}$$

where lev_t is the aggregate financial leverage ratio. Panel B of Table 2 and 3 shows that, consistently with Belo et al. (2014), financial leverage, as the only independent variable, positively explains dividend growth at both short and long horizons –when it is measured using net dividends– but only at very long horizons –when it is measured using after tax corporate profits. However, panel C of Table 2 and 3 reports the estimation results from the case in which both financial leverage and dividend-share are used as independent variables. When the dependent variable is measured by net dividends, both financial leverage and dividend-share are statistically significant at medium and long horizons and feature respectively a positive and negative coefficient in accord with the theory. Moreover, the adjusted R^2 increases with respect to Panels A and B. Instead, when the dependent variable is measured by after tax corporate profits, financial leverage is significant at short horizons only and features a negative coefficient in contrast with the positive theoretical relation of Belo et al. (2014)'s model; the dividend-share is still significant at both short and long horizons and its negative relation with dividend growth is preserved.

These results suggest that financial leverage is not the only determinant of the short-term risk of dividends and that the dividend-share conveys substantially more information about the dividend dynamics. Such a conclusion is not surprising if we consider the empirical evidence documented by Belo et al. (2014) about aggregate EBIT. Their model implies that financial leverage shifts risk toward the short-run from EBIT to dividends. Indeed they assume that EBIT risk is upward-sloping and, as a result of stationary financial leverage, dividend risk is downward-sloping. However, they also document that the term-structure of the variance ratios of aggregate EBIT is decreasing with a shape similar to that of dividends. Figure 1 reports the term-structures of variance-ratios of aggrgeate EBIT and two measures of aggregate dividends from Belo et al. (2014). Moreover, Figure 1 also shows the variance-ratios of aggregate consumption.

While the variance-ratios of consumption are larger than unity and increase with the horizon, those of EBIT and dividends are both decreasing and lower than one. Therefore,

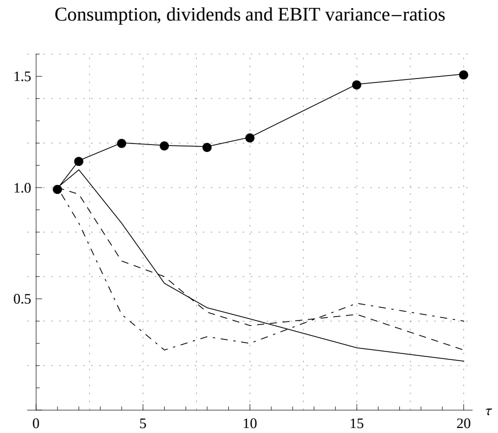

Fig. 1. The term structures of aggregate consumption, dividends and EBIT variance-ratios. The variance-ratios of net dividends (dot-dashed line), dividends with repurchases (dashed line) and EBIT (solid line) are from Belo et al. (2014). The variance-ratios of consumption (marked solid line) are computed using data from Beeler and Campbell (2012). Horizon τ denotes years. The variance-ratios procedure uses the theoretical exposition of Campbell, Lo and MacKinlay (1997, pp. 48-55), which accounts for heteroscedasticity and overlapping observations.

at least at the aggregate level, the financial leverage channel seems to be small and most of the short-term risk of dividends is determined by something else than financial leverage. Marfè (2013) provides empirical evidence and theoretical support to the idea that the shift of long-run consumption (or GDP) risk toward short-run dividend risk should be imputed to labor relations and to the determination of aggregate wages.

In conclusion, this empirical analysis supports the idea that various economic channels contribute to the determination of the short-run risk of dividends. The co-integrating relationship between dividends and consumption (or GDP) embeds such an economic mechanism. And, hence, the modelling of the dynamics of the dividend-share is a crucial ingredient of an endowment economy equilibrium model which aims to capture the short-run properties of dividends growth rates and equity returns.

3. The Economy

3.1 PREFERENCES

A representative agent features recursive preferences in spirit of Kreps and Porteus (1979), Epstein and Zin (1989) and Weil (1989). These preferences allow for the separation between the coefficient of relative risk aversion (RRA) and the elasticity of intertemporal substitution (EIS). For the sake of tractability, I assume their continuous time counterpart which takes the form of stochastic differential utility, as in Duffie and Epstein (1992). Given an initial consumption C, the utility at each time t is defined as $U(C_t) = J_t$ where

J is the unique solution to the SDE:

$$dJ_t = \left(-f(C_t, J_t) - \tfrac{1}{2}A(J_t)\sigma_J\sigma_J'\right)dt + \sigma_J dB_t. \tag{3}$$

Under certain technical conditions, Eq. (3) is well defined and hence the utility function exists and is monotonic and risk averse for $A \leq 0$.[6] As usual, I consider the case $A = 0$ with normalized aggregator given by

$$f(C, J) = \beta\theta J\left(C^{1-\frac{1}{\psi}}((1-\gamma)J)^{-\frac{1}{\theta}} - 1\right), \tag{4}$$

where $\theta = (1-\gamma)/(1-1/\psi)$, γ is RRA, ψ is EIS and β is a discount rate. In the special case $\psi \to 1$, the aggregator reduces to

$$f(C, J) = \beta(1-\gamma)J\left(\log C - \tfrac{\log((1-\gamma)J)}{1-\gamma}\right),$$

and for $\psi \to \gamma^{-1}$ CRRA preferences obtain.

3.2 DYNAMICS

The representative agent receives income from two sources, financial and non-financial, and where the mix between these two sources of income varies over time. In particular, I focus on the aggregate consumption and on its fraction funded by market dividends.

Consumption dynamics is modelled in spirit of long-run risk literature.[7] In particular, I assume that aggregate consumption growth follows a geometric process with stochastic growth rate:

$$\frac{dC_t}{C_t} = \mu_t dt + \sigma dB_{C,t}, \tag{5}$$

$$d\mu_t = \lambda(\bar{\mu} - \mu_t)dt + \nu dB_{\mu,t}, \tag{6}$$

where $d\langle B_C, B_\mu\rangle_t = \rho_{C,\mu}dt$. Parameters $\bar{\mu}, \lambda, \sigma$ and ν are positive constants and $\rho_{C,\mu} \in (-1, 1)$. Heteroscedasticity is not included in the model for the sake of simplicity and exposition. Instead, one element of interest is the latent growth rate μ_t, which captures the small predictable component of the expected growth rate of aggregate consumption, as suggested by empirical evidence.[8] In line with the findings of Constantinides and Ghosh (2011), I propose a continuous time version of the one-channel model of Bansal and Yaron (2004). However, conditional variances as well as jumps can be easily added to the framework.

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[6] Namely, for any consumption process $C \in \mathcal{L}^2$, utility $U : \mathcal{L}^2 \to \mathcal{R}$ is a map defined by two primitive functions (f, A), where $f : \mathcal{R}^+ \times \mathcal{R} \to \mathcal{R}$ and $A : \mathcal{R} \to \mathcal{R}$, and J is unique with boundary condition $J_T = 0$. Let U' and U'' be the utilities associated to (f, A') and (f, A''), then U' is more risk averse than U'' if $A' \leq A''$.

[7] A different specification of cash-flows dynamics is proposed in Section 5.2 and different economic implications are analyzed.

[8] Expected growth, μ_t, is assumed to be an observable variable even if it is instead a latent factor. Because of the affine specification, a straightforward application of the Kalman-Bucy filter allows to model Bayesian learning, which is omitted for the sake of exposition since it does not add economic content to the core of the paper.

In spirit of Longstaff and Piazzesi (2004), Bansal, Dittmar, and Lundblad (2005) and Santos and Veronesi (2006), I explicitly model the dividend-share, S_t: at each point in time, the dividend and non-dividend components of consumption are given by:

$$D_t = C_t S_t \qquad \text{and} \qquad L_t = C_t(1 - S_t), \tag{7}$$

where

$$S_t = e^{\bar{\delta}+\delta_t}. \tag{8}$$

The process δ_t leads to fluctuations over time in the component of aggregate consumption funded by market dividends and, hence, to the correlation of the latter with the investor's marginal utility. Henceforth, I refer to δ_t as dividend-share or corporate fraction with a slight abuse of terminology (namely, δ_t represents the deviation from the steady-state of the logarithm of the dividend-share). The dynamics of the dividend-share follows an Ornstein-Uhlenbeck process:

$$d\delta_t = -\kappa\delta_t dt + \eta dB_{\delta,t}, \tag{9}$$

where $d\langle B_C, B_\delta\rangle_t = \rho_{C,\delta}dt$ and $d\langle B_\mu, B_\delta\rangle_t = \rho_{\mu,\delta}dt$. Parameters κ and η are positive constants, $\bar{\delta}$ is a negative constant, $\rho_{C,\delta}, \rho_{\mu,\delta} \in (-1, 1)$ and heteroscedasticity is omitted for the sake of simplicity. Therefore, S_t is a strictly positive process with negligible probability of being above one, once parameters are set to match the empirical data.[9]

By an application of Itô's Lemma, market dividends have dynamics given by

$$\frac{dD_t}{D_t} = (\mu_t - \kappa\delta_t + (\eta/2 + \sigma\rho_{C,\delta})\eta)\, dt + \sigma dB_{C,t} + \eta dB_{\delta,t} \tag{10}$$

and, hence, the instantaneous growth rate of dividends depends on both the expected growth rate and the dividend-share, whereas the instantaneous volatility is constant. The pair $\{\mu_t, \delta_t\}$ characterizes at each point in time dividend growth as well as asset prices: therefore, the role of the corporate fraction δ_t can be understood in terms of its implications over a standard one-channel long run-risk model in continuous time.[10]

The focus now turns on the term structure of cash-flows and in particular on the slope of their volatility with respect to the time horizon. The specification in Eq. (7)-(8)-(9) leads to the following result.

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[9] In real data S_t is quite small (about 5%) and strongly persistent: therefore, reasonable values for $\bar{\delta}, \kappa$ and η allow S_t to belong to the range $(0, 1)$ most of the times. Alternatively, S_t can be modelled as a Wright-Fisher process: $dS_t = \kappa(\bar{S} - S_t)dt + \eta\sqrt{S_t(1 - S_t)}dB_{S,t}$ which guarantees the property $S_t \in (0, 1)$ as long as $\kappa\bar{S} > \eta^2/2$. The choice of Eq. (8)-(9) provides more tractability and intuition to the results of the paper.

[10] Including stochastic volatility in consumption growth would lead to the same consumption dynamics and, hence, the same state-price density of the standard long-run risk model. The scope of the paper is to analyze the term structure of equity in equilibrium: adding stochastic volatility to the model would not be particularly interesting because its effect would be very similar to that of the growth factor μ_t –that is, to increase the uncertainty of cash-flows in the long-run. Instead, the model of Section 5.2 features a different type of stochastic volatility, induced by the share process, which affects the riskiness short-run cash-flows.

Proposition 1. *The moment generating functions of the logarithm of aggregate consumption and market dividends are given by*

$$\mathbb{C}_t(\tau,\alpha) = \mathbb{E}_t[C^{\alpha}_{t+\tau}] = e^{\alpha \log C_t + a_0(\tau,0,\alpha,0) + a_1(\tau,0,\alpha,0)\mu_t + a_2(\tau,0,\alpha,0)\delta_t}, \tag{11}$$

$$\mathbb{D}_t(\tau,\alpha) = \mathbb{E}_t[D^{\alpha}_{t+\tau}] = e^{\alpha \log C_t + a_0(\tau,0,\alpha,\alpha) + a_1(\tau,0,\alpha,\alpha)\mu_t + a_2(\tau,0,\alpha,\alpha)\delta_t}, \tag{12}$$

$$\mathbb{CD}_t(\tau,\alpha,\beta) = \mathbb{E}_t[C^{\alpha}_{t+\tau} D^{\beta}_{t+\tau}] = e^{(\alpha+\beta) \log C_t + a_0(\tau,0,\alpha,\beta) + a_1(\tau,0,\alpha,\beta)\mu_t + a_2(\tau,0,\alpha,\beta)\delta_t}, \tag{13}$$

where $a_0(\tau,x,y,z), a_1(\tau,x,y,z)$ and $a_2(\tau,x,y,z)$ are deterministic functions of time defined in the Appendix B.

With this result in hand, the term structures of the volatility of consumption and dividends growth rates are computed in closed form as:

$$\sigma_{C,t,\tau} = \sqrt{\tfrac{1}{\tau} \log\left(\tfrac{\mathbb{C}_t(\tau,2)}{\mathbb{C}_t(\tau,1)^2}\right)}, \qquad \sigma_{D,t,\tau} = \sqrt{\tfrac{1}{\tau} \log\left(\tfrac{\mathbb{D}_t(\tau,2)}{\mathbb{D}_t(\tau,1)^2}\right)}. \tag{14}$$

The term structure of the correlation among consumption and dividend growth is given by:

$$\rho_{C,D,t,\tau} = \frac{1}{\tau\, \sigma_{C,t,\tau}\sigma_{D,t,\tau}} \log\left(\frac{\mathbb{CD}_t(\tau,1,1)}{\mathbb{C}_t(\tau,1)\mathbb{D}_t(\tau,1)}\right). \tag{15}$$

The term structure of the expected growth rates can be computed using the growth rates

$$g_{C,t,\tau} = \tfrac{1}{\tau} \log\left(\tfrac{\mathbb{C}_t(\tau,1)}{\mathbb{C}_t(0,1)}\right), \qquad g_{D,t,\tau} = \tfrac{1}{\tau} \log\left(\tfrac{\mathbb{D}_t(\tau,1)}{\mathbb{D}_t(0,1)}\right), \tag{16}$$

or the log growth rates

$$\hat{g}_{C,t,\tau} = \tfrac{1}{\tau}\left(\mathbb{E}_t[\log C_{t+\tau}] - \log C_t\right), \qquad \hat{g}_{D,t,\tau} = \tfrac{1}{\tau}\left(\mathbb{E}_t[\log D_{t+\tau}] - \log D_t\right). \tag{17}$$

In the former case a Jensen adjustment term affects the term-structures, whereas in the latter case the steady-state term-structures are flat at any horizon.

To gain intuition consider the case where instantaneous correlations are turned off. The next corollary determines the slopes of the term structures in Eq. (14) at the steady state.

Corollary 1. *When $\rho_{C,\mu} = \rho_{C,\delta} = \rho_{\mu,\delta} = 0$, $\mu_t = \bar{\mu}$ and $\delta_t = 0$, the slopes of the term structures of expected aggregate consumption and market dividends and their volatility are characterized as follows:*

$$\begin{aligned} sign\left(\partial_\tau g_{C,t,\tau}\right) &= sign\left(\partial_\tau \sigma_{C,t,\tau}\right) \\ &= sign\left(3 + \lambda\tau e^{-\lambda\tau} + (1+\lambda\tau)e^{-\lambda\tau}(e^{-\lambda\tau} - 4)\right), \end{aligned} \tag{18}$$

$$\begin{aligned} sign\left(\partial_\tau g_{D,t,\tau}\right) &= sign\left(\partial_\tau \sigma_{D,t,\tau}\right) \\ &= sign\left((e^{-2\kappa\tau}(1+2\kappa\tau) - 1)\eta^2\nu^3 + (e^{-2\lambda\tau}(1+2\lambda\tau) - 4e^{-2\lambda\tau}(1+\lambda\tau) + 3)\kappa\nu^2\right). \end{aligned} \tag{19}$$

The term structures of expected aggregate consumption and market dividends and their volatility have the following long-run limits:

$$\lim_{\tau\to\infty} g_{C,t,\tau} = \lim_{\tau\to\infty} g_{D,t,\tau} = \bar{\mu} + \frac{\nu^2}{2\lambda^2} + \frac{\sigma\nu\rho_{C,\mu}}{\lambda}, \tag{20}$$

and

$$\lim_{\tau\to\infty} \sigma_{C,t,\tau} = \lim_{\tau\to\infty} \sigma_{D,t,\tau} = \sqrt{\sigma^2 + \frac{\nu^2}{\lambda^2} + \frac{2\sigma\nu\rho_{C,\mu}}{\lambda}}. \tag{21}$$

Eq. (18) states that the term structure of aggregate consumption volatility is positive at any horizon, whereas Eq. (19) states that the term structure of dividend volatility can be upward or downward sloping depending on the horizon. Namely, λ, κ and η large enough and ν small enough can capture the negative slope at short or medium horizons –as observed in the empirical data. In other words, a downward sloping term structure can obtain if expected growth is not too persistent and too volatile (relative to the dividend-share).

Corollary 1 characterizes the long-run behavior of the term structures in Eq. (14) in terms of the model parameters. The long-run expected growth of market dividends approaches to the expected growth of consumption and is given by three terms associated respectively to the instantaneous mean of expected growth, its volatility and its correlation with consumption innovations. A similar limit result holds also for the expected log growth rates and the volatilities of both dividend and consumption.

The stationary dynamics of the dividend-share allows fluctuations of the financial and non-financial components of aggregate consumption to affect the dynamics of dividends at short and medium horizons but such an effect diminishes as long as the horizon grows and disappears in the limit. Therefore, the non-financial component of consumption –which is mainly funded by labor income– and its dynamics have a crucial role from an asset pricing perspective. On the one hand, δ_t can generate the high volatile and downward sloping term structure of dividend risk in the short run: this is the key ingredient to model in equilibrium the recent empirical evidence about the pricing of dividends as in van Binsbergen et al. (2012; 2013). On the other hand, since the effect of δ_t vanishes in the long-run, the model can preserve a long-run explanation of standard puzzles such as the smooth dynamics of the risk-free rate, the equity premium, the return excess volatility and predictability, as documented in Bansal and Yaron (2004) among others.

4. The Equilibrium

4.1 VALUE FUNCTION AND STATE PRICE DENSITY

I seek for a model solution which emphasizes the role of the two state-variables and their interaction in the dynamic formation of prices at equilibrium. For EIS $\neq 1$ it is not possible to find an exact model solution and an approximation method is necessary. Differently from the original model by Bansal and Yaron (2004) and its continuous time counterpart by Eraker and Shaliastovich (2008), I do not log-linearize the return process. Instead, similarly to Benzoni, Collin-Dufresne, and Goldstein (2011), I make an approximation around the (endogenous) steady state of the consumption-wealth ratio and provide closed form solutions for prices and return moments up to such an approximation. Recent empirical literature, such as Lustig, Van Nieuwerburgh, and Verdelhan (2013), has documented that the consumption-wealth ratio is a very smooth variable, making the solution approach not only qualitatively convenient but also empirically reasonable. Analytical solutions allow to study the role of the corporate fraction and its asset pricing implications.

The next proposition characterizes the equilibrium utility process and the state price density which has form as in Duffie and Epstein (1992): $\xi_t = \exp(\int_0^t f_J ds) f_c$.

Proposition 2. *Under preferences in Eq. (4) and dynamics in Eq. (5)-(6)-(10), the utility process is given by*

$$J(C_t, \mu_t) = \frac{1}{1-\gamma} C_t^{1-\gamma} \exp(u_0 + u_1 \mu_t), \tag{22}$$

where u_0 and u_1 are endogenous constants depending on the primitive parameters defined in the Appendix A. The consumption-wealth ratio is equal to

$$cw_t = \log C_t / W_t = \log \beta - \frac{1}{\theta}(u_0 + u_1 \mu_t). \tag{23}$$

The equilibrium state price density is given by

$$d\xi_t = -r_t \xi_t dt - \pi_C \xi_t dB_{C,t} - \pi_\mu \xi_t dB_{\mu,t} \tag{24}$$

where the risk-free rate satisfies

$$r_t = r_0 + r_1 \mu_t, \tag{25}$$

with

$$r_0 = \beta - \frac{\gamma(1+1/\psi)\sigma^2}{2} - \frac{(\gamma - 1/\psi)(1-1/\psi)\nu^2}{2(\lambda + e^{cw})^2} - \frac{\sigma\nu\rho_{C,\mu}(\gamma - 1/\psi)}{\lambda}, \tag{26}$$

$$r_1 = \frac{1}{\psi}, \tag{27}$$

and the prices of risk satisfy

$$\pi_C = \gamma\sigma, \tag{28}$$

$$\pi_\mu = \frac{\gamma - 1/\psi}{\lambda + e^{cw}} \nu. \tag{29}$$

As usual, the consumption-wealth ratio reduces to β when EIS $\to 1$ and depends negatively on the growth factor μ_t as long as $\theta < 0$ (i.e. $\gamma > 1, \psi > 1$). Coefficients u_0 and u_1 satisfy the Bellman equation evaluated with Eq. (22). The former determines the unconditional level of the consumption-wealth ratio, whereas the latter determines both the prices of risk and the growth rate of wealth.

The risk-free rate is an affine function of the growth factor μ_t and the corresponding coefficient, r_1, decreases with EIS, as usual under recursive preferences. The second term of r_0 represents precautionary savings and is monotonically decreasing with RRA: therefore, a reasonable value for risk aversion generates an offsetting mechanism which makes low the unconditional level of the risk-free rate (this is not the case under power utility $\gamma = 1/\psi$ which requires a really high γ to produce the same effect). The fourth and the fifth term of r_0 can be interpreted respectively as a second precautionary savings term and a correlation term due to uncertainty in expected growth (which disappears when $\gamma = 1/\psi$).

The prices of risk π_C and π_μ denote the expositions of the state price density to the Brownian shocks B_C and B_μ. The first, which represents transient risk, has the traditional price given by $\gamma\sigma$. Whereas the second, which represents long-run risk –that is the price that recursive preferences attach to the stochastic growth rate of consumption– is given by $\frac{\gamma-1/\psi}{1-\gamma} u_1$ times the volatility of μ_t. namely, the long-run risk is given by $-f_{C,\mu}/f_C$, where the dependence on μ_t comes out from the utility process in Eq. (22): it reduces to J_μ/J when EIS $= 1$ and disappears when $\gamma = 1/\psi$, that is when utility reduces to the power case.

Two main forces determine the magnitude of the price for long-run risk: i) the larger the persistence of expected growth, the larger its unconditional volatility and, therefore, the compensation required by the agent for bearing this risk; ii) the degree of preference for the early resolution of uncertainty, measured by $\gamma - 1/\psi$.

Notice that shock the B_δ does not command a compensation since δ_t does not enter the consumption dynamics. Section 5.2 studies the case in which consumption moments depend on S_t and, hence, fluctuations in the corporate fraction are priced under recursive utility.

4.2 EQUILIBRIUM DIVIDEND STRIPS

Given the state price density, the next proposition establishes the equilibrium price of the market dividend strip.

Proposition 3. *The equilibrium price of the market dividend strip with maturity τ is given by*

$$P_{t,\tau} = \mathbb{E}_t\left[\frac{\xi_{t+\tau}}{\xi_t} D_{t+\tau}\right] = D_t e^{b_0(\tau,1,1,1)-\bar{\delta}+b_1(\tau,1,1,1)\mu_t+(b_2(\tau,1,1,1)-1)\delta_t}, \tag{30}$$

where

$$b_1(\tau, x, y, z) = \frac{y - r_1 x}{\lambda}(1 - e^{-\lambda\tau}), \tag{31}$$

$$b_2(\tau, x, y, z) = e^{-\kappa\tau} z, \tag{32}$$

and $b_0(\tau, x, y, z)$ is a deterministic function of time defined in the Appendix B. The expected excess return and the volatility of the market dividend strip are given by:

$$\mu_{R,\tau} = \sigma\left(\pi_C + \pi_\mu \rho_{C,\mu}\right) + b_1(\tau,1,1,1)\left(\pi_C \nu \rho_{C,\mu} + \pi_\mu \nu\right) + b_2(\tau,1,1,1)\left(\pi_C \eta \rho_{C,\delta} + \pi_\mu \eta \rho_{\mu,\delta}\right), \tag{33}$$

$$\sigma_{R,\tau} = \sqrt{\begin{array}{c}\sigma^2 + b_1(\tau,1,1,1)^2\nu^2 + b_2(\tau,1,1,1)^2\eta^2 + 2\sigma b_1(\tau,1,1,1)\nu\rho_{C,\mu} \\ +2\sigma b_2(\tau,1,1,1)\eta\rho_{C,\delta} + 2b_1(\tau,1,1,1)\nu b_2(\tau,1,1,1)\eta\rho_{\mu,\delta}\end{array}}. \tag{34}$$

The price of the dividend strip relative to the current dividend value is a stationary function of the growth factor, the dividend-share and the maturity. The price is exponential affine in the state-variables and, hence, the conditional volatility is state-independent (as long as μ and δ are homoscedastic) and is a function of the maturity only.

Since the volatility channel is turned off, the dividend strip has both premium and volatility which are state-independent and only depend on the maturity. The premium has two components associated to the shocks to both consumption growth and its expected growth and other four correlation terms. In particular, the premium depends on $b_2(\tau, 1, 1, 1)$ as long as the correlations $\rho_{C,\delta}$ and $\rho_{\mu,\delta}$ are not zero. Consequently, the term structure of premia on the dividend strip is affected by the degree of persistence, κ, of the dividend-share only if these correlations are not null. Instead, the term structure of volatility of the dividend strip depends on $b_2(\tau, 1, 1, 1)$ and, hence, on κ, despite the values of the correlations. This immediately obtains since the price of the dividend strip directly depends on the dividend-share, δ_t.

Notice that the premium and the volatility in Eq. (33) and (34) depend on the maturity through the functions $b_1(\tau,1,1,1)$ and $b_2(\tau,1,1,1)$. The former is increasing (decreasing) with maturity τ if $r_1 < 1$ ($r_1 > 1$), which obtains if $\psi > 1$ ($\psi < 1$); the latter is always decreasing with the maturity and can offset the effect of the former. Therefore, even with EIS > 1 fluctuations in the dividend-share can lead to a downward sloping term structure of the volatility of dividend strip returns. This is not the case in the standard long-run risk model.

To gain intuition consider the case where correlations are turned off.[11] The next corollary determines the slopes of the term structures in Eq. (33) and (34) in terms of the model parameters.

Corollary 2. *When $\rho_{C,\mu} = \rho_{C,\delta} = \rho_{\mu,\delta} = 0$, the slopes of the term structures of expected excess return and volatility of the market dividend strip are characterized as follows:*

$$sign\,(\partial_\tau \mu_{R,t,\tau}) = sign\,((1-1/\psi)(\gamma - 1/\psi)), \tag{35}$$

$$sign\,(\partial_\tau \sigma_{R,t,\tau}) = sign\left(-e^{-2\kappa\tau}\eta^2\kappa\lambda + e^{-\lambda\tau}(1-1/\psi)^2\nu^2(1-e^{-\lambda\tau})\right). \tag{36}$$

The term structures of the expected excess return and the return volatility of the dividend strips of the aggregate consumption and of the market dividends have the following long-run limits:

$$\lim_{\tau\to\infty} \tilde{\mu}_{R,t,\tau} = \lim_{\tau\to\infty} \mu_{R,t,\tau} = \pi_C\left(\sigma + \frac{\nu\rho_{C,\mu}}{\lambda}(1-1/\psi)\right) + \pi_\mu\left(\sigma\rho_{C,\mu} + \frac{\nu}{\lambda}(1-1/\psi)\right), \tag{37}$$

and

$$\lim_{\tau\to\infty} \tilde{\sigma}_{R,t,\tau} = \lim_{\tau\to\infty} \sigma_{R,t,\tau} = \sqrt{\sigma^2 + \frac{\nu^2}{\lambda^2}(1-1/\psi)^2 + \frac{2\sigma\nu\rho_{C,\mu}}{\lambda}(1-1/\psi)}, \tag{38}$$

where $\tilde{\mu}_{R,\tau}$ and $\tilde{\sigma}_{R,\tau}$ are the counterparts of $\mu_{R,\tau}$ and $\sigma_{R,\tau}$ associated to the consumption claim $P^C_{t,\tau}$ and are defined in the Appendix B.

Under the simple assumptions about the dynamics of C and D, the slope of the term structure of premia only depends on the preference parameters since the moments of the aggregate consumption and, in turn, of the state price density are not affected by the dividend-share. In particular, the term structure of premia is upward-sloping if either i) the intertemporal substitution effect dominates the wealth effect and the agent has preference for the early resolution of uncertainty, or ii) the wealth effect dominates the intertemporal substitution effect and the agent has preference for the late resolution of uncertainty. Otherwise, the slope is negative. Therefore, in the usual case $\gamma > \psi > 1$, the term structure of premia is monotonically increasing; whereas it is monotonically decreasing when ψ is smaller than the risk tolerance γ^{-1}, provided $\gamma > 1$.[12]

[11] The general case is reported in the Appendix B.

[12] A different specification of cash-flows dynamics could instead model the dividend-share as a priced factor under recursive utility. In such a case the term structure of equity premia is not necessarily monotone and depends both on preference and cash-flows parameters as well as the horizon (see Section 5.2 for the details). Here I consider a simpler framework to provide a better intuition of the main model mechanism and to show the implications of minimal modifications of the standard long-run risk model.

The term structure of the volatility of the dividend strips is instead more flexible. The volatility of expected growth, ν and the elasticity of intertemporal substitution, ψ, unambiguously increase the slope whereas the volatility of the dividend-share, η, unambiguously reduces the slope. The degree of persistence in the two state-variables, λ and κ, instead has an effect on the slope which depends on the maturity. As long as expected growth is not too volatile and not too persistent relative to the dividend-share, the model endogenously produces a downward sloping term structure of volatility at short and medium horizons, in line with the recent empirical findings.

Corollary 2 characterizes the long-run behavior of the term structures in Eq. (33) and (34) in terms of the model parameters and in comparison with the term structures associated to the claim on the aggregate consumption, $P^C_{t,\tau} = \mathbb{E}_t[\frac{\xi_{t+\tau}}{\xi_t} C_{t+\tau}]$. The long-run expected excess return on the market dividend strip approaches to the corresponding premium on the consumption claim and is given by two terms associated respectively to both the transient and long-run prices of risk. The limit reduces to $\gamma\sigma^2$ if EIS $\to 1$. A similar limit result holds also for the return volatility of the dividend and the consumption strips: it depends on the instantaneous volatility of consumption innovations, the volatility of the expected growth and their correlation, where the last two terms disappear if EIS $\to 1$.

In a similar fashion to the dynamics of cash-flows, variations in the corporate fraction affect the price dynamics of the dividend strip at short and medium horizons but such an impact reduces as long as the horizon increases and disappears in the limit. Hence, the non-financial component of consumption has an important role for asset pricing. On the one hand, δ_t leads to the high volatile and downward sloping term structure of the dividend strips in the short-run: this is the required ingredient to capture in equilibrium the recent empirical evidence about the pricing of dividends, as in van Binsbergen et al. (2012; 2013). On the other hand, δ_t does not rule out the long-run explanation of standard puzzles, such as the smooth dynamics of the risk-free rate, the equity premium, the return excess volatility and predictability, as documented in Bansal and Yaron (2004) among others.

Option pricing on dividend strips and other derivatives on the term-structure of equity could become an important new practice of financial markets. The present model provides a general and tractable framework for the understanding of their properties in equilibrium. Indeed, the affine specification of both the state-price density and the price of the dividend strip allow to adapt, for instance, the pricing formula by Lewis (2000) and, hence, to obtain option prices up to a single numerical integration.

4.3 EQUILIBRIUM BOND AND EQUITY YIELDS

Given the state price density, the next proposition establishes the equilibrium price of risk-less zero-coupon bonds.

Proposition 4. *The equilibrium price of the zero-coupon bond with maturity τ is given by*

$$B_{t,\tau} = \mathbb{E}_t\left[\frac{\xi_{t+\tau}}{\xi_t}\right] = e^{b_0(\tau,1,0,0)+b_1(\tau,1,0,0)\mu_t+b_2(\tau,1,0,0)\delta_t}, \tag{39}$$

where and $b_0(\tau,x,y,z), b_1(\tau,x,y,z)$ and $b_2(\tau,x,y,z)$ are deterministic functions of time from Proposition 3.

Notice that $b_2(\tau,1,0,0)=0$ for any τ and, hence, the dividend-share disappears from the bond price, which is instead a function of expected growth and maturity only. The zero-coupon bond has price which is exponential affine in expected growth such that the real bond yield:

$$\varepsilon_{t,\tau}=-\frac{1}{\tau}\left(b_0(\tau,1,0,0)+b_1(\tau,1,0,0)\mu_t\right), \tag{40}$$

is linear in μ_t and its conditional volatility is a state-independent function of the maturity only. Instead, a bond which continuously pays a coupon $c(\tau)$ over a finite or infinite horizon T, $B_t=\int_0^T c(\tau)B_{t,\tau}d\tau$, leads to a nonlinear real yields and to state-dependent conditional volatilities.

Armed with these results, I turn on the equity yields as introduced by van Binsbergen, Hueskes, Koijen, and Vrugt (2013). The model equity yield is defined as

$$p_{t,\tau}=-\frac{1}{\tau}\log\left(\frac{P_{t,\tau}}{D_t}\right)=-\frac{1}{\tau}\left(b_0(\tau,1,1,1)-\bar{\delta}+b_1(\tau,1,1,1)\mu_t+(b_2(\tau,1,1,1)-1)\delta_t\right), \tag{41}$$

and can be decomposed as follows:

$$p_{t,\tau}=\varepsilon_{t,\tau}-\hat{g}_{D,t,\tau}+\varrho_{t,\tau}. \tag{42}$$

The equity yield is given by the difference among the yield on the risk-less bond, $\varepsilon_{t,\tau}$, and the dividend expected growth, $\hat{g}_{D,t,\tau}$, plus a premium $\varrho_{t,\tau}$. The latter is implicitly defined by Eq. (14)-(40)-(41) and is a state-independent function of the maturity.

To gain intuition consider the case where correlations are turned off. The next corollary characterizes the term structure of the premium on the equity yield.

Corollary 3. *When $\rho_{C,\mu}=\rho_{C,\delta}=\rho_{\mu,\delta}=0$, the premium on the equity yield is given by*

$$\varrho_{t,\tau}=m_{C,\tau}\sigma+m_{\mu,\tau}\nu+m_{\delta,\tau}\eta, \tag{43}$$

where

$$\begin{aligned}
m_{C,\tau}&=\pi_C-\frac{\sigma}{2},\\
m_{\mu,\tau}&=\pi_\mu\frac{e^{-\lambda\tau}+\lambda\tau-1}{\lambda^2\tau}+\nu\frac{\psi-2}{4\lambda^3\tau\psi}(e^{-2\lambda\tau}-4e^{-\lambda\tau}+3-2\lambda\tau),\\
m_{\delta,\tau}&=-\eta\frac{1-e^{-2\kappa\tau}}{4\kappa\tau},
\end{aligned}$$

and has limit equal to

$$\lim_{\tau\to\infty}\varrho_{t,\tau}=\pi_C\sigma-\frac{\sigma^2}{2}+\left(\frac{\pi_\mu}{\lambda}+\nu\frac{2-\psi}{2\lambda^2\psi}\right)\nu. \tag{44}$$

The premium is state-independent but moves with the maturity. The first term is constant and affine in the price of transient risk, whereas the second and the third term vary with the maturity and depend on the rates of mean-reversion of both μ_t and δ_t. The time limit of the premium on the equity yield in Eq. (44) has two terms. The first is again affine in the price of transient risk; the second term is non-monotone in the EIS. The latter term is positive and decreasing with λ for $\gamma>1/\psi>1/2$.

4.4 EQUILIBRIUM MARKET ASSET

Given the state price density, the next proposition establishes the equilibrium price of the market asset.

Proposition 5. *The equilibrium price of the market asset is given by*

$$P_t = \mathbb{E}_t\left[\int_t^\infty \frac{\xi_u}{\xi_t} D_u du\right] = D_t \int_0^\infty e^{b_0(\tau,1,1,1)-\bar{\delta}+b_1(\tau,1,1,1)\mu_t+(b_2(\tau,1,1,1)-1)\delta_t} d\tau, \tag{45}$$

where $b_0(\tau,x,y,z), b_1(\tau,x,y,z)$ and $b_2(\tau,x,y,z)$ are deterministic functions of time from Proposition 3. The excess return dynamics for the market asset is given by

$$\begin{aligned} dR_t &= P_t^{-1}(dP_t + D_t dt) - r_t dt, \\ &= \mu_{R,t}dt + \sigma_{R,C,t}dB_{C,t} + \sigma_{R,\mu,t}dB_{\mu,t} + \sigma_{R,\delta,t}dB_{\delta,t}. \end{aligned} \tag{46}$$

The instantaneous return volatility and equity premium are equal to

$$\sigma_{R,t} = \sqrt{\sigma^2_{R,C,t} + \sigma^2_{R,\mu,t} + \sigma^2_{R,\delta,t} + 2\rho_{C,\mu}\sigma_{R,C,t}\sigma_{R,\mu,t} + 2\rho_{C,\delta}\sigma_{R,C,t}\sigma_{R,\delta,t} + 2\rho_{\mu,\delta}\sigma_{R,\mu,t}\sigma_{R,\delta,t}}, \tag{47}$$

$$\mu_{R,t} = \sigma_{R,C,t}\left(\pi_C + \pi_\mu\rho_{C,\mu}\right) + \sigma_{R,\mu,t}\left(\pi_C\rho_{C,\mu} + \pi_\mu\right) + \sigma_{R,\delta,t}\left(\pi_C\rho_{C,\delta} + \pi_\mu\rho_{\mu,\delta}\right), \tag{48}$$

where

$$\sigma_{R,C,t} = \sigma, \tag{49}$$

$$\sigma_{R,\mu,t} = \nu \tfrac{\partial}{\partial\mu}\log P_t, \tag{50}$$

$$\sigma_{R,\delta,t} = \eta \tfrac{\partial}{\partial\delta}\log P_t. \tag{51}$$

Notice that the market asset price is given by the time integral of the dividend strip prices over the infinite horizon: $P_t = \int_0^\infty P_{t,\tau} d\tau$. The price-dividend ratio is a stationary function of the growth factor and the dividend-share. The price-dividend ratio is not exponentially affine in the state-variables but is highly nonlinear in μ_t and δ_t. Therefore, the conditional volatility of the price-dividend ratio is state-dependent even under homoscedasticity of fundamentals. The instantaneous return volatility in Eq. (47) has three components which are associated to the three Brownian shocks and three correlation terms. The expositions to $B_{C,t}$ and $B_{\delta,t}$ lead to transient risk, while the exposition to $B_{\mu,t}$ leads to long-run risk. Namely, $\sigma_{R,C,t}$ denotes the constant volatility of aggregate consumption, whereas $\sigma_{R,\delta,t}$ denotes the time-varying volatility due to fluctuations in the dividend-share. Instead, $\sigma_{R,\mu,t}$ captures the sensitivity of the market asset price to the expected growth factor. The terms $\sigma_{R,\mu,t}$ and $\sigma_{R,\delta,t}$ are time-varying since the model generates endogenously stochastic volatility despite the homoscedasticity of the state-variables.

The effects of μ_t and δ_t on the price-dividend ratio are the key channels driving the asset pricing implications of the market return. Notice that:

$$\frac{\partial(P_t/D_t)}{\partial\mu} = \tfrac{1-1/\psi}{\lambda}\int_0^\infty (1-e^{-\lambda\tau})\frac{P_{t,\tau}}{D_t}d\tau \qquad \text{and} \qquad \frac{\partial(P_t/D_t)}{\partial\delta} = -\int_0^\infty (1-e^{-\kappa\tau})\frac{P_{t,\tau}}{D_t}d\tau. \tag{52}$$

The price-dividend ratio moves positively with expected growth if the intertemporal substitution effect dominates the wealth effect –that is, EIS > 1– and vice-versa. The magnitude of such sensitivity depends on both the expected growth and the dividend-share.

Instead, the price-dividend ratio moves negatively with the dividend-share despite the values of preference parameters. This obtains because in the present framework aggregate consumption has dynamics independent of the dividend-share. The negative and time-varying sensitivity of P_t/D_t to δ has the following rationale. The relation between the dividend-share and asset prices operates through the cash-flows channel. Expected dividend growth negatively depends on the dividend-share: when δ is high, dividends are high but expect to decrease in the future; the opposite holds when δ is low. Therefore, high (low) dividend-share is a bad (good) news for expected dividend growth. This depresses (raises) equity prices and raises (depresses) dividend yields.

The equity premium varies with both the state-variables. If $\rho_{C,\mu} = \rho_{C,\delta} = \rho_{\mu,\delta} = 0$, the expected excess return increases with μ_t if the representative investor features preferences for the early resolution of uncertainty, $\gamma > 1/\psi$. Intuitively, an increase in expected growth induces the investors to buy more the stock, whose price rises relative to the dividends. Therefore, investors face a higher risk which requires a higher premium since the price of risk is constant.

The effect of the dividend-share on the equity premium is less obvious. As long as $\rho_{C,\mu} = \rho_{C,\delta} = \rho_{\mu,\delta} = 0$, the dividend-share affects the equity premium through the pricing of long-run risk (the second term of Eq. (48)).

Corollary 4. *When $\rho_{C,\mu} = \rho_{C,\delta} = \rho_{\mu,\delta} = 0$, the sensitivity of the equity premium on the dividend-share has sign given by*

$$sign(\partial_\delta \, \mu_{R,t}) = sign((\gamma - 1/\psi)(1 - \psi)).$$

Under the usual parametrization $\gamma > \psi > 1$, the equity premium is negatively related to the dividend-share. The rationale is as follows. A high (low) dividend-share negatively (positively) impacts expected dividend growth. This quantitatively weakens (strengthens) the sensitivity of prices with respect to long-run growth, that is the channel through which long-run risk is priced under preference for the early resolution of uncertainty. Intuitively, we have

$$\partial_\mu \mathbb{E}_t[\xi_{t+\tau} D_{t+\tau}] = (\partial_\mu \mathbb{E}_t[\xi_{t+\tau} C_{t+\tau}]) \times \mathbb{E}_t[S_{t+\tau}],$$

where the first term on the right hand side can be interpreted as the long-run risk channel, that is the sensitivity of discounted cash-flows with respect to expected growth. The second term is the dividend-share channel. The latter works as a multiplier of the former and is negatively related to the current dividend-share because of the co-integrating relationship between consumption and dividends. As a consequence, the equity premium decreases with the dividend-share.

Instead, if $\rho_{C,\delta} \neq 0, \rho_{\mu,\delta} \neq 0$, the third term in Eq. (48) has sign determined by the product of these correlations and $\sigma_{R,\delta,t}$, that is the semi-elasticity of the price-dividend ratio with respect to δ_t. The latter is negative and, hence, the equity premium increases if δ is negatively correlated with C and μ for $\gamma > 1/\psi$.

5. Discussion

5.1 QUANTITATIVE IMPLICATIONS

5.1.1 Model calibration and main results

Model parameters are set by choosing cash-flows parameters in order to match some moments from the time-series of consumption and dividends growth rates and by choosing preference parameters to provide a good fit of standard asset pricing moments.

This paper for the first time uses the information from the term-structures of cash-flows to calibrate an equilibrium asset pricing model. Namely, I exploit analytical solutions to set the cash-flows parameters. The model has seven parameters $\Theta = \{\mu, \sigma, \lambda, \nu, \bar{\delta}, \kappa, \eta\}$ which characterize the joint dynamics of consumption and dividends.[13] I choose seven empirical moments: the first two moments of consumption and dividends growth rates, the average level of the dividend-share and the variance-ratios of dividends at two and fifteen years. The latter two moments capture the short- and long-run properties of dividend risk. The model counterparts of these moments are:

$$m(\Theta) = \left\{\hat{g}_C(t,1),\, \sigma_C(t,1),\, \hat{g}_D(t,1),\, \sigma_D(t,1),\, \bar{S},\, VR_D(t,2),\, VR_D(t,15)\right\},$$

at the steady-state $\mu_t = \bar{\mu}$ and $\delta_t = 0$. The variance ratios are defined as the ratio of annualized variances over the horizon τ relative to the one year variance:

$$VR_D(t,\tau) = \frac{\sigma_D^2(t,\tau)}{\sigma_D^2(t,1)}.$$

Then, I obtain the parameter vector Θ by minimizing the root-mean-square-error (RMSE):

$$\Theta = \arg\min_{\theta} \text{RMSE}(\theta) = \arg\min_{\theta} \sqrt{\frac{1}{7}\sum_{i=1}^{7}(m_i(\theta) - m_i^{\text{empirical}})^2}\,.$$

The empirical moments are as follows: I set the expected growth rate of consumption and dividends to 2% and the volatility of consumption to 3%, which are the usual values from the literature; the volatility of dividends is set to 15%, which is the value reported in Belo, Collin-Dufresne, and Goldstein (2014); the average value of the dividend-share is set to 5%, which is the average value in Table 1 and is close to the values considered in Longstaff and Piazzesi (2004), Lettau and Ludvigson (2005) and Santos and Veronesi (2006); finally, the variance-ratios at 2 and 15 years are about 85% and 50% as in Belo, Collin-Dufresne, and Goldstein (2014).

Table 4 reports the model parameters and Table 5 reports both the empirical and the model-implied moments of cash-flows as well as the calibration errors.

[13] Correlation parameters are set to zero for the sake of simplicity and comparability with most of long-run risk literature.

Table 4 Model parameters.

		Preferences
β	$\in$[.015,.043]	time-discount rate
γ	$\{5, 7.5, 10\}$	relative risk aversion
ψ	$\{1.5, 2, 2.5\}$	elasticity of intertemporal substitution

		Cash-flows
$\bar{\mu}$	.0204	unconditional mean of consumption growth
σ	.0292	volatility of consumption growth
λ	.0637	speed of reversion of expected growth
ν	.0124	volatility of expected growth
$\exp(\bar{\delta})$	.050	unconditional dividend-share
κ	.1967	speed of reversion of log dividend-share
η	.1616	volatility of log dividend-share

A number of insights are noteworthy. First, the RMSE is about $5.7{\times}10^{-7}$, which is very small and essentially means that the model dynamics are flexible enough to capture the main properties of the empirical data. Indeed, the maximum relative error (i.e. $|m_i(\Theta) - m_i^{\text{empirical}}|/m_i^{\text{empirical}}$) is about $1.0{\times}10^{-4}$ and, hence, all the 7 moments are exactly matched.

Second, in order to further support these results I look at additional moment conditions which are related to the co-integrating relationship between consumption and dividends and their model-implied counterparts. Results are reported in Table 5. Namely, Constantinides and Ghosh (2011) document that the correlation among consumption and dividends growth rates is between 16% and 25%: the model-implied correlation is about 20% and, hence, is consistent with the data. The volatility of the dividend-share is between 0.9% and 1.8% and its first-order autocorrelation is between 0.68 and 0.86 as documented in Table 1:[14] the model-implied volatility and autocorrelation are respectively 1.35% and 0.80 and, hence, are consistent with the data.

Table 5 Cash-flows moments. Unconditional statistics of yearly moments are computed from simulations of the model. All parameters are from Table 4.

Moments used for calibration		Data	Model	Rel. error
$g_{C,1}$	consumption growth	.02	.02	$1.0{\times}10^{-4}$
$\sigma_{C,1}$	consumption volatility	.03	.03	$7.8{\times}10^{-5}$
$g_{D,1}$	dividends growth	.02	.02	$1.0{\times}10^{-4}$
$\sigma_{D,1}$	dividends volatility	.15	.15	$4.1{\times}10^{-5}$
$\bar{S}$	unconditional dividend-share	.05	.05	$7.6{\times}10^{-7}$
$\mathrm{VR}_D(2)$	2-years variance ratio of dividends	.85	.85	$4.5{\times}10^{-6}$
$\mathrm{VR}_D(15)$	15-years variance ratio of dividends	.50	.50	$8.2{\times}10^{-7}$

Implied moments		Data	Model
$\rho_{C,D,1}$	consumption and dividends correlation	(.16,.25)	.200
$\sigma_{S,1}$	dividend-share volatility	(.009,.018)	.014
$AC_{S,1}$	dividend-share autocorrelation	(.68,.86)	.803

Third, the long-run growth factor μ_t has very persistent and smooth dynamics ($\lambda = 6.37\%$ and $\kappa = 1.24\%$). These values are quite in line with most of long-run risk literature.

[14] These results are similar to those in Longstaff and Piazzesi (2004), Lettau and Ludvigson (2005) and Santos and Veronesi (2006).

Hence, the information from the term-structure of dividend risk seems to be consistent with the long-run risk channel, although the model-implied term-structure of dividend volatilities is substantially different from that implied by the standard long-run risk model. To point out this result I consider an alternative dynamics for dividends:

$$\frac{d\tilde{D}_t}{\tilde{D}_t} = \mu_t dt + \alpha\sigma dB_{C,t},$$

which can be interpreted as a levered version of consumption: the leverage parameter $\alpha = 5.15$ is set to produce the same dividend volatility of 15%. This is the usual model choice in the long-run risk literature, which disregards the mean-reverting dynamics of the dividend-share. Figure 2 shows the model implied term-structures of variance-ratios for both the two dividends dynamics. Dividends in the model (solid line) feature a U-shaped term-

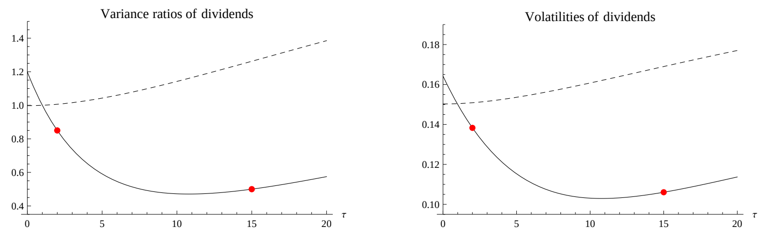

Fig. 2. The term structure of dividend risk. The left and right panels show the term structure of respectively variance-ratios and volatilities of dividends D (solid lines) and $\tilde{D}$ (dashed lines) as a function of time horizon. Markers denote the 2 and 15 years horizons. All parameters are from Table 4.

structure, which is strongly downward-sloping at short horizons and where the two marked points denote the empirical variance-ratios $VR_D(t,2)$ and $VR_D(t,15)$ from Belo, Collin-Dufresne, and Goldstein (2014). Instead, the alternative dividends dynamics (dashed line) feature a monotone increasing term-structure, which fails to match the variance-ratios from the real data. Notice that the above dynamics for $\tilde{D}$ loads the whole excess-volatility of dividends relative to consumption on the transient risk. Alternatively, I could have load the excess volatility on the dividend drift, through a levered exposition to the expected growth factor (e.g. $d\tilde{D}_t = \tilde{D}_t(\alpha\mu_t dt + \sigma dB_{C,t})$ with $\alpha > 1$). In such a case the upward-sloping term structure of variance-ratios would have been steeper and, hence, even more distant from the real data.

Fourth, the shape of the term-structure of dividend risk is the result of the combination of a downward-sloping effect due to δ_t and an upward-sloping effect due to μ_t. One issue with long-run risk models is that the main model mechanism essentially relies on a latent factor which is difficult to estimate. However, the dividend-share is observable and, hence, allows to fix the downward-sloping effect. Therefore, exploiting the information implied by the term-structure of dividend risk i) not only allows to calibrate a model consistently with additional empirical moments but ii) also offers a way to infer about the strength of the long-run risk channel. The above calibration shows that a good match of both the dynamics of the dividend-share as well as the term-structure of variance-ratios is compatible with a dynamics of μ_t, which is in line with most of long-run risk literature.

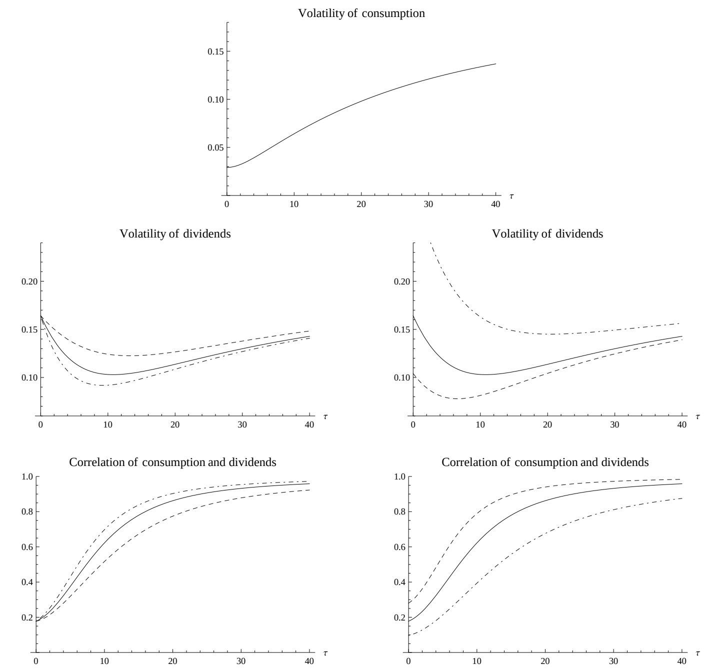

Fig. 3. The term structures of aggregate consumption and dividends. The upper panel shows the term structure of the volatility of aggregate consumption growth as a function of the time horizon. The middle panels show the term structure of the volatility of dividend growth as a function of the time horizon. The lower panels show the term structure of the correlation between aggregate consumption and dividend growth as a function of the time horizon. Solid lines denote the baseline calibration from Table 4. Dashed lines denote the case of either $\kappa = .1$ (left) or $\eta = .1$ (right) and dot-dashed lines denote the case of either $\kappa = .3$ (left) or $\eta = .3$. The state-variables are set $\mu = \bar{\mu}$ and $\delta = 0$.

Figure 3 shows $\sigma_{C,t,\tau}$ and $\sigma_{D,t,\tau}$ as functions of the maturity. The term structure of consumption volatility is monotonically upward sloping since expected growth is stochastic. Instead, $\sigma_{D,t,\tau}$ can be both increasing and decreasing with the maturity depending on the horizon. In particular, a steeper negative slope obtains in the short-run by decreasing κ or increasing η, whereas less pronounced slopes obtain for either κ or η respectively large or small enough. Figure 3 also shows $\rho_{C,D,t,\tau}$ as a function of the maturity: the correlation can be low in the short-run and monotonically increases with the horizon. A lower rate of reversion or a large volatility of δ_t reduce the rate of convergence towards one. The short-run level of correlation is decreasing with the dividend-share volatility η.

Preference parameters are set as follows. Asset pricing implications are investigated under several pairs of relative risk aversion and elasticity of intertemporal substitution, in the range of values considered in the literature: $\gamma = \{5, 7.5, 10\}$ and $\psi = \{1.5, 2, 2.5\}$. These preferences are consistent with most of long-run risk literature and imply that i) the intertemporal substitution effect dominates the wealth effect and ii) the agent has preference for the early resolution of uncertainty. For the sake of comparison, for each pair (γ, ψ), the time-discount rate β is set to match a steady-state risk-free rate of 0.8% (Constantinides and Ghosh (2011) document that the risk-free rate is in the range (0.6%,1.0%)). Results are reported in Table 6.

The model generates a sizeable equity premium of about 4.6% under the usual parametrization $\gamma = 10$ and $\psi = 1.5$. Such a result is quite remarkable since it obtains without heteroscedasticity in fundamentals. As a consequence, the model leads to a return volatility of about 14.4% which is somewhat lower than in the real data. Moreover, the model captures quite well the levels of the Sharpe ratio and of the price-dividend ratio. However, the latter is less volatile than in the real data. Similar results obtain also under lower risk aversion but higher elasticity of intertemporal substitution. For instance, the pairs $\gamma = 7.5, \psi = 2$ and $\gamma = 5, \psi = 2.5$ generate an equity premium of about 4.5%, whereas the empirical level of about 6% obtains for the pair $\gamma = 10, \psi = 2.5$. A well known shortcoming of increasing the elasticity of intertemporal substitution consists of a too low implied volatility of the risk-free rate.

Table 6 Steady-state asset pricing moments. Steady-state yearly moments for the risk-free rate (r) and its volatility (σ_r), the excess stock return (μ_R) and volatility (σ_R), the Sharpe ratio (SR) and the log price-dividend ratio ($\log P/D$) and its volatility ($\sigma_{\log P/D}$) for both the dividend claim and the consumption levered claim are compared with empirical moments from Constantinides and Ghosh (2011). All unreported parameters are from Table 4.

Data

	r	σ_r	μ_R	σ_R	SR	$\log P/D$	$\sigma_{\log P/D}$
1931-2009	.006	.030	.062	.198	.313	3.38	.450
1947-2009	.010	.027	.063	.176	.358	3.47	.429

Model

Preferences			Risk-less claim		Dividend claim					Levered consumption claim				
γ	ψ	β^*	r^*	σ_r	μ_R	σ_R	SR	$\log P/D$	$\sigma_{\log P/D}$	μ_R	σ_R	SR	$\log P/D$	$\sigma_{\log P/D}$
5	1.5	.015	.008	.009	.034	.156	.216	3.94	.153	.047	.156	.303	3.39	.040
7.5	1.5	.021	.008	.009	.043	.148	.289	3.56	.145	.063	.154	.407	3.00	.034
10	1.5	.026	.008	.009	.050	.142	.352	3.34	.139	.077	.153	.501	2.75	.030
5	2	.023	.008	.007	.042	.157	.267	3.64	.154	.055	.160	.341	3.21	.056
7.5	2	.031	.008	.007	.052	.147	.351	3.32	.144	.071	.157	.450	2.86	.047
10	2	.037	.008	.007	.060	.140	.424	3.12	.137	.085	.156	.548	2.63	.041
5	2.5	.029	.008	.005	.046	.159	.291	3.52	.156	.059	.163	.359	3.13	.064
7.5	2.5	.037	.008	.005	.056	.148	.379	3.22	.145	.075	.160	.470	2.80	.054
10	2.5	.043	.008	.005	.064	.141	.457	3.03	.138	.090	.158	.569	2.58	.048

For the sake of comparison, Table 6 also reports the moments associated to the claim on the alternative specification of dividends $\tilde{D}$, which rules out the mean-reverting dynamics of the dividend-share. Overall, the steady-state moments are quite similar: the claim on $\tilde{D}$ improves somewhat on the description of the first two moments of stock returns but is somewhat poorer in the description of the first two moments of the price-dividend ratio.

These results are interesting because they imply that the standard asset pricing moments of a long-run risk model do not deteriorate by modelling the co-integrating relationship between consumption and dividends. Therefore, those standard moments are consistent with the downward-sloping term-structures of dividend and equity risk. In a nutshell, it is possible to reconcile both the short- and long-run patterns of equity returns.

The three panels of Figure 4 report respectively the price-dividend ratio, the return volatility and the equity premium as functions of both the expected growth and the dividend-share, when instantaneous correlations are turned off and the representative agent has preferences for the early resolution of uncertainty.

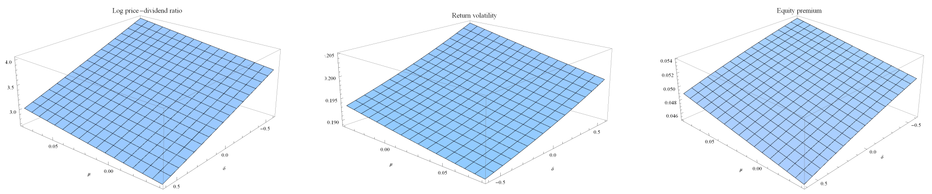

Fig. 4. The price-dividend ratio, return volatility and premium of the market asset. The left, middle and right panels show respectively the log price-dividend ratio, the return volatility and the premium of the market asset as a function of expected growth μ and log dividend-share δ. Preferences are set $\gamma = 10$ and $\psi = 1.5$. All parameters are from Table 4.

The term-structures of the returns of the dividend strips are reported in Figure 5 for the case $\gamma = 10, \psi = 1.5$. Namely, the three panels show the premia, the volatilities and

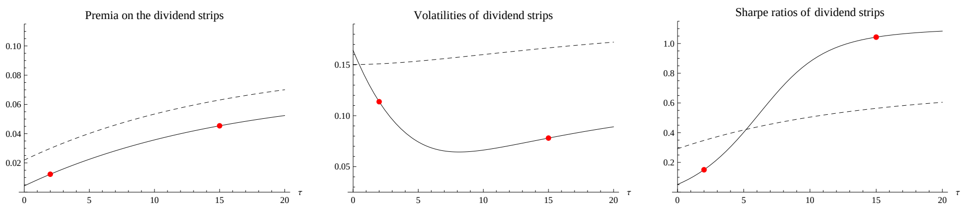

Fig. 5. The term structures of equity returns. The left, middle and right panels show the term structure of respectively premia, volatilities and Sharpe ratios of dividend strip returns associated to D (solid lines) and $\tilde{D}$ (dashed lines) as a function of time horizon. Markers denote the 2 and 15 years horizons. Preference parameters are: $\gamma = 10$ and $\psi = 1.5$. All unreported parameters are from Table 4.

the Sharpe ratios of the dividend strips for both the two dynamics of dividends D and $\tilde{D}$. Under preference for the early resolution of uncertainty, premia are upward-sloping. Since fluctuations in the dividend-share are not priced, the slope of the term-structures of premia associated to D and $\tilde{D}$ are essentially equal and the only difference concerns the level of premia. Instead, the term-structure of return volatilities are quite different: the co-integrating relationship between consumption and dividends leads to downward-sloping equity risk for about ten years and then a slightly positive slope. Instead, the alternative dynamics of dividends implies upward-sloping equity risk at any horizon. As a consequence, the term-structure of Sharpe ratios is increasing for both D and $\tilde{D}$: the co-integrating relationship between consumption and dividends leads to a steeper slope up to about ten years.

Figure 6 reports the term-structure of equity risk associated to D for three pairs of preference parameters: $\gamma = 10, \psi = 1.5$, $\gamma = 7.5, \psi = 2$ and $\gamma = 5, \psi = 2.5$. As commented above, these pairs lead to similar implications for the standard asset pricing moments. However, these pairs imply different combinations of aversion for transient risk (captured by γ) and for long-run risk (captured by $\gamma - 1/\psi$). Figure 6 shows that the term-structures of equity risk are essentially equal up to about 5 years and then diverge at longer horizons. The interpretation is that the three combinations of aversion for transient and long-run risks compensate each other in the short-run. Instead, the degree of preference for the resolution of uncertainty ($\gamma - 1/\psi$) dominates the determination of the slope of equity risk in the long-run. The larger ψ, the steeper the positive long-run slope of equity risk. Notice that all the above combinations are consistent with a dynamics of dividends which nicely fits the term-structure of dividend risk. Therefore, the correct specification of downward-sloping dividend risk is not inconsistent with the idea that long-run risk of fundamentals is a main determinant of equity risk.

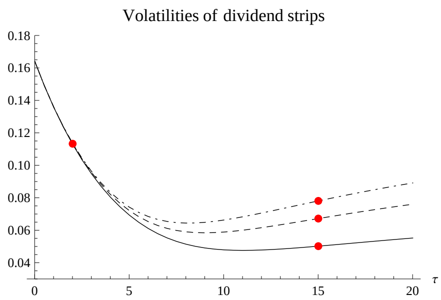

Fig. 6. Preferences and the term structures of equity returns. The figure shows the term structure of volatilities of dividend strip returns associated to D as a function of time horizon. Markers denote the 2 and 15 years horizons. Preference parameters are: $\gamma = 10$, $\psi = 1.5$ (solid line), $\gamma = 7.5$, $\psi = 2$ (dashed line) and $\gamma = 5$, $\psi = 2.5$ (dotted line),. All unreported parameters are from Table 4.

The upper panels of Figure 7 show that $\sigma_{R,t,\tau}$ can be decreasing with the horizon in the short-run and then upward sloping for τ large enough. The level of the volatility is positively and negatively related with η and κ, but the larger the speed of reversion of the dividend-share, the larger the horizon at which the volatility reaches its minimum. The lower panels of Figure 7 show $\mu_{R,t,\tau}$ as a function of the maturity. The term structure of the premium on the dividend strip is upward sloping if EIS > 1 and $\rho_{C,\delta} = \rho_{\mu,\delta} = 0$. However, in such a case reversion and volatility of the dividend-share do not affect the premium.

To assess whether the model quantitatively captures the declining volatilities of equity yields, I compare the empirical evidence from van Binsbergen, Hueskes, Koijen, and Vrugt (2013) with the model counterparts under various preference settings. Namely, van Binsbergen, Hueskes, Koijen, and Vrugt (2013) document that the volatilities of US forward equity yields decrease with the maturity in the range 10%-3% from one to seven years.

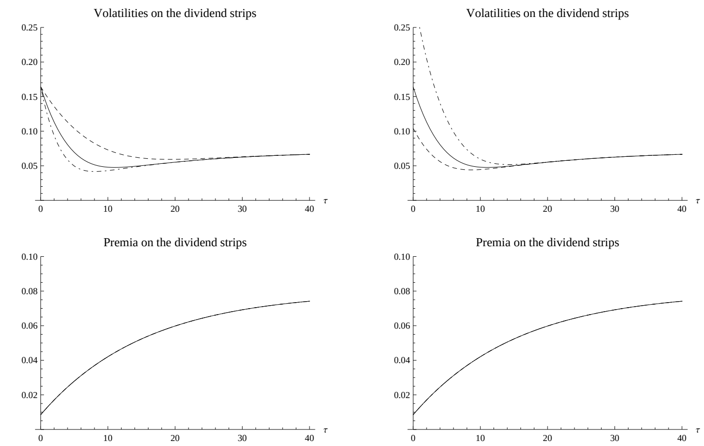

Fig. 7. The term structures of dividend strip returns. The upper panels show the term structure of the dividend strip return volatility as a function of the time horizon. The lower panels show the term structure of the dividend strip return premia as a function of the time horizon. Solid lines denote the baseline calibration from Table 4. Dashed lines denote the case of either $\kappa = .1$ (left) or $\eta = .1$ (right) and dot-dashed lines denote the case of either $\kappa = .3$ (left) or $\eta = .3$. The state-variables are set $\mu = \bar{\mu}$ and $\delta = 0$. Preferences are set $\gamma = 10$ and $\psi = 1.5$.

The model forward equity yield is defined as $p_{t,\tau} - \varepsilon_{t,\tau}$ or, equivalently, as $\varrho_{t,\tau} - \hat{g}_{D,t,\tau}$ (see Eq. (42)). Results are reported in Table 7.

The baseline model calibration captures the declining pattern of volatilities as well as the long-run level, whereas it underestimates the short-run level. This can be eventually due to the simplifying assumption of homoscedastic state-variables.

Table 7 The volatility of forward equity yields. Unconditional yearly volatilities of forward equity yields with maturity τ (in years) are computed from simulations of the model. All parameters are from Table 4. Empirical data are from van Binsbergen, Hueskes, Koijen, and Vrugt (2013).

τ	1	2	3	4	5	6	7
Data	.102	.080	.057	.047	.040	.035	.032
Model	.057	.053	.049	.047	.044	.041	.039

The left upper panel of Figure 8 shows the equity yield as a function of the maturity when μ_t and δ_t are set at the steady-state. The other three panels report the three components of the equity yield: namely, the real yield of the bond, the growth rate of dividends and the premium on the equity yield. In the long-run, the upward-sloping equity yield is due entirely to the premium since the real yield decreases with the horizon.

Figure 9 shows the equity yield as a function of the dividend-share and the maturity. The term structure approaches to a flat long-run limit from above or from below depend-

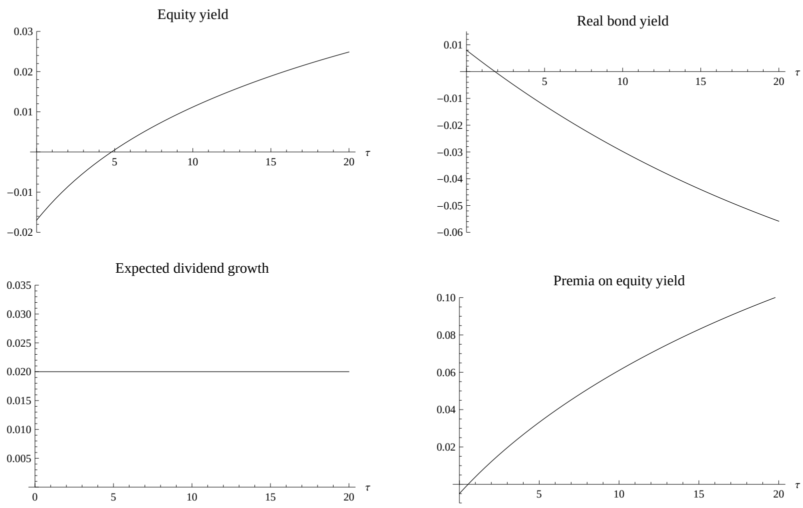

Fig. 8. The term structure of the equity yield and its components. The term structure of the equity yield $p_{t,\tau}$ (upper left panel), of the bond yield $\varepsilon_{t,\tau}$ (upper right panel), of the dividend growth $g_{D,t,\tau}$ (lower left panel) and of the premium on the equity yield $\varrho_{t,\tau}$ (lower right panel) are plotted as functions of the maturity τ. The state-variables are set $\mu = \bar{\mu}$ and $\delta = 0$. Preferences are set $\gamma = 10$ and $\psi = 1.5$. All parameters are from Table 4.

ing if the current dividend-share is respectively above or below its steady-state. Therefore,

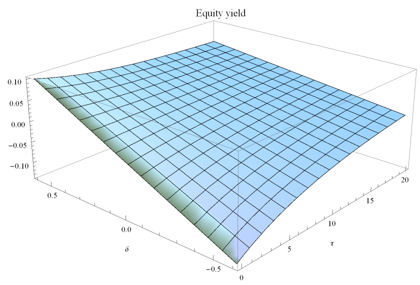

Fig. 9. The dynamic term structure of the equity yield. The term structure of the equity yield $p_{t,\tau}$ is plotted as a function of the maturity τ and of the log dividend-share δ. The state-variables are set $\mu = \bar{\mu}$ and $\delta = 0$. Preferences are set $\gamma = 10$ and $\psi = 1.5$. All parameters are from Table 4.

the equity yield moves positively with the dividend-share and, hence, moves negatively with the price-dividend ratio of the market asset (if $\gamma > \psi > 1$). Intuitively, the larger the fraction of aggregate consumption funded by dividends, the lower the expected growth of dividends, given the stationary dynamics of the dividend-share. This bad cash-flows news pushes down prices relative to dividends and, hence, increases equity yields. Such a countercyclical behavior of the equity yield is strong at short horizons whereas it disappears at longer ones. Consequently, the slope of the term structure of equity yields is countercyclical: that is, the term structure is upward sloping when the market price-dividend ratio is high and, vice-versa, it is downward sloping when the market price-dividend ratio is low. Then such an equilibrium dynamics of the term structure seems to be consistent with the recent empirical findings in van Binsbergen, Hueskes, Koijen, and Vrugt (2013).

5.1.2 Additional testable implications

In order to better understand the properties of the model, I focus on three additional testable implications: i) the predictability of dividend growth; ii) the variance decomposition of equity yields; iii) the term-structure of conditional CAPM betas.

Dividend growth predictability. I study the long-horizon predictability of dividend growth and cumulative excess returns by the price-dividend ratio, the equity yields as well as the state-variables μ_t and δ_t. Table 8 reports the results.

Prices predict both dividend and consumption growth: such an effect is likely larger than in the real data but this is a well known shortcoming of long-run risk models. Moreover, the price-dividend ratio negatively covaries with cumulative excess returns but statistical significance and explanatory power are very limited. This is likely due to, first, an offsetting mechanism between the two sources of predictability μ_t and δ_t and, second, state-independent prices of risk, due to the simplicity of the framework. In line with the empirical evidence by van Binsbergen, Hueskes, Koijen, and Vrugt (2013), the equity yields with both short (one year) and long (ten years) maturity predict both dividend and consumption growth rates: the explanatory power does not change substantially with the maturity of the yield. The expected growth factor μ_t predicts positively consumption and dividend growth but, similarly to the price dividend ratio, its correlation with excess returns is not statistically significant.

The role of δ_t is crucial to the model implications: since prices move with δ_t, we expect that the corporate fraction would predict either returns or dividends growth rates or both. Namely, the dividend-share δ_t barely predicts excess returns and strongly predicts dividend growth rates with negative sign, whereas by construction it cannot predict consumption growth rates. The negative relation among the corporate fraction and future dividend growth is consistent with the empirical findings in Section 2. and Table 2 and 3.

Notice that the cointegration among consumption and dividends mechanically leads to a source of dividend growth predictability. However the model captures the fact that, in line with the data, such a source of predictability is not offset by other sources of predictability, such as time-variation in long-run growth. Notice that the model calibration exploits information from the term-structure of dividend variance ratios. Therefore, this result provides further support to the idea that the cointegration among consumption and dividends is closely related to the timing of dividend risk.

Table 8 Long horizon regressions. Coefficients, t-statistics and R^2 from OLS regressions of one to ten years cumulative excess returns ($R - r_f$), dividends ($\Delta \log D$) and consumption ($\Delta \log C$) growth rates on respectively the logarithm of the price-dividend ratio ($\log P/D$), the equity yields with one (p_1) and ten (p_{10}) years maturity, the logarithm of the dividend share (δ) and the expected growth (μ). Regressions are based on one thousand simulations each one accounting for one hundred years of data. Model parameters are from Table 4.

$\log P/D$	$R-r_f$					$\Delta \log D$					$\Delta \log C$				
	1	2	3	5	7	1	2	3	5	7	1	2	3	5	7
Coeff	-0.01	-0.01	-0.02	-0.02	-0.02	0.27	0.50	0.68	0.95	1.29	0.05	0.09	0.12	0.18	0.27
t-stat	-0.33	-0.38	-0.41	-0.42	-0.25	3.73	5.28	6.42	7.91	8.73	2.63	2.99	3.06	2.97	2.43
R^2 (%)	1.19	2.15	2.94	3.94	5.53	12.70	22.56	30.03	39.23	44.22	9.31	11.76	12.60	12.92	11.51
p_1	$R-r_f$					$\Delta \log D$					$\Delta \log C$				
	1	2	3	5	7	1	2	3	5	7	1	2	3	5	7
Coeff	0.02	0.03	0.04	0.05	0.02	-1.26	-2.28	-3.10	-4.30	-5.77	-0.11	-0.21	-0.29	-0.43	-0.63
t-stat	0.24	0.25	0.26	0.23	0.00	-3.56	-4.99	-6.00	-7.26	-7.84	-1.28	-1.44	-1.47	-1.42	-1.15
R^2 (%)	1.12	2.01	2.73	3.62	5.17	11.78	20.76	27.42	35.43	39.28	5.10	6.71	7.43	8.17	8.34
p_{10}	$R-r_f$					$\Delta \log D$					$\Delta \log C$				
	1	2	3	5	7	1	2	3	5	7	1	2	3	5	7
Coeff	0.06	0.09	0.12	0.17	0.14	-2.66	-4.84	-6.62	-9.22	-12.48	-0.36	-0.68	-0.97	-1.44	-2.14
t-stat	0.30	0.33	0.35	0.34	0.15	-3.68	-5.19	-6.29	-7.69	-8.41	-2.06	-2.34	-2.39	-2.32	-1.90
R^2 (%)	1.16	2.09	2.85	3.80	5.35	12.43	22.01	29.20	37.96	42.46	7.25	9.29	10.06	10.58	9.92
δ	$R-r_f$					$\Delta \log D$					$\Delta \log C$				
	1	2	3	5	7	1	2	3	5	7	1	2	3	5	7
Coeff	0.00	0.00	0.00	-0.00	-0.02	-0.22	-0.39	-0.53	-0.72	-0.95	0.00	0.00	0.00	0.01	0.02
t-stat	0.15	0.12	0.10	0.03	-0.25	-3.24	-4.47	-5.31	-6.28	-6.63	0.07	0.09	0.10	0.11	0.13
R^2 (%)	1.03	1.86	2.52	3.38	5.08	10.08	17.61	23.07	29.52	32.45	3.70	5.07	5.79	6.75	7.61
μ	$R-r_f$					$\Delta \log D$					$\Delta \log C$				
	1	2	3	5	7	1	2	3	5	7	1	2	3	5	7
Coeff	-0.14	-0.27	-0.38	-0.59	-0.96	0.95	1.83	2.61	3.88	5.91	0.95	1.79	2.53	3.76	5.67
t-stat	-0.57	-0.78	-0.92	-1.14	-1.46	1.65	2.35	2.86	3.54	4.27	8.79	11.09	11.77	11.24	8.25
R^2 (%)	1.40	2.60	3.72	5.65	9.28	3.31	6.35	9.12	13.39	18.88	42.84	53.37	55.86	53.66	40.77

Variance decomposition. The variance decomposition of the price-dividend ratio of the market asset is a well known approach to understand the sources of variation of prices. In the model, the price-dividend ratio covaries more with the expected dividend growth than with the discount rates. The recent empirical evidence is quite controversial because of empirical issues in the proper measurement of shareholders' remunerations.

A similar variance decomposition can also be used to understand the variation in the equity yields, $p_{t,\tau}$. Namely, for any τ, I look at the relative role of expected returns and expected dividend growth:

$$\frac{\operatorname{cov}(p_{t,\tau}, \varepsilon_{t,\tau} + \varrho_{t,\tau})}{\operatorname{var}(p_{t,\tau})}, \qquad \text{and} \qquad -\frac{\operatorname{cov}(p_{t,\tau}, g_{D,t,\tau})}{\operatorname{var}(p_{t,\tau})}.$$

Table 9 shows the the two above quantities implied by the model for the maturity 2, 5 and 10 years. Similarly to the case of the price-dividend ratio of the market asset, expected dividend growth contributes more than discount rates to the variance of the equity yields. Moreover such an effect is increasing with the horizon. The term-structure of the variance decomposition of equity yields is barely insensitive to different specifications of the pair (γ, ψ): changing the importance of long-run risk relative to transient risk does not alter the relative contribution of cash-flows and discount rates news about the variation of equity yields.

Table 9 Variance decomposition of equity yields. The table reports the variance decomposition of 2, 5 and 10 years equity yields by means of cash-flows news (CF) and discount rates news (DR), computed as respectively $-\frac{\text{cov}(p_{t,\tau}, g_{D,t,\tau})}{\text{var}(p_{t,\tau})}$ and $\frac{\text{cov}(p_{t,\tau}, \varepsilon_{t,\tau}+\varrho_{t,\tau})}{\text{var}(p_{t,\tau})}$. All unreported parameters are from Table 4. Empirical data are from van Binsbergen, Hueskes, Koijen, and Vrugt (2013).

Data

	US		EU		Japan	
τ	CF	DR	CF	DR	CF	DR
2	.80	.20	.73	.27	.62	.38
5	.73	.27	.78	.22	.78	.21

Model

	$\gamma = 10, \psi = 1.5$		$\gamma = 7.5, \psi = 2$		$\gamma = 5, \psi = 2.5$	
τ	CF	DR	CF	DR	CF	DR
2	1.10	-.10	1.10	-.10	1.09	-.09
5	1.13	-.13	1.13	-.13	1.12	-.12
10	1.21	-.21	1.20	-.20	1.17	-.17

These results are in line with the empirical findings of van Binsbergen, Hueskes, Koijen, and Vrugt (2013), reported in Table 9. They provide international evidence about the variance decomposition of equity yields and document that: i) news about expected dividend growth dominate those about expected returns; ii) in two regions (EU and Japan) out of three (US) the distance in the contribution of the two components increases with the maturity. The model accounts for both these stylized facts, but produces somewhat more extreme numbers.

Term-structure of betas. It is worth noting how premia on the dividend strips relate with the premium on the market asset. The CAPM does not hold in general in the economy under analysis. Namely, the CAPM would hold if preferences reduce to the logarithmic utility and the wealth portfolio (i.e. the claim on aggregate consumption) substitutes for the market asset. However, I test for a conditional CAPM in spirit of van Binsbergen, Hueskes, Koijen, and Vrugt (2013). They show that the coefficient of market excess returns is positive, lower than one and increasing with the maturity of the dividend strip. Moreover, the coefficient of market excess returns multiplied by a measure of economic conditions implies countercyclical conditional betas.

To verify whether such stylized facts also hold within the model, I run the following regression from simulated data:

$$rx_{t,\tau} \;=\; \alpha_\tau \,+\, \beta_{0,\tau}\, rx_t \,+\, \beta_{1,\tau}\, rx_t \times D/P_{t-1} \,+\, \epsilon_t \tag{53}$$

where $rx_{t,\tau}$ and rx_t are the excess returns respectively on the dividend strip with maturity τ and on the market asset. The lagged dividend-to-price ratio of the market asset is used as a measure of economic conditions.

I verify the model implications under the following pairs of preference parameters: $\gamma = 10$, $\psi = 1.5$, $\gamma = 7.5$, $\psi = 2$ and $\gamma = 5$, $\psi = 2.5$. This allows to infer about the role of both transient and long-run risks. Results are reported in Table 10.

Table 10 The term-structure of conditional CAPM beta. Coefficients and t-statistics from OLS regressions of dividend strip excess returns with 2, 5 and 10 years maturity on the market asset excess return and on the market asset excess return multiplied by the lagged dividend yield, as in Eq. (53). Regressions are based on one thousand simulations each one accounting for one hundred years of data. All unreported parameters are from Table 4.

	$\gamma = 10, \psi = 1.5$		$\gamma = 7.5, \psi = 2$		$\gamma = 5, \psi = 2.5$	
τ	$\beta_{0,\tau}$ (t)	$\beta_{1,\tau}$ (t)	$\beta_{0,\tau}$ (t)	$\beta_{1,\tau}$ (t)	$\beta_{0,\tau}$ (t)	$\beta_{1,\tau}$ (t)
2	.47	22.18	.44	13.41	.28	13.12
	(.75)	(1.16)	(.84)	(.90)	(.65)	(.84)
5	.67	9.46	.61	6.23	.49	6.89
	(2.25)	(1.09)	(2.53)	(.91)	(2.28)	(.92)
10	.91	-1.05	.84	.37	.75	2.07
	(20.50)	(-.90)	(21.81)	(.35)	(18.54)	(1.56)

Three results are noteworthy. First, short equity claims covary with the market asset less than long equity claims. Indeed, the coefficient $\beta_{0,\tau}$ is positive, lower than unity and increasing with the horizon under all the pairs of preference parameters. These properties of the betas are in line with the real data. In particular, van Binsbergen, Hueskes, Koijen, and Vrugt (2013) document coefficients ranging between .41 and .48 for $\tau = 2$ and between .60 and .81 for $\tau = 5$. These numbers are quite well matched by the model for $\gamma = 10$ and $\psi = 1.5$.

Second, equity claims covary with the market asset more in bad times than in good times. Indeed, the coefficient $\beta_{1,\tau}$ is positive, implying a countercyclical dynamics of conditional betas. Also this result is consistent with the empirical findings of van Binsbergen, Hueskes, Koijen, and Vrugt (2013).[15]

Third, the three pairs of preference parameters show that the positive slope of the term-structure of beta is not very sensitive to the importance of long-run risk relative to transient risk. Instead, the stronger the long-run risk channel, the lower the level of the term-structure. Namely, shifting preferences from $\gamma = 10$, $\psi = 1.5$ to $\gamma = 5$, $\psi = 2.5$ leads to substantially smaller coefficients $\beta_{0,\tau}$ for both short and long equity claims. Therefore, the long-run risk channel seems to be consistent with the stylized facts documented by van Binsbergen, Hueskes, Koijen, and Vrugt (2013). The larger the compensation that the agent requires for variation in expected long-run growth, the lower the covariance between

[15] van Binsbergen, Hueskes, Koijen, and Vrugt (2013) use the lagged 2-years equity yield as a (countercyclical) measure of economic conditions: such a choice appears somewhat unconventional and I substitute it with the more usual market dividend-to-price ratio.

the market asset and short equity claims. Indeed, the latter are similar to dividends and, hence, have dynamics dominated by the short-run fluctuations of the dividend-share.

5.2 PRICING CORPORATE FRACTION FLUCTUATIONS

The cash-flows specification in Eq. (5)-(7)-(8) is somewhat simplistic but allows for a tractable characterization of all important formulas about dividends, dividend strips and the market asset. However, on the one hand, from a general equilibrium perspective it is difficult to imagine why the dividend-share dynamics could be independent from the consumption dynamics; on the other hand, such a dynamics of the dividend-share seems at odds with either downward-sloping premia on the dividend strips or a high equity premium on the market asset.

This section investigates the equilibrium implications of a different specification of the cash-flows. Namely, the dividend-share is defined in such a way that it positively affects both consumption and dividends dynamics. This is consistent with the real data: the fraction of total resources devoted to shareholders' remuneration is procyclical. The main implications are as follows: i) the term structures of volatility of both consumption and dividends are not necessarily monotone increasing with the horizon; ii) both consumption and dividends growth rates feature stochastic volatility, due to fluctuations in the dividend-share; iii) since the dividend-share affects consumption dynamics, it also enters the dynamics of the state-price density and, hence, its innovations command a price of risk due to recursive utility.

Assume to directly model the non-financial components of aggregate consumption $L = C - D$ as follows:

$$dL_t = \mu_t L_t dt + \sigma_L L_t dB_{L,t}, \tag{54}$$

where μ_t has dynamics as in Eq. (6). Then, define the dividend-share as given by

$$S_t = \frac{D_t}{C_t} = 1 - e^{-\ell_t}, \tag{55}$$

where

$$d\ell_t = \kappa(\bar{\ell} - \ell_t)dt + \eta\sqrt{\ell_t} dB_{\ell,t}, \tag{56}$$

such that

$$\frac{dC_t}{C_t} = (\mu_t - (\kappa - \eta^2/2)\ell_t + \kappa\bar{\ell})dt + \sigma_L dB_{L,t} + \eta\sqrt{\ell_t} dB_{\ell,t}. \tag{57}$$

As long as $\bar{\ell}, \ell_0 > 0$, then $C > L$ and both the instantaneous drift and volatility of aggregate consumption move with ℓ_t. All Brownian motions are assumed to be independent for the sake of exposition. Consumption $C_t = L_t e^{\ell_t}$ is given by the product of an integrated process L_t and a stationary one e^{ℓ_t}: such a dynamics is consistent with Bansal, Kiku, and Yaron (2010), but here the cyclical component ℓ_t also governs the cointegrating relationship between consumption and dividends and their stochastic volatility.

On a technical side, notice that consumption dynamics belongs to the affine class and, hence, a solution methodology similar to that of the previous sections is still available.[16] Instead, dividends dynamics is not any more in the affine class but dividends can be

[16] All the results of this Section are derived in Appendix C.

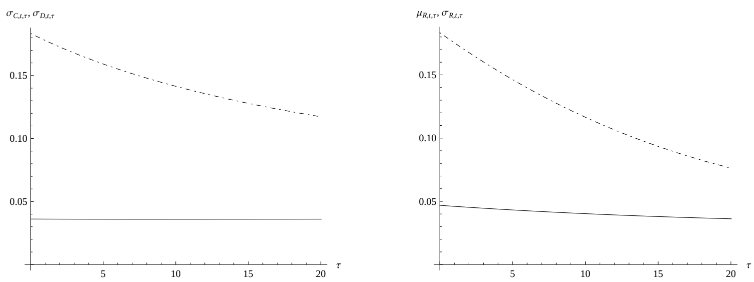

Fig. 10. The term structures of cash-flows and equity. The left panel shows the term structure of the volatility of aggregate consumption (solid line) and dividends (dashed line) growth as a function of the horizon. The right panel shows the term structure of dividend strip return premia (solid line) and volatility (dashed line) as a function of the horizon. The model is as in Section 5.2 and parameters are: $\bar{\mu} = .01$, $\sigma_L = .035$, $\lambda = .10$, $\nu = .001$, $\bar{\ell} = -2.996$, $\kappa = .05$, $\eta = .04$, $\beta = .035$, $\gamma = 25$ and $\psi = .75$. The implied steady-state risk-free rate and log price-dividend ratio are $r_t = 1\%$ and $\log P_t/D_t = 3.41$.

written as the difference among two exponential affine terms: $D_t = L_t(e^{\ell_t} - 1)$. Such a specification has two implications. On the one hand, closed form expectations can be easily computed. On the other hand, the price of the dividend strip is not an exponential affine functional of the states. Therefore, premium and return volatility on the dividend strips depend on both the states and the maturity, whereas they are state-independent in the model of the previous sections.

Figure 10 shows the term structures implied by the model. The left panel displays the volatility of consumption and dividends. Consumption risk is barely flat instead of monotone increasing with the horizon, as in the data (see Figure 3). Indeed, long-run risk due to fluctuations in μ_t is offset by short-term risk due to fluctuations in the corporate fraction, driven by ℓ_t. The term structure of dividend risk features a negative slope and higher short-term risk, as suggested by the empirical evidence (see Figure 3).

In equilibrium, the logarithm of the consumption-wealth ratio is affine in the states μ_t and ℓ_t and the state-price density has dynamics:

$$d\xi_t = -\xi_t(r_0 + r_1\mu_t + r_2\ell_t)dt - \xi_t(\pi_L dB_{L,t} + \pi_\mu dB_{\mu,t} + \pi_\ell \sqrt{\ell} dB_{\ell,t}). \tag{58}$$

Notice that the factor ℓ_t enters the risk-free rate and that a price of risk is associated to its innovations $dB_{\ell,t}$. In particular, the prices of risk are given by:

$$\pi_L = \gamma\sigma_L, \tag{59}$$

$$\pi_\mu = \frac{\nu(\gamma - 1/\psi)}{\lambda + e^{cw}}, \tag{60}$$

$$\pi_\ell = \frac{\eta}{\psi} + \frac{\gamma - 1/\psi}{\eta(1-\gamma)}\left(e^{cw} + \kappa - \sqrt{(e^{cw} + \kappa)^2 + 2e^{cw}\eta^2(\gamma - 1)}\right). \tag{61}$$

The first two prices of risk exactly resemble their counterparts in the model of the previous sections. Instead, the price of risk $\pi_\ell\sqrt{\ell_t}$ has the following interpretation. The first term on the right hand side of Eq. (61), η/ψ, is the price of transient risk: that is, it is the price of the contribution of ℓ_t to the instantaneous volatility of consumption. The second term on the right hand side of Eq. (61) is the price of non-transient risk: that is, it is the price for the effect of fluctuations in the corporate fraction on the continuation utility value of the representative agent. Such a type of risk is absent in the model of the previous sections and disappears in the power utility case ($\psi \to \gamma^{-1}$). Notice that the price of transient risk is decreasing with the EIS, whereas the price of non-transient risk is increasing with the EIS as well as with the degree of early resolution of uncertainty, provided $\gamma > 1$.

The price of the dividend strip (relative to the current dividend value) as well as its premium and return volatility are stationary functions of the states μ_t, ℓ_t and the maturity only:

$$P_{t,\tau} = h_0(\mu_t, \ell_t, \tau) D_t, \tag{62}$$

$$\mu_{R,t,\tau} = h_1(\mu_t, \ell_t, \tau), \tag{63}$$

$$\sigma_{R,t,\tau} = h_2(\mu_t, \ell_t, \tau), \tag{64}$$

where h_0, h_1 and h_2 are derived in closed form in the Appendix C.

The right panel of Figure 10 shows the term structures of the premium and the return volatility of the dividend strips. They are both decreasing with the horizon and roughly capture the magnitude of their empirical counterparts. It is worth noting how this result obtains and how the model differs from the standard long-run risk model. The price of risk associated to expected growth, π_μ, allows to price the risk at long horizons through a large degree of early resolution of uncertainty. Hence, the term structure of equity premia is upward sloping when $\psi > 1$ and μ_t is the only priced factor at equilibrium. This is the mechanism driving the equity premium in long-run risk literature as well as in the model of the previous sections. Instead, the current model commands a high premium in equilibrium either by pricing long-horizon cash-flows through fluctuations in μ_t and high EIS or by pricing short-horizon cash-flows through fluctuations in the corporate fraction, driven by ℓ_t, and low EIS. The latter case also leads to term structures of both equity premia and risk which are downward sloping at short and middle horizons and potentially increasing in the long-run, depending on the model parameters. The right panel of Figure 10 captures negative slopes because of a low EIS ($\psi < 1$). In the current model, the corporate fraction fluctuations alter both the covariance between consumption and dividends as well as conditional consumption moments and, in turn, the equilibrium discount rates.

Similarly to the dividend strips, the price of the market asset (relative to the current dividend value) as well as its premium and return volatility are stationary functions of the states μ_t and ℓ_t:

$$P_t = w_0(\mu_t, \ell_t) D_t, \tag{65}$$

$$\mu_{R,t} = w_1(\mu_t, \ell_t), \tag{66}$$

$$\sigma_{R,t} = w_2(\mu_t, \ell_t), \tag{67}$$

where w_0, w_1 and w_2 are derived in closed form in the Appendix C. Market returns feature stochastic volatility which is partly exogenous (due to the effect of ℓ_t on cash-flows dynamics) and partly endogenous (due to the pricing at equilibrium of both μ_t and ℓ_t). Such a form of stochastic volatility, due to ℓ_t, differs from the usual channel of economic uncertainty included in most of long-run risk literature. Indeed, here, time-varying volatility leads to uncertainty which disappears with the horizon, whereas in standard long-run risk models it integrates over the horizon, generating highly risky long-term cash-flows.

6. Conclusion

This paper shows that a one-channel long-run risk model can accommodate for both short-term and long-term patterns of equity returns as long as dividends are modelled as a stochastic fraction of aggregate consumption. The model leads to a good fit of standard asset pricing moments together with downward sloping term-structures of volatility of both dividends and dividend strip returns at short and medium horizons. Moreover, such results obtain in absence of stochastic volatility in fundamentals and under standard preferences.

An empirical analysis supports the main model mechanism and the close connection between the dividend-share dynamics, the timing of dividend risk and the predictability of dividend growth. The model calibration exploits the information from the term-structure of dividend risk and helps to infer about the strength of the long-run risk channel, which is consistent with most of long-run risk literature.

Analytic tractability and the possibility of extending the present framework to the jump-diffusion affine class allow to develop the model in a number of directions. Dividend strips are receiving growing attention by on-going empirical research: a general and tractable equilibrium framework could provide the starting point for a deep understanding of such new results.

A Value function and state price density

Proof of Proposition 2: Under the infinite horizon, the utility process J satisfies the following Bellman equation:

$$\mathcal{D}J(C,\mu) + f(C,J) = 0, \tag{A1}$$

where $\mathcal{D}$ denotes the differential operator. The Bellman equation can be written as

$$J_C \mu C + \tfrac{1}{2} J_{C,C}\sigma^2 C^2 + J_\mu \lambda(\bar{\mu}-\mu) + \tfrac{1}{2} J_{\mu,\mu}\nu^2 + J_{C,\mu}\sigma C \nu \rho_{C,\mu} + f(C,J) = 0. \tag{A2}$$

Guess a solution of the form $J(C,\mu) = \frac{1}{1-\gamma} C^{1-\gamma} g(\mu)$. The Bellman equation reduces to

$$\mu - \tfrac{\gamma}{2}\sigma^2 + \tfrac{g_\mu}{g}\tfrac{\lambda(\bar{\mu}-\mu)}{1-\gamma} + \tfrac{g_{\mu,\mu}}{g}\tfrac{\nu^2}{2(1-\gamma)} + \tfrac{g_\mu}{g}\sigma\nu\rho_{C,\mu} + \tfrac{\beta}{1-1/\psi}\left(g(\mu)^{-1/\theta} - 1\right) = 0. \tag{A3}$$

The pricing kernel for stochastic differential utility can be written as

$$d\xi = \xi \tfrac{df_C}{f_C} + \xi f_J dt = -r\xi - \pi_C \xi dB_C - \pi_\mu \xi dB_\mu, \tag{A4}$$

where, by use of Itô's Lemma and Eq. (A3), we get

$$r = \beta + \tfrac{\mu}{\psi} - \tfrac{\gamma(1+\psi)\sigma^2}{2} - \tfrac{(\gamma-1/\psi)(1-1/\psi)\nu^2}{2(1-\gamma)^2}\left(\tfrac{g_\mu}{g}\right)^2 - \tfrac{(\gamma-1/\psi)\sigma\nu\rho_{C,\mu}}{\lambda}, \tag{A5}$$

$$\pi_C = \sigma\gamma, \tag{A6}$$

$$\pi_\mu = \tfrac{\gamma-1/\psi}{1-\gamma}\tfrac{g_\mu}{g}\nu. \tag{A7}$$

An exact solution for $g(\mu)$ satisfying Eq. (A3) does not exist for $\psi \neq 1$. Therefore, I look for a solution of $g(\mu)$ around the unconditional mean of the consumption-wealth ratio. Aggregate wealth is given by

$$W_t = \mathbb{E}_t\left[\int_t^\infty \tfrac{\xi_u}{\xi_t} C_u du\right],$$

and, applying Fubini's Theorem and taking standard limits, the consumption-wealth ratio satisfies

$$\frac{C_t}{W_t} = r_t - \frac{1}{dt}\mathbb{E}_t\left[\frac{dW}{W}\right] - \frac{1}{dt}\mathbb{E}_t\left[\frac{d\xi}{\xi}\frac{dW}{W}\right]. \tag{A8}$$

Guess $W_t = C_t\beta^{-1}g(\mu)^{1/\theta}$ and apply Itô's Lemma:

$$\frac{dW}{W} = \left(\mu - \frac{1}{\theta}\frac{g_\mu}{g}\lambda(\bar{\mu}-\mu) + \frac{1}{\theta}\frac{\nu^2}{2}\left(\frac{g_{\mu,\mu}}{g} + \frac{\gamma-1/\psi}{1-\gamma}\left(\frac{g_\mu}{g}\right)^2\right)\right)dt + \sigma dB_C + \frac{1}{\theta}\frac{g_\mu}{g}\nu dB_\mu.$$

Plugging in wealth dynamics, risk-free rate and the pricing kernel into Eq. (A55), we get

$$\begin{aligned}\frac{C_t}{W_t} &= \beta + (\tfrac{1}{\psi}-1)\left(\mu - \frac{\gamma}{2}\sigma^2 + \frac{g_\mu}{g}\frac{\lambda(\bar{\mu}-\mu)}{1-\gamma} + \frac{g_{\mu,\mu}}{g}\frac{\nu^2}{2(1-\gamma)} + \frac{g_\mu}{g}\sigma\nu\rho_{C,\mu}\right) &\text{(A9)}\\ &= \beta + (\tfrac{1}{\psi}-1)\frac{\beta}{1-1/\psi}\left(1-g(\mu)^{-1/\theta}\right) &\text{(A10)}\\ &= \beta g(\mu)^{-1/\theta}. &\text{(A11)}\end{aligned}$$

The second equality comes from the Bellman equation (A3) and the third equality confirms the guess. Notice that the consumption-wealth ratio approaches to β when $\psi \to 1$ as usual.

Denote $cw = \mathbb{E}[\log C - \log W]$, hence, a first-order approximation of the consumption-wealth ratio around cw produces

$$\frac{C}{W} = \beta g(\mu)^{-1/\theta} \approx e^{cw}\left(1 - cw + \log\beta - \frac{1}{\theta}\log g(\mu)\right).$$

Using such an approximation in the Bellman equation (A3) leads to

$$\begin{aligned}0 &= \mu - \frac{\gamma}{2}\sigma^2 + \frac{g_\mu}{g}\frac{\lambda(\bar{\mu}-\mu)}{1-\gamma} + \frac{g_{\mu,\mu}}{g}\frac{\nu^2}{2(1-\gamma)} + \frac{g_\mu}{g}\sigma\nu\rho_{C,\mu}\\ &\quad + \frac{1}{1-1/\psi}\left(e^{cw}\left(1 - cw + \log\beta - \frac{1}{\theta}\log g(\mu)\right) - \beta\right), \end{aligned}\tag{A12}$$

which has exponentially affine solution $g(\mu) = e^{u_0+u_1\mu}$ where

$$u_0 = e^{-cw}\left(\lambda\bar{\mu}u_1 + \nu^2u_1^2/2 + (\gamma-1)\gamma\sigma^2/2 - (\gamma-1)\sigma\nu\rho_{C,\mu}u_1 - \theta\beta\right) - \theta\left(cw - 1 - \log\beta\right), \tag{A13}$$

$$u_1 = \frac{1-\gamma}{\lambda+e^{cw}}, \tag{A14}$$

and the endogenous constant cw satisfies $cw = \log\beta - \frac{1}{\theta}(u_0 + u_1\bar{\mu})$.

The risk-free rate and the prices of risk take the form:

$$\begin{aligned}r &= \beta + \frac{\mu}{\psi} - \frac{\gamma(1+\psi)\sigma^2}{2} - \frac{(\gamma-1/\psi)(1-1/\psi)\nu^2}{2(\lambda+e^{cw})^2} - \frac{(\gamma-1/\psi)\sigma\nu\rho_{C,\mu}}{\lambda}, &\text{(A15)}\\ \pi_C &= \sigma\gamma, &\text{(A16)}\\ \pi_\mu &= \frac{\gamma-1/\psi}{\lambda+e^{cw}}\nu. &\text{(A17)}\end{aligned}$$

■

B Cash-flows and asset prices

Consider the following conditional expectation:

$$M_{t,u}(\bar{c}) = \mathbb{E}_t[e^{c_0+c_1\log\xi_u+c_2\log C_u+c_3\mu_u+c_4\delta_u}], \tag{A18}$$

where $\bar{c} = (c_0, c_1, c_2, c_3, c_4)$ is a coefficient vector such that the expectation exists, and guess an exponential affine solution of the kind:

$$M_{t,u}(\bar{c}) = e^{c_1 \log \xi_t + c_2 \log C_t + \upsilon(u-t,\bar{c}) + \phi(u-t,\bar{c})\mu_t + \varphi(u-t,\bar{c})\delta_t}. \tag{A19}$$

Given dynamics in Eq. (5)-(6)-(9)-(24), Feynman-Kac gives that M has to meet the following partial differential equation:

$$\begin{aligned} M_t &+ M_C \mu C + M_{CC}\tfrac{1}{2}\sigma^2 C^2 + M_\mu \lambda(\bar{\mu} - \mu) + M_{\mu\mu}\tfrac{1}{2}\nu^2 - M_\delta \kappa\delta \\ &+ M_{\delta\delta}\tfrac{1}{2}\eta^2 - M_\xi(r_0 + r_1\mu)\xi + M_{\xi\xi}\tfrac{1}{2}(\pi_C^2 + \pi_\mu^2 + 2\pi_C\pi_\mu\rho_{C,\mu})\xi^2 \\ &+ M_{C\mu}\sigma C\nu\rho_{C,\mu} + M_{C\delta}\sigma C\eta\rho_{C,\delta} - M_{C\xi}\sigma C(\pi_C + \pi_\mu\rho_{C,\mu})\xi \\ &+ M_{\mu\delta}\nu\eta\rho_{\mu,\delta} - M_{\mu\xi}\nu(\pi_C\rho_{C,\mu} + \pi_\mu)\xi - M_{\delta\xi}\eta(\pi_C\rho_{C,\delta} + \pi_\mu\rho_{\mu,\delta})\xi = 0 \end{aligned} \tag{A20}$$

where the arguments have been omitted for ease of notation. Plugging the resulting partial derivatives from the guess solution into the pde and simplifying gives

$$\begin{aligned} \upsilon_t &+ \phi_t\mu + \varphi_t\delta + c_2\mu + \tfrac{1}{2}c_2(c_2 - 1)\sigma^2 + \phi\lambda(\bar{\mu} - \mu) + \tfrac{1}{2}\nu^2\phi^2 - \varphi\kappa\delta \\ &+ \tfrac{1}{2}\eta^2\varphi^2 - c_1(r_0 + r_1\mu) + \tfrac{1}{2}c_1(c_1 - 1)(\pi_C^2 + \pi_\mu^2 + 2\pi_C\pi_\mu\rho_{C,\mu}) \\ &+ c_2\phi\sigma\nu\rho_{C,\mu} + c_2\varphi\sigma\eta\rho_{C,\delta} - c_1c_2\sigma(\pi_C + \pi_\mu\rho_{C,\mu}) \\ &+ \phi\varphi\nu\eta\rho_{\mu,\delta} - c_1\phi\nu(\pi_C\rho_{C,\mu} + \pi_\mu) - c_1\varphi\eta(\pi_C\rho_{C,\delta} + \pi_\mu\rho_{\mu,\delta}) = 0. \end{aligned} \tag{A21}$$

This equation has to hold for all μ and δ. We thus get three ordinary differential equations for υ, ϕ and φ:

$$\begin{aligned} \upsilon_t =& \tfrac{1}{2}c_2(c_2 - 1)\sigma^2 + \phi\lambda\bar{\mu} + \tfrac{1}{2}\nu^2\phi^2 + \tfrac{1}{2}\eta^2\varphi^2 - c_1r_0 + \tfrac{1}{2}c_1(c_1 - 1)(\pi_C^2 + \pi_\mu^2 + 2\pi_C\pi_\mu\rho_{C,\mu}) \\ &+ c_2\phi\sigma\nu\rho_{C,\mu} + c_2\varphi\sigma\eta\rho_{C,\delta} - c_1c_2\sigma(\pi_C + \pi_\mu\rho_{C,\mu}) + \phi\varphi\nu\eta\rho_{\mu,\delta} \\ &- c_1\phi\nu(\pi_C\rho_{C,\mu} + \pi_\mu) - c_1\varphi\eta(\pi_C\rho_{C,\delta} + \pi_\mu\rho_{\mu,\delta}), \end{aligned} \tag{A22}$$

$$\phi_t = c_2 - c_1r_1 - \lambda\phi, \tag{A23}$$

$$\varphi_t = -\kappa\varphi, \tag{A24}$$

with initial conditions $\upsilon(0,\bar{c}) = c_0$, $\phi(0,\bar{c}) = c_3$ and $\varphi(0,\bar{c}) = c_4$. The solution is

$$\phi(\tau,\bar{c}) = \tfrac{c_2 - c_1r_1}{\lambda}(1 - e^{-\lambda\tau}) + c_3e^{-\lambda\tau}, \tag{A25}$$

$$\varphi(\tau,\bar{c}) = c_4e^{-\kappa\tau}, \tag{A26}$$

and $\upsilon(\tau,\bar{c})$ can be computed in closed form but it is too long to be reported.

Proof of Proposition 1: To compute the moment generating function of aggregate consumption growth, recall that

$$\mathbb{C}_t(\tau,\alpha) = \mathbb{E}_t\left[e^{\alpha \log C_{t+\tau}}\right],$$

where the latter expectation is a conditional Laplace transform of the type in Eq. (A18). It has solution as in Eq. (A19) where the system (A22)-(A23)-(A24) is solved for $\bar{c} = (0, 0, \alpha, 0, 0)$. Therefore, the expectation becomes

$$\mathbb{C}_t(\tau,\alpha) = e^{\alpha \log C_t + \upsilon(\tau,\bar{c}) + \phi(\tau,\bar{c})\mu_t + \varphi(\tau,\bar{c})\delta_t}$$

which leads to Eq. (11) when $a_0(\tau, 0, \alpha, 0), a_1(\tau, 0, \alpha, 0)$ and $a_2(\tau, 0, \alpha, 0)$ replace respectively $\upsilon(\tau,\bar{c})$, $\phi(\tau,\bar{c})$ and $\varphi(\tau,\bar{c})$ for $\bar{c} = (0, 0, \alpha, 0, 0)$. Consequently, the moment generating function of aggregate consumption growth relative to the current consumption value is a stationary function of μ_t and the maturity, since $a_2(\tau, 0, \alpha, 0) = 0$ for any τ.

To compute the moment generating function of market dividends growth, recall that

$$\mathbb{D}_t(\tau,\alpha) = \mathbb{E}_t\left[e^{\alpha\log D_{t+\tau}}\right],$$

where the latter expectation is a conditional Laplace transform of the type in Eq. (A18). It has solution as in Eq. (A19) where the system (A22)-(A23)-(A24) is solved for $\bar{c} = (\alpha\bar{\delta}, 0, \alpha, 0, \alpha)$. Therefore, the expectation becomes

$$\mathbb{D}_t(\tau,\alpha) = e^{\alpha\log C_t + \upsilon(\tau,\bar{c}) + \phi(\tau,\bar{c})\mu_t + \varphi(\tau,\bar{c})\delta_t}$$

which leads to Eq. (11) when $a_0(\tau, 0, \alpha, \alpha), a_1(\tau, 0, \alpha, \alpha)$ and $a_2(\tau, 0, \alpha, \alpha)$ replace respectively $\upsilon(\tau,\bar{c})$, $\phi(\tau,\bar{c})$ and $\varphi(\tau,\bar{c})$ for $\bar{c} = (\alpha\bar{\delta}, 0, \alpha, 0, \alpha)$. Consequently, the moment generating function of market dividends growth relative to the current dividend value is a stationary function of μ_t, δ_t and the maturity.

To compute the joint moment generating function of consumption and dividend growth, recall that

$$\mathbb{CD}_t(\tau,\alpha,\beta) = \mathbb{E}_t\left[e^{\alpha\log C_{t+\tau} + \beta\log D_{t+\tau}}\right],$$

where the latter expectation is a conditional Laplace transform of the type in Eq. (A18). It has solution as in Eq. (A19) where the system (A22)-(A23)-(A24) is solved for $\bar{c} = (\beta\bar{\delta}, 0, \alpha+\beta, 0, \beta)$. Therefore, the expectation becomes

$$\mathbb{CD}_t(\tau,\alpha,\beta) = e^{(\alpha+\beta)\log C_t + \upsilon(\tau,\bar{c}) + \phi(\tau,\bar{c})\mu_t + \varphi(\tau,\bar{c})\delta_t}$$

which leads to Eq. (13) when $a_0(\tau, 0, \alpha+\beta, \beta), a_1(\tau, 0, \alpha+\beta, \beta)$ and $a_2(\tau, 0, \alpha+\beta, \beta)$ replace respectively $\upsilon(\tau,\bar{c})$, $\phi(\tau,\bar{c})$ and $\varphi(\tau,\bar{c})$ for $\bar{c} = (\beta\bar{\delta}, 0, \alpha+\beta, 0, \beta)$. Consequently, the joint moment generating function of consumption and dividend growth relative to the current cash-flows value is a stationary function of μ_t, δ_t and the maturity. ■

Proof of Corollary 1: Given the moment generating function in Eq. (11), it is easy to compute the term structures in Eq. (14). Armed with the pairs $g_{C,\tau}, g_{D,\tau}$ and $\sigma_{C,\tau}, \sigma_{D,\tau}$, straightforward calculus leads to the results in Eq. (18) and (19) as well as in Eq. (20) and (21). ■

Proof of Proposition 3: To compute the price of the dividend strip, recall that

$$P_{t,\tau} = \mathbb{E}_t\left[\tfrac{\xi_{t+\tau}}{\xi_t}D_{t+\tau}\right] = \tfrac{1}{\xi_t}\mathbb{E}_t\left[e^{\log\xi_{t+\tau} + \log C_{t+\tau} + \bar{\delta} + \delta_{t+\tau}}\right],$$

where the latter expectation is a conditional Laplace transform of the type in Eq. (A18). It has solution as in Eq. (A19) where the system (A22)-(A23)-(A24) is solved for $\bar{c} = (\bar{\delta}, 1, 1, 0, 1)$. Therefore, the price becomes

$$P_{t,\tau} = \tfrac{1}{\xi_t}e^{\log\xi_t + \log C_t + \upsilon(\tau,\bar{c}) + \phi(\tau,\bar{c})\mu_t + \varphi(\tau,\bar{c})\delta_t}$$

which leads to Eq. (30) when $b_0(\tau, 1, 1, 1), b_1(\tau, 1, 1, 1)$ and $b_2(\tau, 1, 1, 1)$ replace respectively $\upsilon(\tau,\bar{c})$, $\phi(\tau,\bar{c})$ and $\varphi(\tau,\bar{c})$ for $\bar{c} = (\bar{\delta}, 1, 1, 0, 1)$. Consequently, the price of the dividend strip relative to the current dividend value is a stationary function of μ_t, δ_t and the maturity.

The dynamics of $P_{t,\tau}$ obtains by applying Itô's Lemma:

$$dP_{t,\tau} = [\cdot]dt + \tfrac{\partial P_{t,\tau}}{\partial C}\sigma C_t dB_{C,t} + \tfrac{\partial P_{t,\tau}}{\partial \mu}\nu dB_{\mu,t} + \tfrac{\partial P_{t,\tau}}{\partial \delta}\eta dB_{\delta,t},$$

and therefore its return volatility is given by

$$\sigma_{R,t,\tau} = P_{t,\tau}^{-1}\sqrt{\begin{array}{c}\left(\frac{\partial P_{t,\tau}}{\partial C}\sigma C_t\right)^2 + \left(\frac{\partial P_{t,\tau}}{\partial \mu}\nu\right)^2 + \left(\frac{\partial P_{t,\tau}}{\partial \delta}\eta\right)^2 + 2\frac{\partial P_{t,\tau}}{\partial C}\frac{\partial P_{t,\tau}}{\partial \mu}\sigma C_t\nu\rho_{C,\mu} \\ +2\frac{\partial P_{t,\tau}}{\partial C}\frac{\partial P_{t,\tau}}{\partial \delta}\sigma C_t\eta\rho_{C,\delta} + 2\frac{\partial P_{t,\tau}}{\partial \mu}\frac{\partial P_{t,\tau}}{\partial \delta}\nu\eta\rho_{\mu,\delta}\end{array}},$$

which leads to Eq. (34). The premium on the dividend strip is given by $-\frac{1}{dt}\left\langle \frac{dP_\tau}{P_\tau}, \frac{d\xi}{\xi}\right\rangle_t$:

$$\mu_{R,t,\tau} = P_{t,\tau}^{-1}\left(\frac{\partial P_{t,\tau}}{\partial C}\sigma C_t(\pi_C + \pi_\mu\rho_{C,\mu}) + \frac{\partial P_{t,\tau}}{\partial \mu}\nu(\pi_C\rho_{C,\mu} + \pi_\mu) + \frac{\partial P_{t,\tau}}{\partial \delta}\eta(\pi_C\rho_{C,\delta} + \pi_\mu\rho_{\mu,\delta})\right),$$

which leads to Eq. (33). ■

Proof of Corollary 2: Straightforward calculus leads to

$$\partial_\tau\mu_{R,t,\tau} = e^{-\lambda\tau}\nu\left(1 - 1/\psi\right)\left(\pi_C\rho_{C,\mu} + \pi_\mu\right) - e^{-\kappa\tau}\eta\kappa\left(\pi_C\rho_{C,\delta} + \pi_\mu\rho_{\mu,\delta}\right),$$

whose sign reduces to $(1-1/\psi)(\gamma - 1/\psi)$ as in Eq. (35) when $\rho_{C,\mu} = \rho_{C,\delta} = \rho_{\mu,\delta} = 0$. The slope of the dividend strip volatility is given by

$$\partial_\tau\sigma_{R,t,\tau} = \frac{1}{\lambda\sigma_{R,t,\tau}}\times \left(\begin{array}{c} -e^{-2\kappa\tau}\eta^2\kappa\lambda + e^{-(\lambda+\kappa)\tau}(1-1/\psi)\eta\nu(\lambda+\kappa)\rho_{\mu,\delta} - e^{-2\lambda\tau}\nu^2(1-1/\psi)^2 \\ -e^{-\lambda\tau}(1-1/\psi)\nu((1-1/\psi)\nu - \lambda\sigma\rho_{C,\mu}) - e^{-\kappa\tau}\eta\kappa((1-1/\psi)\rho_{\mu,\delta} + \lambda\sigma\rho_{C,\delta})\end{array}\right)$$

whose sign reduces to Eq. (36) when $\rho_{C,\mu} = \rho_{C,\delta} = \rho_{\mu,\delta} = 0$. Following the proof of Proposition 3, it automatically follows that the price of the consumption strip is given by

$$P_{t,\tau}^C = \mathbb{E}_t\left[\frac{\xi_{t+\tau}}{\xi_t}C_{t+\tau}\right] = C_t e^{b_0(\tau,1,1,0) + b_1(\tau,1,1,0)\mu_t + b_2(\tau,1,1,0)\delta_t},$$

where when $b_0(\tau,1,1,0), b_1(\tau,1,1,0)$ and $b_2(\tau,1,1,0)$ replace respectively $\upsilon(\tau,\bar{c})$, $\phi(\tau,\bar{c})$ and $\varphi(\tau,\bar{c})$ for $\bar{c} = (0,1,1,0,0)$. Moreover, $b_2(\tau,1,1,0) = 0$ for any τ. Let $\tilde{\mu}_{R,\tau}$ and $\tilde{\sigma}_{R,\tau}$ denote respectively the expected excess return and the return volatility of the consumption strip, which can be computed as in Proposition 3. Armed with the pairs $\mu_{R,t,\tau}, \sigma_{R,t,\tau}$ and $\tilde{\mu}_{R,t,\tau}, \tilde{\sigma}_{R,t,\tau}$, straightforward calculus leads to the limits of Eq. (37) and (38). ■

Proof of Proposition 4: To compute the price of the non-defaultable zero-coupon bond, recall that

$$B_{t,\tau} = \mathbb{E}_t\left[\frac{\xi_{t+\tau}}{\xi_t}\right] = \frac{1}{\xi_t}\mathbb{E}_t\left[e^{\log\xi_{t+\tau}}\right],$$

where the latter expectation is a conditional Laplace transform of the type in Eq. (A18). It has solution as in Eq. (A19) where the system (A22)-(A23)-(A24) is solved for $\bar{c} = (0,1,0,0,0)$. Therefore, the price becomes

$$B_{t,\tau} = \frac{1}{\xi_t}e^{\log\xi_t + \upsilon(\tau,\bar{c}) + \phi(\tau,\bar{c})\mu_t + \varphi(\tau,\bar{c})\delta_t}$$

which leads to Eq. (39) when $b_0(\tau,1,0,0), b_1(\tau,1,0,0)$ and $b_2(\tau,1,0,0)$ replace respectively $\upsilon(\tau,\bar{c})$, $\phi(\tau,\bar{c})$ and $\varphi(\tau,\bar{c})$ for $\bar{c} = (0,1,0,0,0)$. Consequently, the price of the zero-coupon bond is a stationary function of μ_t and the maturity, since $b_2(\tau,1,0,0) = 0$ for any τ. ■

Proof of Corollary 3: Given the expressions from previous results in Eq. (14), (40), (41) and (42), the formula for the premium on the equity yield in Eq. (43) automatically obtains. ■

Proof of Proposition 5: To compute the price of the market asset, recall that

$$P_t = \mathbb{E}_t\left[\int_t^\infty \tfrac{\xi_u}{\xi_t} D_u du\right] = \int_0^\infty P_{t,\tau} d\tau$$

and, hence, the price automatically obtains as in Eq. (45). Consequently, the price of the market asset relative to the current dividend value is a stationary function of μ_t and δ_t.

The dynamics of P_t obtains by applying Itô's Lemma:

$$dP_t = [\cdot]dt + \tfrac{\partial P_t}{\partial C}\sigma C_t dB_{C,t} + \tfrac{\partial P_t}{\partial \mu}\nu dB_{\mu,t} + \tfrac{\partial P_t}{\partial \delta}\eta dB_{\delta,t},$$

and therefore its return volatility is given by

$$\sigma_{R,t} = P_t^{-1}\sqrt{\begin{array}{c}\left(\tfrac{\partial P_t}{\partial C}\sigma C_t\right)^2 + \left(\tfrac{\partial P_t}{\partial \mu}\nu\right)^2 + \left(\tfrac{\partial P_t}{\partial \delta}\eta\right)^2 + 2\tfrac{\partial P_t}{\partial C}\tfrac{\partial P_t}{\partial \mu}\sigma C_t \nu \rho_{C,\mu} \\ +2\tfrac{\partial P_t}{\partial C}\tfrac{\partial P_t}{\partial \delta}\sigma C_t \eta \rho_{C,\delta} + 2\tfrac{\partial P_t}{\partial \mu}\tfrac{\partial P_t}{\partial \delta}\nu\eta\rho_{\mu,\delta}\end{array}},$$

which leads to Eq. (47). The premium on the market asset is given by $-\frac{1}{dt}\left\langle \frac{dP}{P}, \frac{d\xi}{\xi}\right\rangle_t$:

$$\mu_{R,t} = P_t^{-1}\left(\tfrac{\partial P_t}{\partial C}\sigma C_t(\pi_C + \pi_\mu\rho_{C,\mu}) + \tfrac{\partial P_t}{\partial \mu}\nu(\pi_C\rho_{C,\mu} + \pi_\mu) + \tfrac{\partial P_t}{\partial \delta}\eta(\pi_C\rho_{C,\delta} + \pi_\mu\rho_{\mu,\delta})\right),$$

which leads to Eq. (48). ■

Proof of Corollary 4: For $\rho_{C,\mu} = \rho_{C,\delta} = \rho_{\mu,\delta} = 0$, the equity premium reduces to

$$\begin{aligned}\mu_{R,t} &= \sigma_{R,C,t}\pi_C + \sigma_{R,\mu,t}\pi_\mu \\ &= \sigma_C\pi_C + \nu\frac{D_t}{P_t}\frac{\partial}{\partial\mu}\frac{P_t}{D_t}\pi_\mu \\ &= \sigma_C\pi_C + \pi_\mu\nu\frac{1-1\psi}{\lambda}\int_0^\infty \frac{P_{t,\tau}}{P_t}(1-e^{-\lambda\tau})d\tau.\end{aligned}$$

Therefore, the sign of $\partial_\delta\,\mu_{R,t}$ is given by the sign of

$$sign\left(\pi_\mu(\psi-1)\frac{\partial}{\partial\delta}\int_0^\infty \frac{P_{t,\tau}}{P_t}(1-e^{-\lambda\tau})d\tau\right).$$

Denote g, h and m as

$$g(\delta_t) = \int_0^\infty \frac{P_{t,\tau}}{P_t}(1-e^{-\lambda\tau})d\tau = \frac{\int_0^\infty P_{t,\tau}(1-e^{-\lambda\tau})d\tau}{\int_0^\infty P_{t,\tau}d\tau} = \frac{h(\delta_t)}{m(\delta_t)}$$

and, hence, we have

$$\begin{aligned}
\frac{\partial g(\delta)}{\partial \delta} &= \frac{m(\delta)h'(\delta) - h(\delta)m'(\delta)}{m(\delta)^2} \\
&= \frac{-P_t \int_0^\infty (1-e^{-\lambda\tau})(1-e^{-\kappa\tau})P_{t,\tau}d\tau + \int_0^\infty (1-e^{-\lambda\tau})P_{t,\tau}d\tau \int_0^\infty (1-e^{-\kappa\tau})P_{t,\tau}d\tau}{P_t^2} \\
&= \frac{[\int_0^\infty a(\tau)P_{t,\tau}d\tau][\int_0^\infty b(\tau)P_{t,\tau}d\tau] - [\int_0^\infty P_{t,\tau}d\tau][\int_0^\infty a(\tau)b(\tau)P_{t,\tau}d\tau]}{P_t^2} \\
&= \frac{[\int_0^\infty a(\tau)P_{t,\tau}d\tau][\int_0^\infty b(\tau)P_{t,\tau}d\tau] - [\int_0^\infty P_{t,\tau}d\tau][\int_0^\infty a(\tau)b(\tau)P_{t,\tau}d\tau]}{P_t^2}
\end{aligned}$$

where $a(\tau) = (1-e^{-\lambda\tau})$ and $b(\tau) = (1-e^{-\kappa\tau})$. The numerator can be written as

$$\frac{1}{2}\int_0^\infty \int_0^\infty P_{t,\tau_1}P_{t,\tau_2}[a(\tau_1) - a(\tau_2)][b(\tau_2) - b(\tau_1)]d\tau_1 d\tau_2.$$

Note that $a(\tau)$ and $b(\tau)$ are both positive and monotonically increasing. The integration region can be divided into two parts: $\tau_1 < \tau_2$ and $\tau_1 > \tau_2$. In each one the integrand is negative. Consequently, $\partial_\delta\, g < 0$ and

$$sign(\partial_\delta\, \mu_{R,t}) = sign\left((\gamma - 1/\psi)(1-\psi)\right).$$

■

C Pricing corporate fraction fluctuations

Recall the model dynamics for cash-flows:

$$dL_t = \mu_t dt L_t dt + \sigma_L L_t dB_{L,t}, \tag{A27}$$

$$d\mu_t = \lambda(\bar{\mu} - \mu_t)dt + \nu dB_{\mu,t}, \tag{A28}$$

with labor-share $L_t/C_t = 1 - S_t = e^{-\ell_t}$:

$$d\ell_t = \kappa(\bar{\ell} - \ell_t)dt + \eta\sqrt{\ell_t}dB_{\ell,t}, \tag{A29}$$

such that

$$dC_t = (\mu_t + \kappa\bar{\ell} - (\kappa - \eta^2/2)\ell_t)C_t dt + \sigma_L C_t dB_{L,t} + \eta\sqrt{\ell_t}C_t dB_{\ell,t}. \tag{A30}$$

The representative agent is equipped with stochastic differential utility, as in Duffie and Epstein (1992). Given an initial consumption C, the utility at each time t is defined as $U(C_t) = J_t$ where J is the unique solution to the SDE:

$$dJ_t = \left(-f(C_t, J_t) - \tfrac{1}{2}A(J_t)\sigma_J\sigma_J'\right)dt + \sigma_J dB_t \tag{A31}$$

where

$$f(C,J) = \beta\theta J\left(C^{1-\frac{1}{\psi}}((1-\gamma)J)^{-\frac{1}{\theta}} - 1\right). \tag{A32}$$

Under the infinite horizon, the utility process J satisfies the following Bellman equation:

$$\mathcal{D}J(C,\mu) + f(C,J) = 0, \tag{A33}$$

where $\mathcal{D}$ denotes the differential operator. The solution to Eq. (A33) is given by

$$J(L_t,\mu_t,\ell_t)=\frac{1}{1-\gamma}L_t^{1-\gamma}g(\mu_t,\ell_t)=\frac{1}{1-\gamma}L_t^{1-\gamma}\exp(u_0+u_1\mu_t+u_2\ell_t), \tag{A34}$$

where u_0, u_1 and u_2 are endogenous constants depending on the primitive parameters.

Indeed, the Bellman equation can be written as

$$J_L\mu L+\tfrac{1}{2}J_{L,L}\sigma_L^2L^2+J_\mu\lambda(\bar\mu-\mu)+\tfrac{1}{2}J_{\mu,\mu}\nu^2+J_\ell\kappa(\bar\ell-\ell)+\tfrac{1}{2}J_{\ell,\ell}\eta^2\ell+f(C,J)=0. \tag{A35}$$

Guessing the above solution, the Bellman equation reduces to

$$\mu-\tfrac{\gamma}{2}\sigma_L^2+\tfrac{g_\mu}{g}\tfrac{\lambda(\bar\mu-\mu)}{1-\gamma}+\tfrac{g_{\mu,\mu}}{g}\tfrac{\nu^2}{2(1-\gamma)}+\tfrac{g_\ell}{g}\tfrac{\kappa(\bar\ell-\ell)}{1-\gamma}+\tfrac{g_{\ell,\ell}}{g}\tfrac{\eta^2\ell}{2(1-\gamma)}+\tfrac{\beta}{1-1/\psi}\left(g(\mu,\ell)^{-1/\theta}-1\right)=0. \tag{A36}$$

The pricing kernel for stochastic differential utility can be written as

$$d\xi=\xi\tfrac{df_C}{f_C}+\xi f_J dt=-r\xi-\pi_L\xi dB_L-\pi_\mu\xi dB_\mu-\pi_\ell\sqrt{\ell}\,\xi dB_\ell, \tag{A37}$$

where, by use of Itô's Lemma, we get

$$r=r_0+r_1\mu_t+r_2\ell_t, \tag{A38}$$
$$r_0=-f_{J,0}-\tfrac{1}{2}\tfrac{f_{C,L,L}}{f_C}L_t^2\sigma_L^2-\tfrac{f_{C,\ell}}{f_C}\kappa\bar\ell-\tfrac{f_{C,\mu}}{f_C}\lambda\bar\mu-\tfrac{1}{2}\tfrac{f_{C,\mu,\mu}}{f_C}\nu^2, \tag{A39}$$
$$r_1=-f_{J,\mu}+\tfrac{f_{C,\mu}}{f_C}\lambda-\tfrac{f_{C,L}}{f_C}L_t, \tag{A40}$$
$$r_2=-f_{J,\ell}+\tfrac{f_{C,\ell}}{f_C}\kappa-\tfrac{1}{2}\tfrac{f_{C,\ell,\ell}}{f_C}\eta^2, \tag{A41}$$
$$\pi_L=-\tfrac{f_{C,L}}{f_C}L_t\sigma_L, \tag{A42}$$
$$\pi_\mu=-\tfrac{f_{C,\mu}}{f_C}\nu, \tag{A43}$$
$$\pi_\ell=-\tfrac{f_{C,\ell}}{f_C}\eta. \tag{A44}$$

The partial derivatives of $f(C,J)$ satisfy

$$\tfrac{f_{C,L}}{f_C}=-\gamma/L_t, \tag{A45}$$
$$\tfrac{f_{C,L,L}}{f_C}=\gamma(1+\gamma)/L_t^2, \tag{A46}$$
$$\tfrac{f_{C,\mu}}{f_C}=\tfrac{u_1(\gamma\psi-1)}{(\gamma-1)\psi}, \tag{A47}$$
$$\tfrac{f_{C,\mu,\mu}}{f_C}=\tfrac{(u_1(\gamma\psi-1))^2}{(\gamma-1)^2\psi^2}, \tag{A48}$$
$$\tfrac{f_{C,\ell}}{f_C}=\tfrac{(1-\gamma-u_2(1-\gamma\psi))}{(\gamma-1)\psi}, \tag{A49}$$
$$\tfrac{f_{C,\ell,\ell}}{f_C}=\tfrac{(1-\gamma-u_2(1-\gamma\psi))^2}{(\gamma-1)^2\psi^2}, \tag{A50}$$
$$f_J=f_{J,0}+f_{J,\mu}\mu_t+f_{J,\ell}\ell_t, \tag{A51}$$
$$f_{J,0}=\tfrac{\beta(\gamma-1)\psi+e^{cw}(1-\gamma\psi)\left(1-cw-u_0\theta^{-1}+\log\beta\right)}{\psi-1}, \tag{A52}$$
$$f_{J,\mu}=\tfrac{e^{cw}\left(-u_1\theta^{-1}\right)(1-\gamma\psi)}{\psi-1}, \tag{A53}$$
$$f_{J,\ell}=\tfrac{e^{cw}\left((1-\gamma-u_2)\theta^{-1}\right)(1-\gamma\psi)}{\psi-1}, \tag{A54}$$

where $cw=\mathbb{E}[\log C_t-\log W_t]$.

To recognize the exponential affine form of $g(\mu_t, \ell_t)$ satisfying Eq. (A36), we need to look for a solution around the unconditional mean of the consumption-wealth ratio. Aggregate wealth is given by

$$W_t = \mathbb{E}_t \left[\int_t^\infty \tfrac{\xi_u}{\xi_t} C_u du \right],$$

and, applying Fubini's Theorem and taking standard limits, the consumption-wealth ratio satisfies

$$\tfrac{C_t}{W_t} = r_t - \tfrac{1}{dt}\mathbb{E}_t \left[\tfrac{dW}{W} \right] - \tfrac{1}{dt}\mathbb{E}_t \left[\tfrac{d\xi}{\xi} \tfrac{dW}{W} \right]. \tag{A55}$$

Guessing $W_t = C_t e^{(1/\psi - 1)\ell_t} \beta^{-1} g(\mu_t, \ell_t)^{1/\theta}$ and applying Itô's Lemma, we can compute wealth dynamics $\frac{dW}{W}$. Therefore, plugging the wealth dynamics, the risk-free rate and the pricing kernel into $\frac{C_t}{W_t}$, we find the above guess for W_t. Notice that the consumption-wealth ratio approaches to β when $\psi \to 1$ as usual.

Hence, a first-order approximation of the consumption-wealth ratio around cw produces

$$\tfrac{C_t}{W_t} = \beta g(\mu_t, \ell_t)^{-1/\theta} e^{(1-1/\psi)\ell_t} \approx e^{cw} \left(1 - cw + \log\beta - \tfrac{1}{\theta} \log g(\mu_t, \ell_t) + (1 - 1/\psi)\ell_t \right).$$

Using such approximation in the Bellman equation (A33) we recognize the exponentially affine solution $g(\mu, \ell) = e^{u_0 + u_1\mu + u_2\ell}$ where

$$\begin{aligned}
u_0 =& \tfrac{e^{-cw}(\gamma-1)}{2(\psi-1)} \Bigg(\tfrac{1}{(e^{cw}+\lambda)^2} \Bigg(-2\lambda \left(e^{cw} + \lambda\right) \bar{\mu}(\psi - 1) + \tfrac{1}{\eta^2(\gamma-1)} \Big(2\kappa \left(e^{cw} + \lambda\right)^2 \bar{\ell} \times \qquad (A56) \\
& \left(e^{cw} + \kappa - \sqrt{e^{2cw} + \kappa^2 + 2e^{cw}\left(\kappa + \eta^2(\gamma - 1)\right)} \right) (\psi - 1) + \eta^2(\gamma - 1) \Big(\nu^2(\gamma - 1)(\psi - 1) + \\
& \left(e^{cw} + \lambda\right)^2 \left(\gamma\sigma_L^2(\psi - 1) + 2\left((cw - 1)e^{cw} + \beta\right)\psi \right) \Big) \Big) \Bigg) - 2e^{cw}\psi\log\beta \Bigg), \\
u_1 =& (1 - \gamma)(\lambda + e^{cw})^{-1}, \qquad (A57) \\
u_2 =& \eta^{-2} \left(e^{cw} + \kappa - \sqrt{e^{2cw} + \kappa^2 + 2e^{cw}\left(\kappa + \eta^2(\gamma - 1)\right)} \right), \qquad (A58)
\end{aligned}$$

and the endogenous constant cw satisfies $cw = \log\beta - \frac{1}{\theta}\left(u_0 + u_1\bar{\mu} + (\gamma - 1 + u_2)\bar{\ell}\right)$. Then the above coefficients u_0, u_1 and u_2 can be used to express the risk-free rate and the prices of risk in terms of the primitive parameters.

Consider the following conditional expectation:

$$M_{t,u}(\bar{c}) = \mathbb{E}_t[e^{c_0 + c_1 \log\xi_u + c_2 \log L_u + c_3\mu_u + c_4\ell_u}], \tag{A59}$$

where $\bar{c} = (c_0, c_1, c_2, c_3, c_4)$ is a coefficient vector such that the expectation exists, and guess an exponential affine solution of the kind:

$$M_{t,u}(\bar{c}) = e^{c_1 \log\xi_t + c_2 \log L_t + \upsilon(u-t,\bar{c}) + \phi(u-t,\bar{c})\mu_t + \varphi(u-t,\bar{c})\ell_t}. \tag{A60}$$

Given dynamics in Eq. (A27)-(A28)-(A29)-(A37), Feynman-Kac gives that M has to meet the following partial differential equation:

$$\begin{aligned}
& M_t + M_{\log\xi}(-r_0 - r_1\mu - r_2\ell - \pi_L^2/2 - \pi_\mu^2/2 - \pi_\ell^2\ell/2) + M_{\log\xi,\log\xi}\tfrac{1}{2}(\pi_L^2 + \pi_\mu^2 + \pi_\ell^2\ell) \\
& \quad + M_{\log L}(\mu - \sigma_L^2/2) + M_{\log L,\log L}\tfrac{1}{2}\sigma_L^2 + M_\mu\lambda(\bar{\mu} - \mu) + M_{\mu,\mu}\tfrac{1}{2}\nu^2 \\
& \quad + M_\ell\kappa(\bar{\ell} - \ell) + M_{\ell,\ell}\tfrac{1}{2}\eta^2\ell + M_{\log\xi,\log L}\pi_L\sigma_L + M_{\log\xi,\mu}\pi_\mu\nu + M_{\log\xi,\ell}\pi_\ell\eta\ell = 0, \qquad (A61)
\end{aligned}$$

where the arguments have been omitted for ease of notation. Plugging the resulting partial derivatives from the guess solution into the pde and simplifying gives

$$\begin{aligned} \upsilon_t + \phi_t\mu + \varphi_t\ell + c_1(-r_0 - r_1\mu - r_2\ell - \pi_L^2/2 - \pi_\mu^2/2 - \pi_\ell^2\ell/2) + c_1^2\tfrac{1}{2}(\pi_L^2 + \pi_\mu^2 + \pi_\ell^2\ell) \\ + c_2(\mu - \sigma_L^2/2) + c_2^2\tfrac{1}{2}\sigma_L^2 + \phi\lambda(\bar{\mu} - \mu) + \phi^2\tfrac{1}{2}\nu^2 \\ + \varphi\kappa(\bar{\ell} - \ell) + \varphi^2\tfrac{1}{2}\eta^2\ell + c_1c_2\pi_L\sigma_L + c_1\phi\pi_\mu\nu + c_2\varphi\pi_\ell\eta\ell = 0. \end{aligned} \tag{A62}$$

This equation has to hold for all μ and ℓ. We thus get three ordinary differential equations for υ, ϕ and φ:

$$\begin{aligned} \upsilon_t =& -c_1(r_0 + \pi_L^2/2 + \pi_\mu^2/2 - c_1\pi_L^2/2 - c_1\pi_\mu^2/2) + c_2(c_2 - 1)\sigma_L^2/2 \\ & + \phi\lambda\bar{\mu} + \phi^2\nu^2/2 + \varphi\kappa\bar{\ell} - \pi_L\sigma_L c_1c_2 - \pi_\mu c_1\phi, \end{aligned} \tag{A63}$$

$$\phi_t = c_2 - c_1r_1 - \lambda\phi, \tag{A64}$$

$$\varphi_t = -c_1(r_2 + \pi_\ell^2/2 - c_1\pi_\ell^2/2) - \kappa\varphi + \eta^2\varphi^2/2 - \pi_\ell\eta c_1\varphi, \tag{A65}$$

with initial conditions $\upsilon(0,\bar{c}) = c_0$, $\phi(0,\bar{c}) = c_3$ and $\varphi(0,\bar{c}) = c_4$. The solution is

$$\phi(\tau,\bar{c}) = \tfrac{c_2 - c_1r_1}{\lambda}(1 - e^{-\lambda\tau}) + c_3e^{-\lambda\tau}, \tag{A66}$$

$$\varphi(\tau,\bar{c}) = \eta^{-2}\left(\kappa + c_1\eta\pi_\ell - \Theta\tan\left(-\tfrac{\tau}{2}\Theta + \arctan\left(\tfrac{\kappa + \eta(c_2\pi_\ell - c_4\eta)}{\Theta}\right)\right)\right), \tag{A67}$$

where $\Theta = \sqrt{-\kappa^2 - 2c_1\kappa\eta\pi_\ell - c_1\eta^2(2r_2 + \pi_\ell^2)}$ and $\upsilon(\tau,\bar{c})$ can be computed in closed form but it is too long to be reported.

To compute the term structures of the first two moments of cash-flows growth rates:

$$\begin{aligned} g_{C,t,\tau} &= \tfrac{1}{\tau}\log\left(\tfrac{\mathbb{E}_t[C_{t+\tau}]}{C_t}\right), \\ g_{D,t,\tau} &= \tfrac{1}{\tau}\log\left(\tfrac{\mathbb{E}_t[D_{t+\tau}]}{D_t}\right), \\ \sigma_{C,t,\tau} &= \sqrt{\tfrac{1}{\tau}\log\left(\tfrac{\mathbb{E}_t[C_{t+\tau}^2]}{\mathbb{E}_t[C_{t+\tau}]^2}\right)}, \\ \sigma_{D,t,\tau} &= \sqrt{\tfrac{1}{\tau}\log\left(\tfrac{\mathbb{E}_t[D_{t+\tau}^2]}{\mathbb{E}_t[D_{t+\tau}]^2}\right)}, \end{aligned} \tag{A68}$$

we need the following expectations:

$$\begin{aligned} \mathbb{E}_t[C_{t+\tau}] &= \mathbb{E}_t[L_{t+\tau}e^{\ell_{t+\tau}}] \\ &= L_te^{\upsilon(\tau,\bar{c})+\phi(\tau,\bar{c})\mu_t+\varphi(\tau,\bar{c})\ell_t}, \text{ with } \bar{c} = (0,1,0,1), \end{aligned} \tag{A69}$$

$$\begin{aligned} \mathbb{E}_t[C_{t+\tau}^2] &= \mathbb{E}_t[L_{t+\tau}^2e^{2\ell_{t+\tau}}] \\ &= L_t^2e^{\upsilon(\tau,\bar{c})+\phi(\tau,\bar{c})\mu_t+\varphi(\tau,\bar{c})\ell_t}, \text{ with } \bar{c} = (0,2,0,2), \end{aligned} \tag{A70}$$

$$\begin{aligned} \mathbb{E}_t[D_{t+\tau}] &= \mathbb{E}_t[L_{t+\tau}(e^{\ell_{t+\tau}} - 1)] \\ &= L_t\left(e^{\upsilon(\tau,\bar{c})+\phi(\tau,\bar{c})\mu_t+\varphi(\tau,\bar{c})\ell_t} - e^{\upsilon(\tau,\bar{c}')+\phi(\tau,\bar{c}')\mu_t+\varphi(\tau,\bar{c}')\ell_t}\right), \\ &\quad \text{with } \bar{c} = (0,1,0,1) \text{ and } \bar{c}' = (0,1,0,0), \end{aligned} \tag{A71}$$

$$\begin{aligned} \mathbb{E}_t[D_{t+\tau}^2] &= \mathbb{E}_t[L_{t+\tau}^2(e^{2\ell_{t+\tau}} - 2e^{\ell_{t+\tau}} + 1)] \\ &= L_t^2\left(e^{\upsilon(\tau,\bar{c})+\phi(\tau,\bar{c})\mu_t+\varphi(\tau,\bar{c})\ell_t} - 2e^{\upsilon(\tau,\bar{c}')+\phi(\tau,\bar{c}')\mu_t+\varphi(\tau,\bar{c}')\ell_t} + e^{\upsilon(\tau,\bar{c}'')+\phi(\tau,\bar{c}'')\mu_t+\varphi(\tau,\bar{c}'')\ell_t}\right), \\ &\quad \text{with } \bar{c} = (0,2,0,2),\ \bar{c}' = (0,2,0,1) \text{ and } \bar{c}'' = (0,2,0,0). \end{aligned} \tag{A72}$$

To compute the price of the dividend strips and the zero-coupon bonds:

$$\begin{aligned} P^C_{t,\tau} &= \mathbb{E}_t\left[\tfrac{\xi_{t+\tau}}{\xi_t} C_{t+\tau}\right], \\ P_{t,\tau} &= \mathbb{E}_t\left[\tfrac{\xi_{t+\tau}}{\xi_t} D_{t+\tau}\right], \\ B_{t,\tau} &= \mathbb{E}_t\left[\tfrac{\xi_{t+\tau}}{\xi_t}\right], \end{aligned} \tag{A73}$$

we need the following expectations:

$$\begin{aligned} \mathbb{E}_t\left[\xi_{t+\tau} C_{t+\tau}\right] &= \mathbb{E}_t[\xi_{t+\tau} L_{t+\tau} e^{\ell_{t+\tau}}] && \text{(A74)} \\ &= \xi_t L_t e^{\upsilon(\tau,\bar{c})+\phi(\tau,\bar{c})\mu_t+\varphi(\tau,\bar{c})\ell_t}, \text{ with } \bar{c}=(1,1,0,1), \\ \mathbb{E}_t\left[\xi_{t+\tau} D_{t+\tau}\right] &= \mathbb{E}_t[\xi_{t+\tau} L_{t+\tau}(e^{\ell_{t+\tau}}-1)] && \text{(A75)} \\ &= \xi_t L_t \left(e^{\upsilon(\tau,\bar{c})+\phi(\tau,\bar{c})\mu_t+\varphi(\tau,\bar{c})\ell_t} - e^{\upsilon(\tau,\bar{c}')+\phi(\tau,\bar{c}')\mu_t+\varphi(\tau,\bar{c}')\ell_t}\right), \\ &\quad \text{with } \bar{c}=(1,1,0,1) \text{ and } \bar{c}'=(1,1,0,0), \\ \mathbb{E}_t\left[\xi_{t+\tau}\right] &= \xi_t e^{\upsilon(\tau,\bar{c})+\phi(\tau,\bar{c})\mu_t+\varphi(\tau,\bar{c})\ell_t}, \text{ with } \bar{c}=(1,0,0,0). \end{aligned}$$

It easily follows that the bond price and the valuation ratios $P^C_{t+\tau}/C_t$ and $P_{t+\tau}/D_t$ are stationary functions of μ_t, ℓ_t and the maturity only.

The dynamics of $P_{t,\tau}$ obtains by applying Itô's Lemma:

$$dP_{t,\tau} = [\cdot]dt + \tfrac{\partial P_{t,\tau}}{\partial L}\sigma_L L_t dB_{L,t} + \tfrac{\partial P_{t,\tau}}{\partial \mu}\nu dB_{\mu,t} + \tfrac{\partial P_{t,\tau}}{\partial \ell}\eta\sqrt{\ell_t}dB_{\ell,t},$$

and therefore its return volatility is given by

$$\sigma_{R,t,\tau} = P^{-1}_{t,\tau}\sqrt{\left(\tfrac{\partial P_{t,\tau}}{\partial L}\sigma_L L_t\right)^2 + \left(\tfrac{\partial P_{t,\tau}}{\partial \mu}\nu\right)^2 + \left(\tfrac{\partial P_{t,\tau}}{\partial \ell}\eta\right)^2 \ell_t},$$

which is a function of μ_t, ℓ_t and the maturity only. The premium on the dividend strip is given by $-\frac{1}{dt}\left\langle \frac{dP_\tau}{P_\tau}, \frac{d\xi}{\xi}\right\rangle_t$:

$$\mu_{R,t,\tau} = P^{-1}_{t,\tau}\left(\tfrac{\partial P_{t,\tau}}{\partial L}\sigma_L L_t \pi_L + \tfrac{\partial P_{t,\tau}}{\partial \mu}\nu\pi_\mu + \tfrac{\partial P_{t,\tau}}{\partial \ell}\eta\pi_\ell\ell_t\right),$$

which is a function of μ_t, ℓ_t and the maturity only. Indeed, we have

$$\begin{aligned} P^{-1}_{t,\tau}\left(\tfrac{\partial P_{t,\tau}}{\partial L}\right)L_t &= 1, \\ P^{-1}_{t,\tau}\left(\tfrac{\partial P_{t,\tau}}{\partial \mu}\right) &= \tfrac{\phi(\tau,\bar{c})e^{\upsilon(\tau,\bar{c})+\phi(\tau,\bar{c})\mu_t+\varphi(\tau,\bar{c})\ell_t} - \phi(\tau,\bar{c}')e^{\upsilon(\tau,\bar{c}')+\phi(\tau,\bar{c}')\mu_t+\varphi(\tau,\bar{c}')\ell_t}}{e^{\upsilon(\tau,\bar{c})+\phi(\tau,\bar{c})\mu_t+\varphi(\tau,\bar{c})\ell_t} - e^{\upsilon(\tau,\bar{c}')+\phi(\tau,\bar{c}')\mu_t+\varphi(\tau,\bar{c}')\ell_t}}, \\ P^{-1}_{t,\tau}\left(\tfrac{\partial P_{t,\tau}}{\partial \ell}\right) &= \tfrac{\varphi(\tau,\bar{c})e^{\upsilon(\tau,\bar{c})+\phi(\tau,\bar{c})\mu_t+\varphi(\tau,\bar{c})\ell_t} - \varphi(\tau,\bar{c}')e^{\upsilon(\tau,\bar{c}')+\phi(\tau,\bar{c}')\mu_t+\varphi(\tau,\bar{c}')\ell_t}}{e^{\upsilon(\tau,\bar{c})+\phi(\tau,\bar{c})\mu_t+\varphi(\tau,\bar{c})\ell_t} - e^{\upsilon(\tau,\bar{c}')+\phi(\tau,\bar{c}')\mu_t+\varphi(\tau,\bar{c}')\ell_t}}, \end{aligned}$$

with $\bar{c}=(1,1,0,1)$ and $\bar{c}'=(1,1,0,0)$. The functions h_0, h_1 and h_2 in Eq. (62)-(63)-(64) automatically follow.

The price of the market asset easily follows from that of the dividend strip:

$$P_t = \mathbb{E}_t\left[\int_t^\infty \tfrac{\xi_u}{\xi_t} D_u du\right] = \int_0^\infty P_{t,\tau} d\tau, \tag{A76}$$

and, hence, the premium, the return volatility and the valuation ratio, P_t/D_t, are stationary functions of μ_t and ℓ_t only. Indeed, we have

$$\begin{aligned}
&P_t^{-1}\left(\tfrac{\partial P_t}{\partial L}\right)L_t = 1,\\
&P_t^{-1}\left(\tfrac{\partial P_t}{\partial \mu}\right) = \frac{\int_0^\infty \left(\phi(\tau,\bar{c})e^{\upsilon(\tau,\bar{c})+\phi(\tau,\bar{c})\mu_t+\varphi(\tau,\bar{c})\ell_t} - \phi(\tau,\bar{c}')e^{\upsilon(\tau,\bar{c}')+\phi(\tau,\bar{c}')\mu_t+\varphi(\tau,\bar{c}')\ell_t}\right)d\tau}{\int_0^\infty \left(e^{\upsilon(\tau,\bar{c})+\phi(\tau,\bar{c})\mu_t+\varphi(\tau,\bar{c})\ell_t} - e^{\upsilon(\tau,\bar{c}')+\phi(\tau,\bar{c}')\mu_t+\varphi(\tau,\bar{c}')\ell_t}\right)d\tau},\\
&P_t^{-1}\left(\tfrac{\partial P_t}{\partial \ell}\right) = \frac{\int_0^\infty \left(\varphi(\tau,\bar{c})e^{\upsilon(\tau,\bar{c})+\phi(\tau,\bar{c})\mu_t+\varphi(\tau,\bar{c})\ell_t} - \varphi(\tau,\bar{c}')e^{\upsilon(\tau,\bar{c}')+\phi(\tau,\bar{c}')\mu_t+\varphi(\tau,\bar{c}')\ell_t}\right)d\tau}{\int_0^\infty \left(e^{\upsilon(\tau,\bar{c})+\phi(\tau,\bar{c})\mu_t+\varphi(\tau,\bar{c})\ell_t} - e^{\upsilon(\tau,\bar{c}')+\phi(\tau,\bar{c}')\mu_t+\varphi(\tau,\bar{c}')\ell_t}\right)d\tau},
\end{aligned}$$

with $\bar{c} = (1,1,0,1)$ and $\bar{c}' = (1,1,0,0)$. The functions w_0, w_1 and w_2 in Eq. (65)-(66)-(67) automatically follow.

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