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Rent-seeking contests with private valuations

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Abstract

We study a rent-seeking contest in which players have heterogeneous and private valuations. In addition to their own type, agents only know that all valuations are drawn from an unspecified distribution, of which they only know the mean. We obtain a closed-form solution for agents' optimal level of investment and subject it to comparative statics analysis. We also investigate the issue of entry in the game and the amount of rent dissipation that results in equilibrium. Finally, we compare our results with those that would emerge in a context of perfect information.

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1 Introduction

A rent-seeking game is a probabilistic contest in which players exert costly effort in order to influence the probability that they will be awarded a prize. Rent-seeking games were first investigated by Tullock (1980). In the following decades, Tullock's seminal model has been extended and generalized in many important directions (see Congleton *et al.*, 2008, for a recent and comprehensive literature review). In particular, an interesting and ongoing line of research aims at relaxing the standard hypothesis according to which all participants in the game share a common and publicly known valuation of the prize.

Indeed, in many typical applications of rent-seeking games (lobbying for political favor, R&D races to secure a patent, fighting among individuals, organizations, or countries to conquer a contested resource), the alternative assumption of asymmetric and private valuations seems more realistic. For instance, in the case of R&D expenditures, different competitors may assess the potential of the patent according to different information or in light of different scenarios, where these may in turn be influenced by agents' personal attitudes and biases. Furthermore, clearly no agent has incentives to truthfully disclose his valuation to rivals.

Initial contributions that pursued this line of research allowed for heterogeneity in players' valuations but maintained the assumption of their common knowledge (see, for instance, Hillman & Riley, 1989; Nti, 1999; Stein, 2002). On top of asymmetry, other papers then investigated the consequences of the privacy of players' valuations in various contexts:[1] two-player games with one-

[1] This line of research, which is the one we pursue in this paper, assumes that agents know their own type (i.e., their own valuation of the prize) but are uncertain about the other

sided private information and continuous types (Hurley & Shogren, 1998a), two-player games with two-sided private information and discrete types (Hurley & Shogren, 1998b, Malueg & Yates, 2004) or continuous types (Ewerhart, 2010), n-player games with one-sided private information and discrete types (Schoonbeek & Winkel, 2006).

This paper contributes to the literature by introducing and studying a rent-seeking game in which $n \geq 2$ players have asymmetric and private valuations. In particular, we study a framework in which the only information available to players (in addition to the knowledge of their own type) is that all valuations are identically and independently distributed according to an unknown probability distribution with mean $\bar{v}$. Note that this is a weaker hypothesis with respect to the standard private values assumption that requires players to know the entire distribution of types and not just its mean.[2]

The information structure that we adopt is thus minimal and can mimic a number of real-life situations. For instance, it can apply to all those cases in which players do not know the identity of the other participants. As such, agents cannot infer or assess their rivals' valuations based on their reputation or observable characteristics and must therefore rely on some very general summary statistic such as the mean or expected value. The common knowledge about this statistic could in turn stem from different sources. For instance, the principal might have superior information and publicly announce the mean

participants' valuations. A different strand of the literature (see, for instance, Wärneryd, 2003) investigates instead rent-seeking games where players are uncertain about their own valuation since they cannot properly ex-ante assess the quality of the prize. More in general, participants in a contest may be heterogeneous also across other dimensions such as the effectiveness of their lobbying efforts on the probability of winning, their cost functions, or their budget constraints (see for instance Yamazaki, 2008).

[2] The private value assumption is commonly used in many contexts (e.g., auction theory; see Krishna, 2002) and its implications have also been explored in the case of rent-seeking games (Hurley & Shogren, 1998a).

valuation before the contest opens; or agents may use as a proxy the mean value that emerged in previous rent-seeking games with similar prizes; or some external player (say an authoritative expert, a think-tank, or a governmental agency) may publicly provide a valuation of the prize that thus becomes a natural focal point agents will use to attribute a valuation to their rivals.[3]

We explicitly solve the model for the case of constant returns to scale success function and obtain closed-form solutions for the agent's optimal level of investment, as well as for his perceived and actual probability of winning and expected profits in equilibrium. We subject these results to comparative statics analysis and investigate the issue of rent dissipation and entry in the game. We find that in equilibrium an agent dissipates in rent-seeking activities at most 25% of his valuation. This upper bound is constant: we show that it does not depend on the agent's type, on how this compares with the mean value, and on the number of potential participants. Concerning entry, we find that an agent invests a strictly positive amount if and only if his private valuation is above a certain threshold that we analytically pin down. In particular, we show that a "strong" player (i.e., a player whose valuation is above the average) always participates while a "weak" player decides to participate or not depending on the number of competitors. While existing literature already highlighted how asymmetric valuations may act as a barrier to entry (Hillman & Riley, 1989; Stein, 2002), our analysis shows how the combination of heterogeneity and imperfect information can sometimes exacerbate and sometimes contrast this effect.

The remainder of the paper is organized as follows: Section 2 introduces the

[3] Concerning this last point, many countries recently set up specific agencies (both at the national and at the local level) whose task is to provide so-called standard costs (i.e., mean valuation) for the supply to the public sector of goods and services that are assigned through procurement auctions.

model and derives agents' optimal level of investment. Section 3 performs some comparative statics exercises on equilibrium results. Section 4 investigates the issue of rent dissipation on the individual and the aggregate level. Section 5 compares the results of the model with those that would emerge in a context of perfect information. Section 6 presents our conclusions.

2 The model

Consider a rent-seeking game in which $n \geq 2$ risk neutral players compete to win a prize. Players' valuations of the prize are heterogeneous and $v_i \in (0, v_{\max}]$ indicates the valuation of player $i \in N$ with $N = \{1, ..., n\}$. The actual realization of v_i is agent i's private information. It is instead common knowledge that all valuations are identically and independently distributed according to an unspecified and unknown probability distribution with mean $\bar{v}$.

Players can exert effort (or devote resources) in order to influence their chances of winning the prize. Call $x_i \in [0, v_i]$ the effort chosen by player i (we measure effort in units commensurate with the rent) and let the vector $x = (x_1, ..., x_n)$ collect the choices of all the players. The probability $P_i(x)$ with which a generic player i wins the prize follows the famous logit specification originally proposed by Tullock (1980). In particular, and in order to obtain closed-form solutions (see Stein, 2002), we adopt the formulation that features constant returns to scale such that $P_i(x) = \frac{x_i}{x_i + \sum_{j \neq i} x_j}$.[4] We also assume that

[4] As is well known, a more general formulation of the success function is given by $P_i(x) = \frac{x_i^r}{x_i^r + \sum_{j \neq i} x_j^r}$ where the parameter $r > 0$ measures the returns to scale of a player's investment on effort. The rent-seeking technology shows decreasing returns to scale if $r \in (0, 1)$, constant returns to scale if $r = 1$, and increasing returns to scale if $r > 1$.

$P_i(x) = \frac{1}{n}$ if $x = (0, ..., 0)$.

Each player must simultaneously choose how much effort to exert. The optimal level, which we label $\hat{x}_i$, is the one that maximizes a player's expected payoff π_i:

$$\max_{x_i} \pi_i(x) = \left(\frac{x_i}{x_i + \sum_{j \neq i} x_j} \right) v_i - x_i \tag{1}$$

In tackling this problem, player i does not know, nor he can infer, the levels of effort that his opponents will choose. In fact, the optimal investment of generic agent $j \neq i$ depends on the valuation v_j (i.e., $\hat{x}_j = \hat{x}_j(v_j)$), which is agent j's private information.

The only information available to agent i is that all valuations are independently drawn from a distribution with mean (or expected value) $\bar{v}$. The agent thus necessarily sets $\hat{x}_j = \hat{x}_j(\bar{v})$ for any of his $(n-1)$ opponents. Therefore, from i's point of view, problem 1 becomes:

$$\max_{x_i} \pi_i(x) = \left(\frac{x_i}{x_i + (n-1)\hat{x}_j(\bar{v})} \right) v_i - x_i \tag{2}$$

Player i can actually explicitly compute $\hat{x}_j(\bar{v})$. The player in fact not only assigns a valuation $\bar{v}$ to every agent $j \neq i$, he also expects every other participant in the game to adopt a similar behavior. More precisely, he expects any player $j \neq i$ to attach a valuation $\bar{v}$ to all his opponents $k \neq j$. However, the set of these players includes agent i himself. In other words, agent i ascribes to every agent $j \neq i$ the same behavior that player j would adopt in a rent-seeking game in which all the participants had a homogeneous valuation $\bar{v}$. More formally, agent i sets $\hat{x}_j(\bar{v}) = x'_j(\bar{v})$ for any $j \neq i$ where $x'_j(\bar{v}) = \left(\frac{n-1}{n^2}\right) \bar{v}$ is the equilibrium solution of a standard rent-seeking game among n players with valuations $v_i = \bar{v}$ for any $i \in N$.[5] Player i's problem thus becomes:

$$\max_{x_i} \pi_i(x) = \left(\frac{x_i}{x_i + (n-1)\left(\frac{n-1}{n^2}\right)\bar{v}} \right) v_i - x_i \tag{3}$$

Necessary and sufficient conditions for an interior solution are:

$$\frac{\partial \pi_i(x)}{\partial x_i} = \left(\frac{\left(\frac{n-1}{n}\right)^2 \bar{v}}{\left(x_i + \left(\frac{n-1}{n}\right)^2 \bar{v}\right)^2} \right) v_i - 1 = 0 \tag{4}$$

$$\frac{\partial^2 \pi_i(x)}{\partial x_i^2} = -\frac{2\left(\frac{n-1}{n}\right)^2 \bar{v}}{\left(x_i + \left(\frac{n-1}{n}\right)^2 \bar{v}\right)^3} v_i < 0 \tag{5}$$

One can immediately see that condition 5 is always verified. Therefore, solving condition 4 with respect to x_i yields $\hat{x}_i$, the optimal level of investment of agent i:[6]

$$\hat{x}_i = \frac{n-1}{n}\sqrt{\bar{v}v_i} - \left(\frac{n-1}{n}\right)^2 \bar{v} \tag{6}$$

Note that $\hat{x}_i > 0$ if and only if $v_i > \lambda$ where $\lambda = \left(\frac{n-1}{n}\right)^2 \bar{v}$. In other words, an agent actively participates in the rent-seeking game (i.e., he invests a strictly positive amount) if and only if his personal valuation is above a certain threshold λ. This threshold is an increasing function of n and $\bar{v}$. Still, the condition $\lambda < \bar{v}$ always holds. This implies that a "strong" player (i.e., a

[5] The standard result $x'_j(\bar{v}) = \left(\frac{n-1}{n^2}\right)\bar{v}$ emerges as the solution to the following problem: $\max_{x_j} \pi_j(x) = \left(\frac{x_j}{x_j + \sum_{k \neq j} x_k}\right)\bar{v} - x_j$ subject to $v_k = \bar{v}$ for any $k \neq j$.

[6] More precisely, problem 3 has at least a real solution whenever $\left(\frac{n-1}{n^2}\right)\bar{v}v_i \neq 0$. In our context, such a condition is always verified as all the terms in the product are strictly positive. The problem then actually has two real solutions: $x_i^1 = \frac{n-1}{n}\sqrt{\bar{v}v_i} - \left(\frac{n-1}{n}\right)^2 \bar{v}$ and $x_i^2 = -\frac{n-1}{n}\sqrt{\bar{v}v_i} - \left(\frac{n-1}{n}\right)^2 \bar{v}$. Note that only the first solution is meaningful given that x_i^2 is always negative. Therefore, $\hat{x}_i = x_i^1$.

player with a valuation $v_i \geq \bar{v}$) always actively participates in the game. On the other hand, a "weak" player may decide to participate ($\lambda < v_i < \bar{v}$) or not ($v_i \leq \lambda < \bar{v}$). In particular, and everything else being equal, a player of type $v_i < \bar{v}$ may invest in rent-seeking activities if the game features only a few competitors but could instead abstain if competition looks tougher.[7]

Combining the optimal solution to the profit maximization problem with the constraint that defines active participation in the game, one obtains a player's optimal strategy, defined by the following proposition:

Proposition 1 *Consider a rent-seeking game among $n \geq 2$ players with heterogeneous and private valuations v_i and common knowledge about the mean valuation $\bar{v}$. An agent's optimal investment strategy x_i^* takes the following form:*

$$x_i^* = \begin{cases} \frac{n-1}{n}\sqrt{\bar{v}v_i} - \left(\frac{n-1}{n}\right)^2 \bar{v} & \text{if } \; v_i > \left(\frac{n-1}{n}\right)^2 \bar{v} \\ 0 & \text{otherwise} \end{cases}$$

3 Comparative statics and equilibrium results

In this section, we perform comparative statics exercises and investigate how an agent's optimal level of investment is influenced by the various parameters of the game. We then study some more general properties that characterize the equilibrium solution.

[7] Note, however, that a player that perceives himself as being very weak (that is, a player whose private valuation is much lower than the mean value) may refuse to participate even when he faces a single opponent (i.e., $n = 2$). This happens when the condition $v_i \leq \frac{1}{4}\bar{v}$ holds. The fact that a player may be inactive even when $n = 2$ is consistent with the analysis of Schoonbeek & Winkel (2006) while can never happen in models of perfect information such as Nti (1999), where the two players always participate, or Stein (2002), where non-entry of some player can occur only when $n \geq 3$.

Consider an agent of type $v_i > \lambda$ where, as defined above, $\lambda = \left(\frac{n-1}{n}\right)^2 \bar{v}$. In line with Proposition 1, agent i thus invests an amount $x_i^* = \frac{n-1}{n}\sqrt{\bar{v}v_i} - \left(\frac{n-1}{n}\right)^2 \bar{v}$ in rent-seeking activities, with $x_i^* > 0$. As a first observation, note that, whenever $v_i = \bar{v}$, the equilibrium solution cleanly reduces to $x_i^* = \left(\frac{n-1}{n^2}\right)\bar{v}$, which is the standard solution of a rent-seeking game among n symmetric players of type $\bar{v}$. The intuition is straightforward: in the model, an agent attributes a valuation $\bar{v}$ to all of his opponents and he also expects them to do the same; it follows that an agent of type $v_i = \bar{v}$ behaves as if he was involved in a rent-seeking game where all of the n players have a homogeneous valuation $\bar{v}$. In other words, the standard model à la Tullock (1980) is nested into our more general framework.

Second, an agent's optimal level of investment is an increasing function of his private valuation. The marginal effect of v_i on x_i^* is given by $\frac{\partial x_i^*}{\partial v_i} = \frac{1}{2n}\frac{\bar{v}}{\sqrt{\bar{v}v_i}}(n-1)$, which is always positive. Still, this marginal effect is decreasing since x_i^* is a concave function of v_i.[8] It follows that x_i^* is more sensitive to changes in v_i when the agent's valuation is low rather than high.

Perhaps less intuitive is the fact that the marginal effect of $\bar{v}$ on x_i^* is non monotonic.[9] Whenever the mean valuation is below the threshold defined by $\tilde{v} = \frac{1}{4}\left(\frac{n}{n-1}\right)^2 v_i$, an increase in $\bar{v}$ boosts the agent i's equilibrium effort x_i^* (although at a decreasing rate): as $\bar{v}$ approaches $\tilde{v}$ from below, agent i expects more aggressive rent-seeking from his opponents but he also continues to perceive himself as the strongest player ($\bar{v} < \tilde{v} < v_i$). Therefore, he increments his investment so as to maintain his good chances of winning the

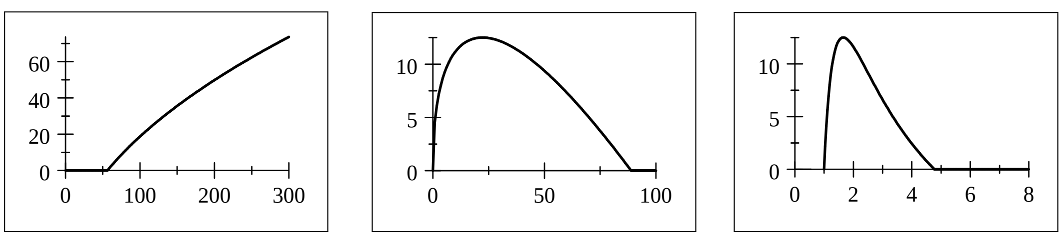
[8] The second derivative is given by $\frac{\partial^2 x_i^*}{\partial v_i^2} = -\frac{1}{4n}\frac{v^2}{(vv_i)^{\frac{3}{2}}}(n-1)$ such that $\frac{\partial^2 x_i^*}{\partial v_i^2} < 0$ for any v_i.

[9] The marginal effect is given by $\frac{\partial x_i^*}{\partial \bar{v}} = \frac{(n-1)}{2n}\frac{v_i}{\sqrt{\bar{v}v_i}} - \left(\frac{n-1}{n}\right)^2$ such that $\frac{\partial x_i^*}{\partial \bar{v}} > 0$ for $\bar{v} < \tilde{v}$ while $\frac{\partial x_i^*}{\partial \bar{v}} < 0$ for $\bar{v} > \tilde{v}$ where $\tilde{v} = \frac{1}{4}\left(\frac{n}{n-1}\right)^2 v_i$.

contest. On the contrary, whenever $\bar{v} > \tilde{v}$, an hypothetical further increase of $\bar{v}$ has a negative (and progressively stronger) effect on x_i^* as agent i now expects a much tougher competition and thus adopts a softer strategy. Indeed, as $\bar{v}$ increases, the agent progressively lowers his equilibrium effort up to the point of nullifying it as soon as $\bar{v} > \left(\frac{n}{n-1}\right)^2 v_i$.

Finally, the marginal effect of n on x_i^* is also potentially non monotonic. The effect is given by $\frac{\partial x_i^*}{\partial n} = \frac{1}{n^3}\left(2\bar{v} + n\sqrt{\bar{v}v_i} - 2n\bar{v}\right)$, which is a concave function that reaches its maximum at $\tilde{n} = \frac{-2\bar{v}}{\sqrt{\bar{v}v_i} - 2\bar{v}}$. Therefore, if $\tilde{n} \leq 2$, the condition $\tilde{n} \leq n$ certainly holds and an agent's optimal level of investment x_i^* monotonically decreases with the number of participants in the game. Whenever $\tilde{n} > 2$, equilibrium effort instead initially increases with n but then monotonically decreases as soon as $n > \tilde{n}$.

The following figures illustrate the effects that v_i (Figure 1.a), $\bar{v}$ (Figure 1.b), and n (Figure 1.c) have on agent i's optimal level of investment x_i^* in some specific examples.[10]



1.a) $x_i^*(v_i), n = 4, \bar{v} = 100$ 1.b) $x_i^*(\bar{v}), n = 4, v_i = 50$ 1.c) $x_i^*(n), v_i = 50, \bar{v} = 80$

It is also interesting to compute the probability with which agent i expects to win the contest given his optimal investment strategy $x_i^* > 0$ and his

[10] For illustrative purposes in figure 1.c (as well as in the derivation of the marginal effect) the number of players n is treated as a continuous variable.

conjecture that $x'_j(\bar{v}) = \left(\frac{n-1}{n^2}\right)\bar{v}$ for any $j \neq i$. This probability is given by:

$$P_i\left(x_i^*, \{x'_j\}_{j\neq i}\right) = 1 - \left(\frac{n-1}{n}\right)\sqrt{\frac{\bar{v}}{v_i}} \tag{7}$$

with $P_i\left(x_i^*, \{x'_j\}_{j\neq i}\right) \in (0,1)$. In particular, agent i's perceived probability is ensured to be strictly positive given that the condition $P_i\left(x_i^*, \{x'_j\}_{j\neq i}\right) > 0$ holds whenever $v_i > \left(\frac{n-1}{n}\right)^2 \bar{v}$, which is the same constraint that defines active participation in the game (see Proposition 1).[11] Obviously, in equilibrium a player exerts positive effort only if his expected profits (which we formally define below) are strictly positive and this necessarily requires $P_i\left(x_i^*, \{x'_j\}_{j\neq i}\right)$ to be positive.[12]

While one can immediately see that $P_i\left(x_i^*, \{x'_j\}_{j\neq i}\right)$ is increasing with v_i and decreasing with n and $\bar{v}$, it is interesting to note how expression 7 relates an agent's expected probability of winning to the concept of relative resolve, as introduced in Hurley & Shogren (1998a, 1998b). The relative resolve of agent i with respect to a generic agent j is defined as $\rho_i = \sqrt{\frac{v_i}{v_j}}$ and thus provides a measure of the relative strength of the player. In particular, agent i is stronger (weaker) than j when $\rho_i > 1$ ($\rho_i < 1$). Expression 7 shows that $P_i\left(x_i^*, \{x'_j\}_{j\neq i}\right)$ decreases with $\sqrt{\frac{\bar{v}}{v_i}}$. It follows that agent i's expected probability of winning increases with $\rho_i = \sqrt{\frac{v_i}{\bar{v}}}$, which can be interpreted as the relative resolve of player i with respect to his "representative rival" of type

[11] Here as well, our results subsume those that arise in a standard rent-seeking game with homogeneous valuations and perfect information. Expression (7) shows in fact that $P_i\left(x_i^*, \{x'_j\}_{j\neq i}\right) = \frac{1}{n}$ whenever $v_i = \bar{v}$.

[12] A positive expected probability of winning is a necessary but not sufficient condition to ensure active participation of an agent. The probability of winning is in fact certainly positive for any strictly positive level of effort $x_i > 0$. Still, this probability and/or the agent's valuation v_i may be too small such that their product (i.e., the agent's expected revenues) may fall short of the cost of exerting effort and the agent thus prefers not to participate.

$\bar{v}$.

Given the expected probability of winning defined in expression 7, one can also compute an agent's expected profits. These are given by $\pi_i\left(x_i^*, \{x_j'\}_{j\neq i}\right) = P_i\left(x_i^*, \{x_j'\}_{j\neq i}\right) v_i - x_i^*$ and can thus be explicitly expressed as:

$$\pi_i\left(x_i^*, \{x_j'\}_{j\neq i}\right) = \frac{1}{n^2}\left(n\sqrt{v_i} - n\sqrt{\bar{v}} + \sqrt{\bar{v}}\right)^2 \qquad (8)$$

Expected profits are thus strictly increasing with v_i and, for the range of admissible parameters, strictly decreasing with $\bar{v}$.[13]

With respect to the expected probability of winning as reported in expression 7, the actual probability of winning (i.e., the probability that emerges in equilibrium when every agent $i \in N$ plays his optimal investment strategy x_i^*) may differ. The latter is given by:

$$P_i(x^*) = \frac{\sqrt{v_i} - \left(\frac{n-1}{n}\right)\sqrt{\bar{v}}}{\sqrt{v_i} - (n-1)\sqrt{\bar{v}} + \sum_{j\neq i}\sqrt{v_j}} \qquad (9)$$

which increases with v_i, decreases with v_j, and it is such that $P_i(x^*) = \frac{1}{n}$ when $v_i = v_{j\neq i} = \bar{v}$. The expected probability of winning (expression 7) and the actual probability of winning (expression 9) agree only when $(n-1)\sqrt{\bar{v}} = \sum_{j\neq i}\sqrt{v_j}$, i.e., when the sum of (the square root of) agent i's expectations about his opponents' valuations equals the sum of the (square root of the) actual valuations of agent i's rivals. If this is not the case, then agent i overestimates his probability of winning whenever he underestimates the aggregate strength of his rivals $\left((n-1)\sqrt{\bar{v}} < \sum_{j\neq i}\sqrt{v_j}\right)$ and he underestimates his

[13] More precisely, function 8 is convex in $\bar{v}$ and reaches its minimum $\pi_i(\cdot) = 0$ at $\bar{v} = \left(\frac{n}{n-1}\right)^2 v_i$. Therefore, whenever the participation constraint $v_i > \left(\frac{n-1}{n}\right)^2 \bar{v}$ holds (i.e., $\bar{v} < \left(\frac{n}{n-1}\right)^2 v_i$), expected profits are positive but strictly decreasing with $\bar{v}$.

probability of winning when the opposite holds true.

The following example illustrates all the results obtained so far in the context of a specific rent-seeking game.

Example 1 *Consider a rent-seeking game with four players with private valuations* $v_1 = 36$, $v_2 = 25$, $v_3 = 16$, *and* $v_4 = 9$. *All players know that the mean valuation in the population is* $\bar{v} = 16$. *Each player* $i \in N$ *thus expects each one of his opponents to play* $x'_{j \neq 1} = 3$. *In line with Proposition 1, it follows that equilibrium levels of investment are* $x_1^* = 9$, $x_2^* = 6$, $x_3^* = 3$, *and* $x_4^* = 0$ *(the participation constraint requires* $v_i > 9$*). Now, for illustrative purposes, focus on player* 2. *The player expects to win the game with probability* $P_2(3, 6, 3, 3) = \frac{2}{5}$ *(see expression 7), which implies an expected payoff of* $\pi_2(3, 6, 3, 3) = 4$ *(see expression 8). Given that* $\sum_{j \neq 2} \sqrt{v_j} > (n-1)\sqrt{\bar{v}}$ *(i.e.,* $13 > 12$*), agent* 2*'s actual probability of winning* $P_1(9, 6, 3, 0) = \frac{1}{3}$ *(see expression 9) is lower than his expected probability of winning as agent* 2 *slightly underestimates the total strength of his rivals.*

4 Rent dissipation in equilibrium

We now investigate the issue of rent dissipation at the individual level (RD_i) and at the aggregate level (RD). We define rent dissipation at the individual level as the fraction of an agent's valuation that is invested in rent-seeking activities. In equilibrium, a player with valuation v_i thus dissipates an amount:

$$RD_i^* = \frac{x_i^*}{v_i} = \left(\frac{n-1}{n}\right)\left(\frac{\sqrt{\bar{v}}}{\sqrt{v_i}} - \left(\frac{n-1}{n}\right)\frac{\bar{v}}{v_i}\right) \tag{10}$$

Once again, this expression subsumes the results of the standard model with homogeneous valuations. In fact, when $v_i = \bar{v}$, expression 10 simplifies to $RD_i^* = \frac{(n-1)}{n^2}$. As is well known, this is a constant ratio that does not depend on v_i. However, a part from this specific case, expression 10 shows that in general RD_i^* does depend on v_i. In particular, rent dissipation at the individual level is increasing in the agent's valuation for $v_i < \tilde{v}_i$ and decreasing for $v_i > \tilde{v}_i$ where $\tilde{v}_i = \left(\frac{n-1}{n}\right)^2 4\bar{v}$.[14] Similarly, and holding v_i fixed, rent dissipation is a concave function of $\bar{v}$ that reaches its maximum at $\tilde{v} = \frac{1}{4}\left(\frac{n}{n-1}\right)^2 v_i$.[15] The shape of the RD_i^* function is driven by the behavior of x_i^* that, as has been shown, is strictly concave in v_i and $\bar{v}$.

One can also relate rent dissipation at the individual level with the relative resolve of agent i with respect to the representative rival of type $\bar{v}$. Defining the relative resolve of i as $\rho_i = \frac{\sqrt{v_i}}{\sqrt{\bar{v}}}$ and rearranging expression 10 one obtains:

$$RD_i^* = \left(\frac{n-1}{n}\right)\left[\frac{1}{\rho_i} - \left(\frac{n-1}{n}\right)\frac{1}{\rho_i^2}\right] \tag{11}$$

In equilibrium, RD_i^* is thus first increasing and then decreasing with ρ_i. Rent dissipation reaches its maximum for $\tilde{\rho}_i = \frac{1}{n}(2n-2)$, in which case $RD_i^*(\tilde{\rho}_i) = \frac{1}{4}$. Note that this maximum is a constant that does not depend on the number of participants n. Therefore, the amount that an agent is willing to dissipate in rent-seeking activities never exceeds 25% of his valuation, no matter the value of his relative resolve (and thus the values of v_i and $\bar{v}$) or the number of participants in the game.

To illustrate this peculiar result, the following graph depicts RD_i^* as a

[14] The marginal effect is given by $\frac{\partial RD_i^*}{\partial v_i} = -\frac{(n-1)}{2n^2 v_i^2}\left(2\bar{v} + n\sqrt{\bar{v}v_i} - 2n\bar{v}\right)$ such that $\frac{\partial RD_i^*}{\partial v_i} > 0$ for $v_i < \left(\frac{n-1}{n}\right)^2 4\bar{v}$ and $\frac{\partial RD_i^*}{\partial v_i} < 0$ for $v_i > \left(\frac{n-1}{n}\right)^2 4\bar{v}$.

[15] In this case, the marginal effect is given by $\frac{\partial RD_i^*}{\partial \bar{v}} = \frac{(n-1)}{n^2 v_i}\left(\frac{1}{2}n\frac{v_i}{\sqrt{\bar{v}v_i}} - n + 1\right)$ such that $\frac{\partial RD_i^*}{\partial \bar{v}} > 0$ for $\bar{v} < \frac{1}{4}\left(\frac{n}{n-1}\right)^2 v_i$ and $\frac{\partial RD_i^*}{\partial \bar{v}} < 0$ for $\bar{v} > \frac{1}{4}\left(\frac{n}{n-1}\right)^2 v_i$.

function of ρ_i in three different rent-seeking games that are characterized by $n = 2$, $n = 4$, and $n = 100$ (left to right). In all cases, the participation constraint implies a positive investment $x_i^* > 0$ for $\rho_i > \left(\frac{n-1}{n}\right)$.

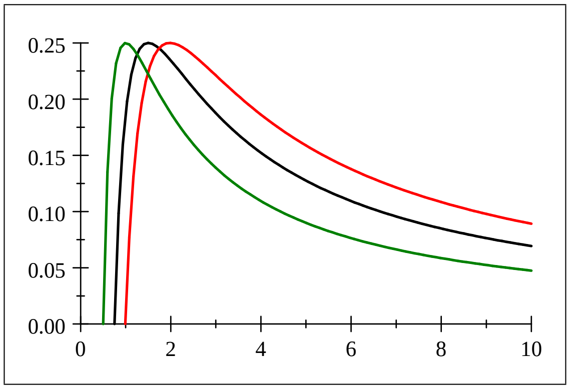


Figure 2: Examples of rent dissipation at the individual level.

In line with the literature (see, among others, Hurley & Shogren, 1998a, 1998b; Stein, 2002), we define rent dissipation at the aggregate level as the total expenditures by all the players.[16] Therefore, in equilibrium, $RD^* = \sum_i x_i^*$.

To obtain an explicit formulation for RD^*, one must consider that in equilibrium not all the players necessarily invest a positive amount (see Proposition 1). We define the set of active players as $M = \left\{ i \in N \mid v_i > \left(\frac{n-1}{n}\right)^2 \bar{v} \right\}$, i.e., those agents that play $x_i^* > 0$. Therefore, $M \subseteq N$, or, equivalently, $m \leq n$. Rent dissipation at the aggregate level is then given by:

$$RD^* = \sum\nolimits_{i \in M} \left[\left(\frac{n-1}{n} \right) \sqrt{\bar{v} v_i} \right] - m \left(\frac{n-1}{n} \right)^2 \bar{v} \tag{12}$$

RD^* is thus weakly increasing and weakly concave in any individual valuation v_i with $i \in N$. The "weakness" of these relations stems from the fact that

[16] Note in fact that whenever agents' valuations are heterogeneous, the sum of individual rent dissipations (i.e., $\sum_i RD_i^*$) makes no sense. Each RD_i^* is in fact computed with respect to the agent's specific valuation v_i.

a small increase in the valuation of an agent that does not participate (i.e., $i \notin M$) may still not be enough to convince him to actually invest a positive amount. If this is the case, then RD^* obviously would not change. On the contrary, the possible positive effect on total rent dissipation of an increase in an individual valuation can flow through two channels: a higher v_i pushes up the optimal amount $x_i^* > 0$ of an agent that would have invested anyway or a higher v_i may convince a player to enter the game and thus change his optimal strategy from $x_i^* = 0$ to $x_i^* > 0$.

The effect of an increase of $\bar{v}$ on RD^* can instead go in both directions. A higher $\bar{v}$ increases the threshold that defines entry. As such, it can negatively affect participation and thus depress total rent dissipation. However, we also showed (see Section 3 and Figure 1.b) that the optimal level of investment of a participating player is not monotonic in the mean valuation. In particular, x_i^* first increases and then decreases with $\bar{v}$. Therefore, the net effect of a change of $\bar{v}$ on RD^* can be positive or negative.

We collect the results about rent dissipation in the following proposition.

Proposition 2 *Consider a rent-seeking game among $n \geq 2$ players with heterogeneous and private valuations v_i and common knowledge about the mean valuation $\bar{v}$. Then, in equilibrium, rent dissipation at the individual level ($RD_i^* = \frac{x_i^*}{v_i}$) and the aggregate level ($RD^* = \sum_i x_i^*$) is such that:*

- RD_i^* is a concave function of v_i, $\bar{v}$, and ρ_i that reaches its global maximum respectively at $\tilde{v}_i = \left(\frac{n-1}{n}\right)^2 4\bar{v}$, $\tilde{v} = \frac{1}{4}\left(\frac{n}{n-1}\right)^2 v_i$, and $\tilde{\rho}_i = \frac{1}{n}(2n-2)$.

- $RD_i^* \in \left[0, \frac{1}{4}\right]$ for any v_i, $\bar{v}$, ρ_i, and n.

- RD^* is weakly increasing and weakly concave in any v_i.

5 A comparison with the perfect information case

To highlight how the specific form of imperfect information that we have modeled influences players' behaviors, we compare our results with those that would emerge in a context of perfect information. We use the results that appear in Stein (2002) as a benchmark.

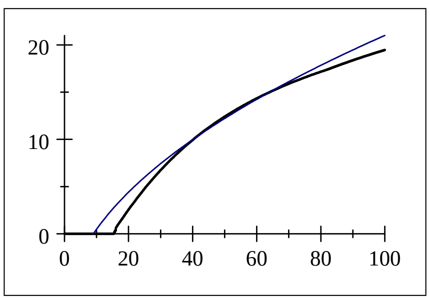
Stein (2002) studies a rent-seeking game among $n \geq 2$ players with heterogeneous and publicly known valuations and derives explicit solutions for the case of constant returns to scale success function. Therefore, the only difference between Stein's model and our model lies in the different information structure agents can rely on. Stein (2002) finds that the optimal strategy of generic agent $i \in N$ takes the following form:

$$x_i^{*Stein02} = \begin{cases} \frac{(p-1)\Phi_p}{p}\left[1 - \frac{(p-1)\Phi_p}{pv_i}\right] & \text{if } i \leq p \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

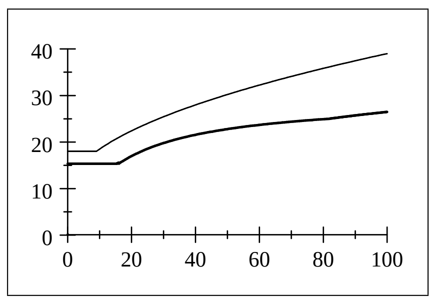
where $p \in \{1, ..., n\}$ is the largest number for which the condition $v_p > \frac{(p-1)}{p}\Phi_p$ holds (players are ordered in terms of their valuations such that $v_1 \geq v_2 \geq ...v_n > 0$) and $\Phi_p = \left[\frac{1}{p}\sum_{i \leq p}\frac{1}{v_i}\right]^{-1}$ is the harmonic mean of the first p values of $\{v_i\}_{i \in N}$. In equilibrium, rent dissipation at the aggregate level is given by $RD^{*Stein02} = \left[(p-1)\Phi_p\right]/p$ and the relation $v_{p+1} \leq RD^{*Stein02} < v_p$ always holds.

Note that in the special case of homogeneous valuations, Stein's solution reduces to the standard solution (Tullock, 1980). In fact, when $v_i = v$ for all $i \in N$, then $p = n$ and $\Phi_p = v$. Therefore, $x_i^{*Stein02} = \left(\frac{n-1}{n^2}\right) v$. We have already shown in Section 3 that also our solution (see Proposition 1) simplifies to the standard one as $x_i^* = \left(\frac{n-1}{n^2}\right)\bar{v}$ when $v_i = \bar{v}$. Therefore, both Stein's model and our model subsume the standard framework of a symmetric rent-seeking game with common knowledge and they thus lead to the same solution under those specific assumptions.

However, apart from this peculiar case, the two models usually differ both in terms of individual optimal investment and rent dissipation. To highlight these aspects, consider a slight generalization of the situation presented in Example 1. In particular, let $n = 4$ with $v_1 = 36$, $v_2 = 25$, $v_3 = 16$, $v_4 \in (0, 100]$, and $\bar{v} = 16$. Figure 3.a below shows how the optimal level of investment of player 4 changes as a function of his own valuation in the two models. Figure 3.b illustrates instead the evolution of rent dissipation at the aggregate level.



3.a) x_4^* (thin) and $x_4^{*Stein02}$ (thick)



3.b) RD^* (thin) and $RD^{*Stein02}$ (thick)

Focusing on Figure 3.a, the pattern of the two functions appears to be qualitatively similar. The only noticeable difference concerns the threshold that triggers the agent's entry: in Stein's model, agent 4 invests in rent-seeking activities when his valuation is such that $v_4 > 15.352$; in our model, entry occurs for $v_4 > 9$. The reason for this different threshold is that in our framework agent 4 assigns a valuation of $\bar{v} = 16$ to any of his opponents. Therefore, the agent underestimates the actual strength of his opponents, expects to face less fierce competition, and thus more easily enters the game.

Still, there is a more subtle difference between the two models. The two information structures can in fact lead to a different number of active players and this can in turn have important implications for the aggregate level of rent dissipation. The following table reports the number $m \leq n$ and the identity of the agents that invest a positive amount in rent-seeking activities in the two models, as v_4 varies in the interval $(0, 100]$.

	Our model		Stein02 model	
	m	active pl.	m	active pl.
$v_4 \in (0, 9]$	3	$\{1, 2, 3\}$	3	$\{1, 2, 3\}$
$v_4 \in (9, 15.132]$	4	$\{1, 2, 3, 4\}$	3	$\{1, 2, 3\}$
$v_4 \in (15.352, 16]$	4	$\{1, 2, 3, 4\}$	4	$\{1, 2, 3, 4\}$
$v_4 \in (16, 81.818]$	4	$\{1, 2, 3, 4\}$	3	$\{1, 2, 4\}$
$v_4 \in (81.818, 100]$	4	$\{1, 2, 3, 4\}$	2	$\{1, 4\}$

Table 1: number and identity of active players in the rent-seeking game.

With respect to the perfect information benchmark, the specific form of imperfect information that we have modeled influences therefore not only the individual optimal investment strategies but also the number of participating players. It follows that the differences between the two models in terms of total rent dissipation (Figure 3.b) are more pronounced with respect to those that emerge at the level of the individual equilibrium strategy (Figure 3.a).[17]

[17] Obviously, the results illustrated in Figures 3.a and 3.b are specific to the example being examined and cannot be generalized. The situation would be reversed (i.e., agent 4 would more easily enter the game and aggregate rent dissipation would be higher in a context of perfect information) if players overestimate the actual strength of their opponents (this for instance would happen if $\bar{v} = 30$).

6 Conclusions

We have studied some properties of rent-seeking games characterized by the lack of common knowledge about players' heterogeneous types. More precisely, we investigated a framework plagued by a severe form of imperfect information: in addition to the knowledge of their own type, agents only know that all valuations are drawn from an unknown distribution with mean $\bar{v}$. The key passage of the model is that a player necessarily uses this summary statistic not only to attach a valuation to his rivals but also to attribute a beliefs system to them. This allows participants to conjecture/approximate the level of effort that the other players will exert and thus to implement the investment strategy that best responds to such a conjecture.

Focusing on contests with constant returns to scale, we obtained closed-form solutions for agents' optimal level of effort as well as for the amount of rent dissipation that emerges in equilibrium. Comparative statics analysis then led to some interesting results. We showed, for instance, that an agent's optimal level of investment is not monotonic in the mean valuation $\bar{v}$ and that the amount that an agent dissipates in rent-seeking activities is bounded above by a threshold that is independent of the agent's valuation, his degree of relative resolve, and the number of opponents.

In general, we have highlighted how agents' behaviors and equilibrium results may be shaped by small pieces of shared information that become focal among all participants. This consideration opens interesting paths for future research. For instance, it suggests the possibility that the principal may strategically release some specific information (say some selected summary statistics) with the goal of influencing some of the outcomes of the game such as the number of active players or the aggregate level of rent dissipation.

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