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# Bankruptcy, Value Puzzles and the Survivorship Bias

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**Abstract**

This paper argues that a survivorship bias distorts upwards the measurement of the average ex-post firm value, because bankruptcy cancels firms with low realized cash flows from databases. This survivorship bias, that increases in bankruptcy probability, generates known pricing puzzles across types of firms. For instance, it turns a *true ex-ante* diversification premium, due to lower expected bankruptcy costs in conglomerates, into an *apparent ex post* diversification discount. Similarly, it makes a parent company appear to trade at a discount relative to its stand-alone counterpart because the parent survives to recessions more often than the stand-alone firm.

**JEL Classification**: G32, D23, K19.

**Keywords**: diversification discount, survivorship bias, parent company discount, bankruptcy, coinsurance, contagion.

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# 1 Introduction

Despite the real-world relevance of diversified firms, the link between their economic role and their value remains an open question. Some argue that diversification reduces firm efficiency, thereby lowering the value of diversified firms below that of specialized, stand-alone ones. Others question the causal link between the observed diversification discount and inefficiency, showing that it disappears once self-selection and accounting definitions enter the analysis. In a similar vein, other scholars show that the discount may simply reflect the efficient acquisition of lower value firms by conglomerates.[1]

The model in this paper sheds new light on value paradoxes, including the diversification discount, by focusing on bankruptcy. First, we show that a survivorship bias distorts upwards the measurement of the average ex-post firm value, because bankruptcy cancels low value firms from databases. This bias increases in bankruptcy probability. An implication is that this bias distorts the comparison of the average ex-post value of diversified and focused firms. If the former survive more often in adverse states of the world than the latter,[2] then their average ex-post value will appear to be lower. Thus, bankruptcy and the survivorship bias contribute to explain the diversification discount.

This paper departs from pricing models that overlook bankruptcy in order to preserve linearity. This allows to price a well known function of diversification, that of reducing bankruptcy costs through coinsurance (as in Banal, Ottaviani and Winton (2013), Boot and Schmeits, 2000; Leland, 2007; Lewellen, 1971). Our model will predict a true ex-ante premium for firms that save more bankruptcy costs, indicating this reason for the real-world relevance of diversified organizations. Interestingly, it will also predict the highest apparent average ex post discount for these firms due to the survivorship bias. Our second contribution is to bring to the foreground this paradox and the wrong inference on relative firm efficiency it implies.

Our third contribution is to highlight other pricing puzzles, across types of diversified organizations, due to the survivorship bias. Following Leland (2014) and Nicodano and Regis (2017), we juxtapose diversified business groups, where parent and subsidiary firms have sepa-

[1] A summary of the early, but ongoing, debate on the conglomerate discount appears in Maksimovic and Phillips (2007).

[2] This is the case in Borghesi, Houston and Naranjo, 2007; and Santioni Schiantarelli and Strahan, 2017. Moreover, firm age correlates with higher discounts (see Borghesi et al., 2007; and Hund, Monk and Tice, 2010.) and only surviving firms grow older.

rate legal liability, to diversified conglomerate mergers, where units are jointly liable for their debt obligations. In a conglomerate, an unprofitable unit may drag the profitable units into bankruptcy. As a consequence, the true value of groups exceeds the conglomerates'. The reason is that groups avoid contagion thanks to separate incorporation, but allow a group affiliate to support other affiliates thus providing the same coinsurance benefits of the conglomerate. Since groups are able to survive more often, their apparent value will be lower.[3]

The survivorship bias may contribute to explain another pricing puzzle, namely the parent company discount relative to its stand-alone valuation.[4] According to our model, this apparent discount originates from the parent surviving more often thanks to the dividend receipts from its subsidiaries. Since a listed subsidiary provides a smaller dividend to its parent company than a private one, taking the firm public reduces the parent company survival in industry downturns. The pricing implication of this reasoning is that the apparent discount of the parent company, induced by the survivorship bias, is lower in pyramidal groups than in private groups, and disappears when subsidiaries are spun-off.

We present a calibrated numerical example to assess the extent of value distortions due to the survivorship bias, by adapting to our state space the baseline parameters in Leland (2007). We first compute the distortion within types of organizations. A parent company, a conglomerate and a stand alone firm trade at an apparent premium of 8%, 11% and 18% with respect to their true value, respectively. We then compute apparent discounts across organizations. The parent company seems to trade at a discount of 10.35% with respect to the matched stand-alone firm. The group appears to trade at a discount of 1.01% with respect to conglomerates, that in turn suffer from a discount of 2.55% with respect to stand-alone firms. These simulations imply that the apparent conglomerate discount in our baseline calibration (2.55%) accounts for 20% of the discount (13%) reported, for instance, by Berger and Ofek (1995). Our sensitivity analysis shows that the discount may reach up to 7%, when contagion within conglomerates is less likely to occur.

Our analysis implies that the diversification discount originates from a survivorship bias, which is positively related to bankruptcy and inversely to economic efficiency. In our setting,

[3]See Almeida, Park, Subrahmanhyam and Wolfenzon (2011) and references in Khanna and Yafeh (2007) for empirical evidence on relative value and profitability of group affiliates.

[4]See Cornell and Liu (2001) on the parent company discount.

the amount of debt is exogenous - as if firms were subject to credit rationing - so as to allow for straightforward value comparisons. The implications carry over to models that allow for an endogenous choice of debt (as in Leland, 2007; Luciano and Nicodano, 2014) and ownership (Nicodano and Regis, 2018), conditional on the level of debt. Our model overlooks agency costs that have been the focus of much previous literature on the diversification puzzle. However, in a robustness section, we also investigate a setting with unobserved effort provision where coinsurance distorts effort incentives of conglomerate managers (as in Boot and Schmeits, 2000).

The rest of the paper proceeds as follow. Section 1.1 discusses closely related literature. Section 2 presents our simple model with bankruptcy costs. Section 3 derives the true and apparent firm values and the relationship between the diversification discount and the survival skills of conglomerates and groups. Section 4 presents a numerical example. In Section 5 we discuss the robustness of our results. Conclusions follow. All proofs are in Appendix A.1, while Appendix A.2 provides further details of the numerical example.

### 1.1 Related Literature

This paper provides new insight into the diversification-value controversy, bringing into this arena the known problem of survivorship bias in empirical finance. We know that databases either exclude companies that filed for bankruptcy or have otherwise ceased to exist, or report of their past existence but are obviously unable to report their post-bankruptcy prices and balance sheet data. Banz and Breen (1986) observe that this induces an ex-post-selection bias that disturbs the comparison of returns between different types of firms, such as firms with low and high price/earnings. Kothari, Shanken and Sloan (1995) show that ex-post selection overstates the excess return on high book-to-market portfolios. Brown, Goetzmann and Ross (1995) highlight how survival distorts return predictability and the equity premium. We show that it biases upwards the average ex post value of firms. Moreover, the bias may distort our inference about the relative efficiency of diversified and focused organizations, because it is inversely related to economic efficiency when there are no market imperfections beside bankruptcy costs. We thus suggest a plausible explanation for the most puzzling aspect of the controversy, namely the existence of a large discount for diversified firms in the raw data together with the difficulty in associating it to a blatant inefficiency.

Gomes and Livdan (2004) argue that the conglomerate discount reflects the endogenous, efficient selection of less productive firms into diversified conglomerates. In a somewhat similar vein, our model suggests that the efficiency of firms with the largest average, ex-post discounts is actually the highest. The difference is that, in our setting, the discount is a pure artifact of the data. However, our argument concerning the survivorship bias holds in case there is a true, ex-ante discount (or premium) for conglomerates. This may stem from the functioning of the internal capital market (Almeida et al., 2011; Rajan, Servaes and Zingales, 2000; Stein, 2002), from employee incentives (Cestone and Fumagalli, 2005; Fulghieri and Sevilir, 2011), as well as production decisions (Alonso, Dessein and Matouschek, 2015). Our general point is that any true discount (or premium) due to such motives is upward (downward) biased by enhanced survival.

Other papers highlight the role of bankruptcy in explaining the diversification discount. Mansi and Reeb (2002) highlight that diversification brings about a wealth transfer for bondholders at the expense of stockholders by reducing default risk. The excess value measure in the discount literature captures wealth transfer from stock-holders but not the wealth transfer to bondholders, relying on the market value of equity but on the book value of debt. This produces an apparent diversification discount. Our insight indicates the presence of an additional distortion induced by the data, when the analyst does not account for the differential mortality of firms. Hund, Monk and Tice (2010) build on the idea that diversified firms have less uncertainty about future mean profitability. They show that, in the cross-section, diversified firms will trade at a discount relative to single segment firms due to convexity of the discounting function. In our model, diversified firms are less risky but the diversification discount in the cross section is an artifact of the data.

## 2 The Model

This section sets up the essential elements of the model and derives the true cost of debt and the true value of different organizations.

### 2.1 Organizational Structures and Cash Flows

We define below three organizational types for two production units, that produce equal cash-flows irrespective of the organization they belong to. Each production unit funds its investment through a fixed amount of debt. Organizations differ in the extent of support that each unit can provide to the other one, as well as in the extent of liability for the other unit's debt. This affects both credit conditions and profits, net of funding costs, to each organization.

Each unit, indexed by $i = (A, B)$, raises an amount of debt, $D_i = 1$, to invest in a project at the initial time ($t = 0$). The operating profit of each unit is realized in $t = 1$. It will be High $\{H\}$, and equal to $X_i > 0$, with probability $p_i \in (0, 1)$, and it will be Low $\{L\}$, and equal to zero, with probability $(1 - p_i)$. We define four states of the world, $\{HH, LL, HL, LH\}$, where the first (second) letter in each pair refers to the profit of unit A (B).

Our choice of values implies that each unit has insufficient operating profits in state L to honor its own debt obligations. Without support, it defaults, and the future profit of unit i, $K_i > 0$, is lost.[5] Our key assumption is that the profit of unit A, in state $\{HL\}$, exceeds combined debt repayment of the two units, whereas the profit of unit B is lower than the combined service of debt. We will later assess that payoffs satisfy these restrictions.

The entrepreneur chooses among three organizations: stand-alone firms, business groups and conglomerates. Stand-alone firms operate separately. Each is independently liable to competitive lenders, who require a rate $R_i$. Given the assumptions concerning cash flows, firm A defaults in states $\{LH\}$ and $\{LL\}$ while firm B defaults in $\{HL\}$ and $\{LL\}$.

In a conglomerate, segments A and B belong to the same firm. They are therefore jointly liable *vis-à-vis* lenders. The conglomerate defaults in state $\{LL\}$, when both units either have zero profits, and in state $\{LH\}$, when segment A drags the profitable segment B into bankruptcy. However, there are coinsurance benefits in state $\{HL\}$, because profits from segment A save B from insolvency.[6] Thus, the conglomerate organization allows for coinsurance, while stand-alone companies do not, but it suffers from contagion. So far, we are following the setup of Boot and Schmeits (2000) without incentive problems, adding instead the assumption of asymmetric profits. This assumption makes contagion possible, a feature that is prominent in other analyses

[5] Hennessy and Whited (2007) estimate bankruptcy costs in the range of 8%-15% of capital for US Compustat firms, while Leland (2007) calibrates them to 23% for BBB-rated firms.

[6] This is the situation when Indian groups fail to provide support to ailing subsidiaries (Gopalan et al., 2007).

of conglomerate mergers such as Banal, Ottaviani and Winton (2013) and Leland (2007).

We now extend the analysis to groups. In a group, the incorporation of companies is separate and lenders fund them individually. The parent company (B) owns its subsidiary (A) and receives dividends that allow the parent to meet its debt obligations when the former is profitable and the latter is not, in state $\{HL\}$. Despite this ownership link, the parent company B enjoys corporate limited liability vis-à-vis the debt obligations of unit A. This limit on liability implies that unit A selectively defaults in state $\{LH\}$. Thus, the group organization allows for diversification, as in conglomerates, without incurring into contagion costs. In the first part of our analysis, we assume that the parent company (B) owns 100% of the shares of its subsidiary (A).

It is worthwhile discussing a few features of our model concerning diversified organizations. First, corporate limited liability is central to the argument that groups save on bankruptcy costs with respect to conglomerates. Courts may occasionally repeal corporate limited liability asking the parent company to meet its subsidiary debt obligations. Appendix A.3, available upon request, reports on court practice in several jurisdictions, that usually 'pierce the corporate veil' in case of fraud, only. Second, the model is simple so as to allow for value comparisons. On the one hand, the amount of debt is exogenous, as in the case of credit rationing. In general, optimal debt responds to both coinsurance and contagion (Leland, 2007; Nicodano and Regis, 2018). Insights on relative survival rates, relative efficiency and relative apparent values carry over to these more complex settings conditional on debt levels. On the other hand, our one-shot model rules out mergers and divestment, that have been the focus of prior research (Fluck and Lynch, 1999; Gomes and Livdan, 2004). Another simplifying assumption is that coinsurance takes the form of a transfer from A to B, only, both in conglomerates and in groups. We could enlarge this minimalist state space to define an additional state in which B rescues A from bankruptcy and more generally to eliminate asymmetries in profits (as in Boot et al., 1993; and Luciano and Nicodano, 2014). These algebraic complications do not affect the key insight concerning true and apparent value in each organization, to which we turn in the next sections.

## 2.2 Coinsurance, Contagion, and the Cost of Debt

We now determine the interest factors charged by the lenders, assuming risk-neutrality and a zero risk-free rate. Lenders of independent firms, $i = A, B$, receive the debt repayment in state

$\{H\}$ and collect nothing in state $\{L\}$. It follows that the interest factor for unit i, $R_i$, satisfying the lenders' zero expected profit condition, $(1 - p_i) \times 0 + p_i R_i$, is equal to:

$$R_i = p_i^{-1} \tag{1}$$

Conglomerate's lenders receive the debt repayment in state $\{HH\}$ and $\{HL\}$. They also recover the cash flow $X_B$ in state $\{LH\}$, when division A drags division B into bankruptcy, but B is profitable. Thus, the interest factor for the Conglomerate (C) is equal to:

$$R_C = p_A^{-1}[2 - p_B(1 - p_A)X_B)] \tag{2}$$

This factor solves the zero profits condition, which requires lenders' expected repayments to equal the 2 units of the loan provided at t=0, i.e.:

$$[p_A p_B + p_A(1 - p_B)]R_C + p_B(1 - p_A)X_B = 2$$

Lenders collect the interest payment when either both divisions are successful, an event that has probability $p_A p_B$, or division A is profitable but B is not, with probability $p_A(1-p_B)$. Moreover, they recover profit, $X_B$, upon the conglomerate default when there is contagion.

Turning to the group, lenders charge to the subsidiary A the same interest rate of the corresponding stand-alone firm (like in equation (1)), $R_{A \in G} = R_A$. This result holds because the subsidiary defaults in the same states of the world as the stand-alone firm. The cost of borrowing for the parent B is, instead, lower than the corresponding cost of unit B when stand-alone, thanks to the possibility of support in state $\{HL\}$:

$$R_{B \in G} = [p_B + p_A(1 - p_B)]^{-1} \tag{3}$$

Indeed the parent, B, defaults in state $\{LL\}$ only. The dividend it receives from its subsidiary, A, in state $\{HL\}$ is sufficient to avoid its own insolvency. It avoids contagion from its subsidiary in state $\{LH\}$ thanks to its corporate limited liability.

We can now rank the borrowing costs across the different organizational structures.

**Proposition 1:** *Assume costly bankruptcy. Then the interest factor of a conglomerate is lower than the one of the group, which is, in turn, lower than in the corresponding stand-alone firms.*

Groups pay a lower interest factor on debt with respect to stand-alone firms. Such overall improvement in credit conditions is due to the positive probability of the coinsurance state ($LH$) when subsidiary dividend allows the parent to survive. In turn, the conglomerate pays a lower interest factor with respect to groups, because lenders anticipate recovering positive cash flow ($X_B$) when the profitable segment B defaults due to contagion. In other words, reduction in the interest factor stems from either lower bankruptcy costs (thanks to coinsurance) or higher recovery upon default due to contagion. Both conglomerates and groups have lower borrowing costs with respect to stand-alone firms, thanks to coinsurance. However, groups have a higher cost of debt than conglomerates because A's lenders have no claim to B's profits.

We can now make explicit the restrictions on asymmetric cash flows, underlying the derivations of the previous equations. The assumption that profits in state $\{H\}$ allow to service debt implies that the cash flow in state H, $X_i$, exceeds the total debt repayment, that is $X_i p_i$ for each unit i. Recall now that the subsidiary, A, selectively defaults in state $\{LH\}$ while the parent company, B, receives support by A in state $\{HL\}$. This holds if the profit of unit A, $X_A$, exceeds both units' debt, that is:

$$X_A \geq max(R_C, R_{A \in G} + R_{B \in G}) \tag{4}$$

A sufficient condition is that $X_i \geq p_i^{-1} + p_i^{-1}$, since the highest cost of debt is the one of the stand-alone firms by Proposition 1. The profit of unit B, $X_B$, must in turn fall short the combined interest factor:

$$X_B < min(R_C, R_{A \in G} + R_{B \in G}) \tag{5}$$

Restriction (5) implies $X_B < 2[p_A + p_B(1 - p_A)]^{-1}$, since the lowest cost of debt is the one in conglomerates, by Proposition 1.

The next subsection computes the value of each organization. We will show that a lower cost of debt in conglomerates does not imply that they have the highest value, completing the reasoning in Hann et al. (2013).

### 2.3 The True Value of Diversification

The value of each organization coincides with profit after the service of debt, thanks to the zero risk-free rate assumption. It is straightforward to show that it is respectively equal to:

$$\pi_I = (X_A + K_A)p_A + p_B(X_B + K_B) - 2 \tag{6}$$

for two stand-alone firms;

$$\pi_C = (X_A + K_A + K_B)p_A + p_B X_B - 2 \tag{7}$$

for a conglomerate; and

$$\pi_G = (X_A + K_A)p_A + (X_B + K_B)p_B + K_B(1 - p_B)p_A - 2 \tag{8}$$

for a group. We now compare them across firm organizations, also highlighting the effects of coinsurance and limited liability on firm value.

**Proposition 2:** *Assume costly bankruptcy. Then*

*a. Groups have the highest true value.*

*b. The group premium relative to conglomerates represents the value of corporate limited liability. The one relative to two stand-alone firms represents the value of coinsurance. They are, respectively, equal to* $K_B(1 - p_A)p_B$ *and* $K_B(1 - p_B)p_A$*.*

*c. Conglomerate excess value relative to stand-alone firms is positive if, and only if, contagion costs are lower than diversification gains* ($p_B < p_A$)*.*

This proposition highlights the economic role of diversified firms, that increases firm value. The ability to save on bankruptcy costs may explain the reason why diversified business groups and conglomerates are common corporate organizations, generating a total value added of 28 US trillion dollars in over 200 countries (Altomonte and Rungi, 2013; Herring and Carmassi, 2009). Part (b) highlights that corporate limited liability saves future profits ($K_B$) from being lost in the bankruptcy procedure due to contagion. This happens with probability $(1 - p_A)p_B$, which is the probability that unit B, while solvent by itself, is unable to provide support to the insolvent one. Coinsurance allows saving unit B from insolvency. This occurs with probability $(1 - p_A)p_B$, which is the probability that B is insolvent but unit A generates enough profits to support it. Part (c) indicates that the excess value of conglomerates is positive only if contagion problems are limited. It is not a new result, reminding of previous insight due to Banal et al. (2013) without tax distortions and Leland (2007) with tax distortions.

The proposition indicates that groups have the highest expected value relative to both conglomerates and focused firms, because they best protect firm activity. The following section explains how the highest true diversification premium becomes the highest apparent diversification discount due to a survivorship bias.

# 3 The Apparent Diversification Discount and Other Pricing Puzzles

This section will argue that the survivorship bias contributes to the explanation of pricing puzzles. The best known is the diversification discount, that is the observation that the average ex post value of diversified firms is lower that the average ex-post value of matched focused firms. The second one is the parent company discount, where a parent company displays lower value than an equivalent portfolio of stand alone firms. The most recent one is the boring company discount, highlighting the lower valuation of firms operating in industries with lower dispersion in profitability. We will first develop the theoretical predictions and then discuss some stylized facts about these pricing puzzles.

## 3.1 Survivorship Bias

We now show that group affiliates and conglomerates appear to trade at a discount relative to stand-alone companies if the econometrician does not account for survival. We thus address the methodology of several papers investigating the excess value of multi-unit firms. These papers typically match single-unit to multi-unit firms, *when all of them are alive* and therefore appear in the database. The survivorship bias originates from three facts. First, datasets cannot contain balance sheet information on bankrupt firms. Second, both group affiliates and conglomerates exploit diversification in order to survive during industry downturns. Third, the econometrician does not control for the survival ability of different organizations.

In order to be able to infer the *apparent* value of each organization, we must ask whether the state is high or low when the econometrician finds firms in her dataset. This will determine the chances of observing a high or a low cash flow. Let us, therefore, start with stand-alone units. The probability of state $\{H\}$, when the econometrician notes that a stand-alone firm is present in her dataset, is 1. Indeed, a stand-alone unit goes bankrupt and exits from the database in state $\{L\}$. It follows that the apparent value of a stand-alone is equal to the high cash flow realizations net of the debt repayment, that is:

$$V_i = X_i + K_i - R_i \tag{9}$$

**Proposition 3:** *Assume costly bankruptcy. Then:*

*a. The apparent value of a stand-alone firm exceeds its true value, due to a survivorship bias.*

*b. The bias increases in the probability of default.*

We now determine the probability of being in a good or bad state when the econometrician observes a conglomerate in the dataset. The probability of state $\{HH\}$, conditional on observing both stand-alone companies alive, is one. On the contrary, the probability of state $\{HH\}$ conditional on observing a conglomerate is lower, because of the conglomerate ability to survive when A rescues B. Such probability is equal to $Pr[HH/(HH + HL)]$, that is:

$$p_A p_B [p_A p_B + (1 - p_B) p_A]^{-1} \tag{10}$$

which simplifies in

$$p_B < 1.$$

The probability of state $Pr[HL/(HH + HL)]$ is equal to $(1 - p_B)$. Thus, the estimated value of the conglomerate equals:

$$
\begin{aligned}
&p_B(X_B + K_B + X_A + K_A - R_C) + (1 - p_B)(K_A + X_A + K_B - R_C) = \\
&= (K_B + X_A + K_A - R_C) + p_B X_B.
\end{aligned}
\tag{11}
$$

Let us now compare equations (11) and (9). The conglomerate appears to have a lower market value with respect to stand-alone firms if :

$$X_B > (R_A + R_B - R_C)(1 - p_B)^{-1} \tag{12}$$

that is, if:

$$X_B > p_B^{-1}$$

This inequality always holds, since the profit in each unit always exceeds the value of its debt by assumption. Thus, conglomerates trade at an apparent discount relative to stand-alone whether or not they save on bankruptcy costs. The survivorship bias is, therefore, tricky when it comes to conglomerates. We know already the rationale for the apparent discount when conglomerates save on bankruptcy costs, that is, when $p_B < p_A$. When the opposite holds, contagion cancels segment B with positive profits more often than it saves segment A with zero profits. Hence, a profitable segment disappears from the database, thereby contributing all the same to the conglomerate discount.

We now carry out a similar comparison between groups and stand-alone units, when an econometrician observes all units in operations. The probability of state H, when the subsidiary A is alive, equals one because affiliation does not influence the default of the subsidiary. On the contrary, the state is $\{H\}$ with probability equal to $[p_B + (1 - p_B)p_A]^{-1} p_B$, and low with probability equal to $[p_B + (1 - p_B)p_A]^{-1}(1 - p_B)p_A$ when the parent company, B, appears in the dataset. This occurs because B survives in low states, when it generates zero cash flows, thanks to the subsidiary support. The econometrician, therefore, estimates the value of the parent company B as being equal to:

$$[p_B + (1 - p_B)p_A]^{-1}[(X_B + K_B - R_{B \in G})p_B + (K_B - R_{B \in G})(1 - p_B)p_A] \tag{13}$$

The parent value in (13) is always lower than the apparent value of an independent unit (9), if:

$$[p_B + (1-p_B)p_A]^{-1}[(X_B + K_B - R_{B\in G})p_B + (K_B - R_{B\in G})(1-p_B)p_A] < K_B + X_B - R_B \quad (14)$$

which simplifies in

$$X_B > p_B^{-1}$$

This inequality always holds. It follows that parent companies appear to trade at a discount, while in reality, they are saving on dissipative bankruptcy.

In summary, the survivorship bias affects both conglomerates and business groups. However, the Appendix (A.1) shows that the diversification discount for business groups is more severe because the group survives more often when profitability is low. The analyst will thus observe a lower value for groups in operations than for conglomerates in operation. We summarize our results as follows:

**Proposition 4:** *Assume an analyst matches groups and conglomerates to the corresponding stand-alone firms, conditional on available data, without accounting for different mortality. Then the value of groups appears lower than the value of conglomerates which appears, in turn, lower than the value of stand-alone firms.*

This proposition implies that comparing average market values of different organizations lead to the wrong inference concerning their relative efficiency. This is not due to distortions in market prices: prices are rational, and there are no limits to arbitrage. The wrong inference derives from the different survival ability of organizations, which investors and analysts alike often fail to consider because of data limitations.

### 3.2 Explaining Observed Pricing Puzzles

This section collects some stylized facts that our model may help explain. It begins with the evidence of a positive value of diversification in experiments that are free from the survivorship bias. It proceeds to discuss the observed relationship between the conglomerate discount, firm age and segment relatedness, connecting it to the survivorship bias.[7] g It then reads through the lenses of our model both the parent company discount and the so called boring company puzzle. Our model posits that diversification creates value, as in Proposition 1, and this should be evident when no survivorship bias disturbs the inference. Indeed, diversification has positive value when the analyst uses survey data to estimate ex-ante returns (see Hann et al., 2013). Survey data should not be subject to selection bias, to the extent that surveyed people remember the occurrence of defaults. Similarly, a regression analysis that accounts for defaulted units - thus getting rid of the survivorship bias - should reveal that diversified firms outperform during industry distress. Gopalan and Xie (2011) measure the average discount on multi-units firms just before and during unexpected industry distress, taking into account the disappearance (through delisting, i.e. bankruptcies, mergers, etc.) of weak stand-alone firms. They find that the average conglomerate discount reduces from 20% the year before industry distress to 6.9% in the three years after industry distress.

We now turn to the diversification discount. While most studies do not control for differential bankruptcy of conglomerates and portfolios of focused firms, the relationship between firm age and the diversification discount is well known. Borghesi, Houston, and Naranjo (2007) show that part of the discount arises from differences between mature diversified firms and young focused firms. After controlling for firm age, the diversification discount falls by 15% to 30%. They also find that conglomerates are less likely to declare bankruptcy than their focused counterparts and that this is not the result of diversified firms refocusing before going bankrupt. They interpret their findings in the light of models suggesting a life-cycle for firm growth opportunities (Matsusaka, 2001; Bernardo and Chowdhry, 2002). Our model (Proposition 4) suggests instead that controlling for age reduces the survivorship bias. The positive association of the conglomerate discount with firm age also appears in Hund, Monk, and Tice (2014). They argue that, as firm ages, asset multiples decrease more quickly for focused firms because uncertainty about their true value resolves more quickly. In the light of our model, mature diversified firms survive to adverse contingencies thanks to cross-subsidization. Without controlling for age, their ex post average profits and average value appear lower due to the survivorship bias. Once the analyst

[7]In the diversification discount literature there is hardly any distinction between conglomerates and groups. This is because databases, such as Compustat, usually refer to conglomerates also when subsidiaries issue debt in their own name - and are therefore groups according to our model.

controls for age, the survivorship bias fades away.

Our model assumes unrelated diversification, in that cash flows are independently distributed. The survivorship bias would diminish in case of more synchronous bankruptcies in conglomerates units, due to higher cash flow correlation. This implication aligns with the observation that firms engaging in unrelated diversification are subject to significantly higher discount compared to conglomerate operating in related business (Berger and Ofek, 1995). This finding holds when using BITS instead of Compustat data (Villalonga, 2004), as well.

In Proposition 4, groups and their parent companies have lower apparent values than their stand-alone counterparts, because they survive more often to industry downturns. Several works document the lower valuation of groups with respect to non-group affiliates, attributing it to tunneling and expropriation of resources by controlling shareholders [Joh (2003); Bae, Kang, and Kim (2002); Johnson, Boone, Breach and Friedman (2000)]. However, Masulis, Pham, and Zein (2008) find that Tobin's Q is higher in subsidiaries of pyramidal groups, where the separation of ownership from control is high, than in firms at the top, after controlling for endogeneity of group membership. Proposition 4 may account for both stylized facts. Similarly, Mitchell et al.(2002), and Cornell and Qiao Liu (2001), find several cases in which the value of the parent company is lower than the value of its subsidiary. [8] In our setting, the healthier is the subsidiary, the lower is the apparent value of the parent because of its higher likelihood of survival thanks to subsidiary's dividends.

Last but not least, Proposition 3 relates the survivorship bias to the probability of default, that is arguably higher for firms operating in industries with higher profit dispersion. The bias may therefore explain why companies in such industries display higher value than firms in "boring" industries. Chen, Hou and Stulz (2015) cannot find a rational explanation to this seeming mispricing and resort to a behavioral one. They also show that firms in less boring industries have lower realized returns. This evidence appears consistent with both a mispricing conjecture and our survivorship bias explanation.

In summary, our model may provide a unifying rationale for puzzling patterns uncovered by different strands of empirical work.

[8] Schill and Zhou (2000), and Lamont and Thaler (2000) find that subsidiaries have higher value than their parent firms in carve outs operations.

### 3.3 Numerical Example

This section develops a numerical example in order to assess the extent of the bias. Our baseline parameters come from an adaptation of those in Leland (2007) that mirror the case of a firm with a BBB rating. The adaptation, described in the Appendix (A.3), sets both tax rates and net interest rate to zero, and normalizes the face value of debt to 100 as in our model. It then matches the first and the second moment of operational cash flows, in such a way as to fit the state space of the model.

Table 1 displays the results of the adaptation. The baseline values for the payoff of firm A in state H, $X_A = 255.56$, exceeds the cost of debt for the two stand-alone firms, $R_A + R_B = 244.44$. The probability of having positive profits for firm A is high, $p_A = 0.90$. Foregone future profits in case of default are comparatively small, $K_A = 27.68$. For firm B, the calibrated values are $X_B = 173.33$, which is lower than the service of debt for a conglomerate, $R_C = 207.78$. The probability of positive payoff is set to $P_B = 0.75$ and foregone future profits are higher than for firm A, $K_B = 49.22$. Coinsurance delivers a lower funding costs for groups ($R_i = 213.68$) with respect to stand-alone firms ($R_i = 244.44$). The lowest cost of debt is the one of conglomerates ($R_i = 207.78$), resulting also from the contagion effect that increases lenders' recovery upon default.

Given these baseline inputs, in Table 2 we report the true [equations (6)-(8)] and apparent values [equations (9)-(11)] of groups, conglomerates and stand-alone firms. By construction, the highest true value (232.90) is the one of groups. The reported difference with respect to conglomerates (1.5%) is due to corporate limited liability. In this example, being $p_B < p_A$, diversification gains exceed contagion costs, and conglomerate excess value relative to stand-alone firms is equal to 2% of the true value of standalone firms.

Turning to apparent values, this table highlights that the survivorship bias distorts upwards the value of each firm, the more the higher is its default probability. For instance, the parent company trades at an apparent premium of 3% with respect to its true value; this low upward bias reflects the low probability of default that characterize parent companies. Moving to conglomerates, the premium increases to 11%. The apparent premium is the same for the stand-alone firm with lower default probability, while it reaches 33% for the one with higher default probability. As for the average bias across organizations, the Table reports an overpricing of 8%

for groups, increasing to 11% for conglomerates and to 18% for stand-alone firms. Against this background, Table 3 highlights the baseline values of the apparent discounts, due to differential survivorship bias. The parent company seems to trade at a discount of 10.35% with respect to the matched stand-alone firm. The group appears to trade at a discount of 1.01% with respect to conglomerates, that in turn suffer from a discount of 2.55% with respect to stand-alone firms.

These simulations imply that the apparent conglomerate discount in our baseline calibration (2.55%) accounts for 20% of the discount (13%) reported, for instance, by Berger and Ofek (1995). As we will see in the next section, this conglomerate discount may reach up to 7% when the volatility of unit A falls. Thus the survivorship bias may explain a significant portion of the conglomerate discount detected in the data.

We now present a sensitivity analysis (*ceteris paribus*) of values across organizations to cash flow volatility. Cash flow volatility depends on the probability of survival. Changing the latter will also affect expected payoff, unless we adjust the level of profits as well. We examine a mean preserving increase in the volatility of cash flows - that is both volatility and payoff change so as to keep the expected value of the payoff constant.

Figure 1 reports the percentage change in the apparent firm value when the cash flow volatility of firm A (panel A) and B (panel B) varies. We standardize the lowest apparent firm value for groups at 0% when the volatility equal its minimum (4% in panel A and 15% in panel B), in order to report the percentage differences between groups and conglomerates, and between conglomerates and stand-alone firms, for incremental percentage differences in the cash flow volatility. The area at the bottom of the figure reports the value of groups, while the white-dashed and the grey-dashed areas represent the excess value (in % changes) of conglomerates and standalones with respect to groups. The values of the volatility varies in an interval with respect to the value of the volatility reported in our base case in Table 1 (respectively, 15% for the volatility of unit A and 26% for the volatility of unit B).

Panel A shows that as the volatility of firm A falls towards 4%, the apparent value of the conglomerate approaches the value of groups, with a widening of the diversification discount reaching 7%. Since unit A is almost risk-free, there is hardly any contagion in the conglomerate. When firm A is riskier (26%, to be equal to the volatility to the unit B), contagion costs grow and they offset the value of coinsurance. In this situation, conglomerates have an apparent value that tends towards the apparent value of stand-alone firms.[9]

Similarly, Figure1 - panel B - reports the sensitivity of firm value to unit B' cash flow volatility. Panel B shows that as the volatility of firm B (the parent) grows to 45%, the group discount relative to stand-alone firms widens to 10%. In fact, since the level of firm A's volatility is 15%, contagion costs are relatively low respect to coinsurance, therefore we do not observe a significant difference between groups and conglomerates. When the parent is less risky (15%, to be equal to the volatility to the unit A), we observe the highest value for conglomerates as probability of contagion is relatively low (cash flow volatility of unit A is 15%), and no differences with respect to the standalone firms, as the benefits of coinsurance reduce.

Overall, the sensitivity analysis confirms that high survival skills will lead to a higher apparent value of standalones with respect to groups and conglomerates, where such apparent discount is more severe for groups.

# 4 Robustness

This section qualifies previous results. It suggests that the apparent discount is larger when unit B is able to support unit A (Section 1). It then suggests other modifications to relative pricing that derive from partial ownership of the subsidiary (Section 2) and non-contractible managerial effort (Section 3).

## 4.1 A Larger Apparent Discount

Our model set-up allows only unit B to support unit A, for simplicity. However, this simplifying assumption compresses the apparent diversification discount. In fact, unit B does not suffer from the survivorship bias. It is, however, easy to add a state of nature with A profits in excess of the debt obligations for both units, as in Leland (2007) and Nicodano and Regis (2018). In that state, unit A would be able to support unit B. This addition is also realistic. For instance, arent companies often support their subsidiaries, without being subject to contractual guarantees. For example, they write "comfort letters" assuring subsidiaries' lenders that they would assist them in distress. These letters do not undermine their limited liability, because they are legally unenforceable, but allow the parent company to choose whether to honor them ex post (Boot

[9] It can be also higher, whenever $p_A < p_B$.

et al., 1993). The survivorship bias and in both groups and conglomerates would then become more severe relative to stand-alone firms. Our numerical examples in Section 3 therefore provide a lower bound on both the group and the conglomerate discounts.

### 4.2 Diversification Benefits in Pyramidal groups

Previous sections deal with groups with fully owned subsidiaries, which are common in several jurisdictions, as mentioned in Appendix A. However, pyramidal groups with listed subsidiaries are also common (see La Porta et al. (1999)). This section investigates the consequences of partial subsidiary ownership for the diversification role of groups and their pricing, continuing to sidestep the expropriation of minority shareholders taking place with separation of ownership from control. A simple argument implies that the listing of affiliates does not, in general, improve, and may worsen, group survival thereby reducing both the true premium and the apparent discount.

Let the parent firm own a percentage,$\gamma$, of subsidiary equity. Then the dividends it receives from the subsidiary reduce to $\gamma X_A$. Lower dividends increase the costs of parent debt, as lenders anticipate a higher default probability. Let variable $\lambda$ account for the bailout probability, we assume that it might assume two values, conditional to the realization of cash flow of units A and B:

$$\lambda = 1 \qquad if \qquad \gamma X_A \geq (R_{A \in G} + R_{B \in G}) \tag{15}$$

$$\lambda = 0 \qquad if \qquad \gamma X_A < (R_{A \in G} + R_{B \in G}) \tag{16}$$

We can determine the threshold value of $\gamma$, which we indicate with $\gamma^*$, such that $\lambda = 1$:

$$\gamma > \gamma^* = (R_A + R_B) X_A^{-1} = p_A^{-1} [p_B + p_A (1 - p_B)]^{-1} X_A^{-1} \tag{17}$$

Since $X_A$ is always greater than $(R_A + R_B)$, the RHS of this equation is always lower than one. This result indicates that coinsurance is no longer possible, in a pyramidal group, if the parent ownership share falls short of $\gamma^*$. In such a case, both the cost of debt and the value are equal across group affiliates and stand-alone firms. Consequently, the group will not suffer from the survivorship bias.

### 4.3 Effort Provision, Contagion, and Outside Funding

The previous sections show that diversified firms suffer from an apparent discount when they reduce bankruptcy costs with respect to stand-alone firms. It also establishes that the discount is larger on groups than on conglomerates, *provided coinsurance and contagion do not distort managerial incentives*.

This section discusses the robustness of this result once the probability of success for unit A becomes endogenous and non-contractible, expanding on the analysis of Boot and Schmeits (2000). They assume that managerial effort increases the success probability of unit A, but imposes on it a monitoring cost. Lenders will exert "market discipline," trying to detect the true probability of success of the unit. Boot and Schmeits (2000) point out that there are negative incentive effects in conglomerates, due to coinsurance. Effort provision in conglomerates is lower than in stand-alone firms for all levels of market discipline because Manager A does not fully internalize the positive consequences of his effort provision on unit B.

Our model reinforces their insight, because unit A may also contaminate unit B with manager A enjoying a lower funding cost rather than incurring a penalty. Such contagion is not present in a group thanks to the limited liability of each unit. These agency costs tend to diminish the survival skills of diversified firms, especially in conglomerates. Thus, the apparent diversification discount will fall as the relevance of managerial moral hazard grows, thereby reducing the true diversification premium.

## 5 Conclusion

There is conflicting evidence on the performance of diversified organizations. While owners choose diversified organizations for their firms, several works find that corporate diversification reduces firm value. This paper proposes a resolution of this conflict going back to an old economic rationale for diversification, which is enhanced firm survival. We point out that the diversification discount may artificially arise in empirical analysis because of the better survival skills of diversified organizations. This pricing paradox is due to a known problem of existing databases, namely the ex-post selection bias. Databases do not contain price information on stand-alone firms that disappeared in a downturn due to defaults and mergers, while they do include the diversified affiliates that survived. Thus, ex-post relative average price does not reflect relative firm value because such measurement does not control for selection bias.

Our pricing model shows that the implied apparent diversification discount, conditional on databases that cancel out bankrupt firms, is even larger for groups than for conglomerates, because of their better survival skills. An extensive literature considers such discount as true, relating it to the possibility of using minority shareholders' resources to the benefit of the controlling party. Our paper points out an alternative reason for group discounts, which rests in the ability to avoid contagion thanks to their corporate limited liability. Corporate limited liability is a persistent legal provision of groups across several jurisdictions, which policy-makers consider a benefit of the group organization.

This reasoning leads to conclude that the economic function of diversification, consisting in limiting dissipative bankruptcy costs, is hard to detect in the data. We leave it to future work to correct the survivorship bias, downsizing the empirical estimates of the diversification discount - thereby explaining owners' preferences for diversification.

# A Appendices

## A.1 Proofs

In this section, we provide the proofs of propositions.

**Proposition 1 - Borrowing costs across organizations.**

We first need to prove that $R_C < R_A + R_B$, that is:

$p_A^{-1}[2 - p_B(1 - p_A)X_B] < p_A^{-1} + p_B^{-1}$ .

This simplifies to:

$p_B[2 - p_B(1 - p_A)X_B] < p_A + p_B$,

that reduces to

$-p_B(1 - p_A)X_B < p_A - p_B$,

that is equal to:

$X_B > (p_B - p_A)[p_B^2(1 - p_A)]^{-1}$

This inequality always holds because the threshold level of the cash flow, $x_B^* = (p_B - p_A)[p_B^2(1 - p_A)]^{-1}$, is lower than the minimum level required for unit B cash flow (Equation 5) when successful, that is equal to $(p_B - p_A)[p_B^2(1 - p_A)]^{-1} < (p_B)^{-1}$ . This inequality reduces to:

$$(p_B - p_A)[p_B(1 - p_A)]^{-1} < 1 \qquad or \qquad (p_B - p_A) < p_B(1 - p_A),$$

that is

$$p_B < 1.$$

This inequality always holds by assumption. We now need to show that the combined cost of debt is lower for business groups, that is:

$$R_G < R_A + R_B$$

Recall that the interest factor in unit A is the same for both organizational forms, $R_{A \in G} = R_A$. Then the following must hold:

$$R_{B\in G} < R_B \equiv [p_B + p_A(1-p_B)]^{-1} < p_B^{-1} \equiv p_A(1-p_B) > 0$$

This inequality is always satisfied. Finally, we need to show that the cost of debt for business groups is lower than the conglomerate case, that is $R_A + R_{B\in G} < R_C$. This inequality becomes:

$$\begin{aligned}
&p_A^{-1}[2 - p_B(1-p_A)X_B] > p_A^{-1} + [p_B + p_A(1-p_B)]^{-1}\\
&\equiv p_A^{-1}[1 - p_B(1-p_A)X_B] > [p_B + p_A(1-p_B)]^{-1}\\
&\equiv p_A^{-1}[-p_B(1-p_A)X_B] > [p_B + p_A(1-p_B)]^{-1} - p_A^{-1}\\
&\equiv -p_A^{-1}[p_B(1-p_A)X_B] > -p_A^{-1}\{1 - p_A[p_B + p_A(1-p_B)]^{-1}\}\\
&\equiv [p_B(1-p_A)X_B] < 1 - p_A[p_B + p_A(1-p_B)]^{-1}\\
&\equiv [p_B(1-p_A)X_B] < p_B(1-p_A)[p_B + p_A(1-p_B)]^{-1}\\
&\equiv X_B < [p_B + p_A(1-p_B)]^{-1}
\end{aligned}$$

This last inequality never holds because profits for unit B, when successful, are sufficient to repay its own debt by assumption. The difference between the cost of borrowing of conglomerates and business groups equals: $p_B(1-p_A)[X_B - R_{B\in G}]$. Recall that the minimum level for $X_B$ is $p_B^{-1}$. It follows that the cost of borrowing in business groups exceeds the one of conglomerates by, at least, by $[p_B + p_A(1-p_B)]^{-1}[p_B + p_A(1-2p_B)]^{-1} > 0$.

**Proposition 2 - Firm values across organizations.**

The combined profits of the two independent units are equal to:

$$\pi_I = (X_A + K_A - R_A)p_A + p_B(X_B + K_B - R_B) = (X_A + K_A)p_A + p_B(X_B + K_B) - 2 \quad \text{(A.1)}$$

In turn, conglomerate profits are equal to:

$$\begin{aligned}
\pi_C &= (X_A + K_A + X_B + K_B - R_C)p_A p_B + (X_A + K_A + K_B - R_C)p_A(1-p_B) =\\
&= p_A p_B X_B + (X_A + K_A + K_B - R_C)p_A = (X_A + K_A + K_B)p_A + p_B X_B - 2.
\end{aligned} \quad \text{(A.2)}$$

Group profits are equal to:

$$\pi_G = (X_A + K_A + X_B + K_B - RA - R_{B\in G})p_A p_B + (X_A + K_A + K_B - R_A - R_{B\in G})(1 - p_B)p_A +$$
$$+ (X_B + K_B - R_{B\in G})p_B(1 - p_A) =$$
$$= (X_A + K_A)p_A - 1 + p_B X_B + (K_B - R_{B\in G})[p_A p_B(1 - p_A)] =$$
$$= (X_A + K_A)p_A + (X_B + K_B)p_B + K_B(1 - p_B)p_A - 2 \tag{A.3}$$

We write Equation (A.3) using Equation (A.1) as:

$$\pi_G = \pi_I + K_B(1 - p_B)p_A \tag{A.4}$$

Similarly, we write Equation (A.3) using Equation (A.2) as:

$$\pi_G = \pi_C + K_B(1 - p_A)p_B \tag{A.5}$$

This proves Part (a) of Proposition 2. As for Part (b), the value increases relative to conglomerates (stand-alone companies) in (A4) ((A5)) determines the benefits of corporate limited liability (coinsurance). Finally, equating (A4) and (A5) and rearranging terms we find that conglomerate profits are higher than stand-alone firms' are if the diversification effect prevails on the contagion effect:

$$p_B K_B < p_A K_B \equiv p_A > p_B$$

This proves Part (c). In turn, group profits when parent B does not own 100% of the subsidiary are:

$\pi_G = (X_A + K_A + X_B + K_B - R_A - R_{B\in G})p_A p_B + (X_A + K_A + K_B - R_A - R_{B\in G})\alpha p_A +$
$-p_B p_A \alpha(X_A + K_A + K_B - R_A - R_{B\in G}) + (X_B + K_B - R_{B\in G})p_B - p_B p_A(X_B + K_B - R_{B\in G})$

This reduces to $\pi_I$ if $\alpha = 0$, and to (A.3) if $\alpha = 1$.

**Proposition 3 - True and apparent value of stand-alone firms.** The apparent value is higher that the true value if:

$$X_i + K_i - R_i > (X_i + K_i - R_i)p_i \tag{A.6}$$

which reduces to the condition:

$p_i < 1$

that is always verified. The bias, $(X_i + K_i - R_i)(1 - p_i)$, increases in the probability of default.

**Proposition 4 - Survivorship Bias and Conglomerate Discount.**

We need to prove that survivorship bias generate a discount for groups and conglomerates. This entails showing that the value of a conglomerate, when alive, exceeds the value of two alive group affiliates. We first prove the inequality in equation (12), implying that conglomerates trade at an apparent discount relative to stand-alone firms:

$(K_B + X_A + K_A - R_C) + p_B X_B < X_B + K_B + X_A + K_A - R_A - R_B$

$\equiv -R_C + p_B X_B < X_B - R_A - R_B$

$\equiv (1 - p_B)X_B > (R_A - \frac{1}{2}R_C) + (R_B - \frac{1}{2}R_C)$

$\equiv (1 - p_B)X_B > p_A^{-1} 2p_A^{-1}[2 - p_B(1 - p_A)X_B] + p_B^{-1} - 2p_A^{-1}[2 - p_B(1 - p_A)X_B]$

$\equiv p_B p_A(1 - p_B)X_B > p_A - p_B[2 - p_B(1 - p_A)X_B] + p_B$

$\equiv p_B p_A X_B - p_B^2 p_A X_B - p_B^2 X_B + p_B^2 p_A X_B > p_A - p_B$

which simplifies to:

$p_B p_A X_B - p_B^2 X_B > p_A - p_B$

$\equiv X_B > p_B^{-1}$

We now need to prove the inequality in (14), implying that groups trade at an apparent discount vis-à-vis stand-alone firms:

$[p_B(1 - p_B)p_A]^{-1}[(X_B + K_B - R_{B\in G})p_B + (K_B - R_{B\in G})(1 - p_B)p_A < K_B + X_B - R_B$

$\equiv p_B X_B + p_B K_B - p_B R_{B\in G} + (1 - p_B)p_A K_B - R_{B\in G}(1 - p_B)p_A < (K_B + X_B - R_B)R_{B\in G}^{-1}$

$\equiv p_B X_B + p_B K_B + (1 - p_B)p_A K_B - R_{B\in G} R_{B\in G}^{-1} < (K_B + X_B - R_B)R_{B\in G}^{-1}$

$\equiv p_B X_B + p_B K_B + (1-p_B)p_A K_B - 1 < p_B K_B + p_A K_B - p_B p_A K_B + p_B X_B + p_A X_B - p_B p_A X_B +$
$-1 - (1-p_B)p_A p_B^{-1}$
$\equiv p_A(1-p_B)X_B - (1-p_B)p_A p_B^{-1} > 0$
$\equiv X_B > p_B^{-1}$

Finally, we compare the price of a group to that of a conglomerate when both are alive. The value of the conglomerate in operation is equal to:

$$p_B(X_B + K_B + X_A + K_A - R_C) + (1-p_B)(K_A + X_A + K_B - R_C + p_B X_B =$$
$$\equiv (X_A + K_A + K_B) - R_C + p_B X_B$$

whereas the value of the group in operations is equal to:
$p_B(X_B + K_B - R_{B\in G}) + (1-p_B)(K_B - R_{B\in G}) + X_A + K_A - R_A = (X_A + K_A + K_B) - R_A - R_{B\in G} + p_B X_B$.

Then the difference between the two expressions is equal to $R_{B\in G} + R_A - R_C$, which is always positive because we know that $R_{B\in G} + R_A > R_C$.

**Equation (18): Apparent discount with partial ownership**

We now determine how the partial ownership affects the apparent value of the parent firm. For each level of the ownership $\gamma$, the probability of bailout is function of the cash flow of unit A and the probability of success of both units, that is:

$$\lambda(X_A, p_A, p_B) > 0 \qquad if \qquad X_A \geq (R_{A\in G} + R_{B\in G})/\gamma$$

The apparent value of the parent firm, corrected by the probability of bailout of unit B ($\lambda$), is equal to:

$$[p_B + (1-p_B)p_A]^{-1}[(X_B + K_B - R_{B\in G})p_B + \lambda(K_B - R_{B\in G})(1-p_B)p_A] \qquad \text{(A.7)}$$

Intuitively, if parent ownership share $\gamma$ is lower than its threshold level,$\gamma^*$, the coinsurance between the parent and its subsidiary is not possible, and $\lambda = 0$. This also implies that that conditional probability of observing a parent in operation is the same as a standalone firm, equal to one, and the interest factors of parents and standalone firms align. It follows that the group will not suffer from the survivorship bias, and the apparent value of parent firms equal the apparent value of standalone firms, ceteris paribus.

## A.2 Calibration

This Appendix describes the process of adaptation of the baseline parameters in Leland (2007) to our model, resulting in Table 1 in the main body of the paper. Table (A1) summarizes the set up of the two models. The process starts by assuming specific (and different for Firm A and B) inputs for Leland model in order to generate coherent inputs for the present model framework. In Leland model, five-year cash flows $X(T)$ are normally distributed with mean and standard deviation (127.63, 49.19), resulting in a present value of expected operational cash flow of 100.00, with annualized volatility of 22%. The annualized operational cash flow volatility $\sigma = 22\%$ is based on (Schaefer and Strebulaev, 2008), who estimate asset volatility from equity volatility for firms with investment grade debt over the period 1996 to 2002.[10]

The hypotheses made for the two Firms A and B within Leland framework are normally distributed cash flow with mean and standard deviation of (230, 75) for Firm A and (130, 75) for Firm B, respectively. The calibration of parameters $X_i$ and $p_i$ of the two-point distribution within the model framework proposed in the present work is performed by matching first and second moment of the cash flow normal distribution. Firm B is riskier than firm A, with annualized cash flow volatility equal to 26% and 15%, respectively. Then, the last exogenous parameter of the current model is $K_i$. In order to calibrate $K_i$, the strategy is to match the present value of equity $(X_i - R_i + K_i)p_i$ within the current model with the value of optimal equity in Leland model, after setting tax rates and net interest rate to zero and the FACE value of debt to 100.

Given the aforementioned inputs, the following step is to calibrate the values of $P^*$ and $\alpha$ in order to maximize total firm value $\nu_0(P)$, subject to a model expected recovery rate $R(P)$ that

[10]This volatility also approximates the 23% asset volatility that (Leland, 2004) finds, using a structural model of debt, to match Moodys observed default rates on long-term investment grade debt over the period 1980 to 2000.

matches observed recovery rates.[11] Leland model yields, then, default costs equal to 25% for firm A and 16% for firm B. They are within (or very close to) the range reported in Andrade and Kaplan (1998). Optimal zero-coupon bond principals are 105 and 140, respectively. The value of equity is 107.06 for firm A and 25.19 for firm B.

Finally the construction of the bottom part of the Table A2 can be completed. The expected payoff and the annualized cash flow volatility are set as in the adapted Leland firms on the upper part. Similarly, the value of equity at $t = 0$ in is set to be equal to the unlevered firm value $E_0(P)$ in Leland model. The two calibrated models are reported in Table (A2). The resulting calibrated values for firm A are $X_A = 255.56$ (greater than $R_A + R_B = 244.44$), $p_A = 0.90$ and $K_A = 27.68$. For firm B, the calibrated values are $X_B = 173.33$ (less than $R_C = 207.78$), $p_B = 0.75$ and $K_B = 49.22$.

[11]The recovery rate depends upon the level of debt as well as other parameters including the default cost fraction $\alpha$. We assume debt principal is equal to its optimal level, $P^*$. Elton et al. (2001) report an average recovery rate on BBB-rated debt of 49.4% for the period 1987 to 1996, while Acharya et al. (2007) estimate median recovery of 49.1% for their 1982 to 1999 sample of defaulted debt. Direct evidence on default costs, $\alpha$, is mixed. Andrade and Kaplan (1998) suggest a range of default costs, from 10% to 23% of firm value at default, based on studies of firms undergoing highly leveraged takeovers (HLTs). However, firms subject to HLTs are likely to have lower-than-average default costs, since high leverage is more likely to be optimal for firms with this characteristic.

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Table 1: **Model Parameters**
This table reports the estimation of the parameters of our model following the apprach in Leland (1997). Our endogenous variables are: payoff, probability of success, and future profits, for units A and B. In column (1), we report the optimal values of the calibrated parameters for firm A, while column (2) reports the values of the calibrated parameter for unit B. The case base parameters (interest rate = 0%, tax rate = 0%, $D_0* = 100$) and the details of the estimation are reported in the Appendix A.3. We set the optimal debt at 100, and we find the interest factors $R_i$ as a function of the calibrated probabilities of success of the units A and B. To calibrate $K_i$ at the optimum, we use the degree of freedom that we have on default cost. We then calibrate a value of default costs that leads $R_i$ to its optimum $R_i^*$, subject to a recovery rate that follows in an an interval (between 44.5% and 49.5%) of the recovery rate found by Leland (49.3%).

| | | Firm A | Firm B |
|---|---|---|---|
| | | (1) | (2) |
| Future profit at time $t > 1$ | $K_i$ | 27,68 | 49,22 |
| Profit at time $t = 1$ | $X_i$ if state $L$ | 0,00 | 0,00 |
| | $X_i$ if state $H$ | 255,56 | 173,33 |
| Probabilities | $1 - p_i$ | 10% | 25% |
| | $p_i$ | 90% | 75% |
| Debt at time $t = 0$ | $D_i$ | 100,00 | 100,00 |
| Std Dev. $X_i$ | $Sd_i$ | 0.15 | 0.26 |
| Exp. Value $X_i$ | $X_ip_i$ | 230.00 | 130.00 |

Table 2: **True and Apparent Value with Base-Case Parameters**
This table reports the estimation of true and apparent values of units A and B. The value of the parameters are retrieved from table 1. Details of parameter estimation are in Appendix A.3. Columns (1)-(2) report the values for each unit (A and B) when standalone firms (Panel A), when belonging to a business group (Panel B), or when Belonging to a conlomerate. Column (3) reports the total value of units for each organizational structure. The variable $\Delta$ and $\Delta\%$ are, respectively, absolute and relative variation between true and apparent value across organizations.

| Panel A | Firm A<br>Stand-alone<br>(1) | Firm B<br>Stand-alone<br>(2) | Total<br><br>(3) |
|---|---|---|---|
| True value | 154,91 | 66,92 | 221,82 |
| Apparent value | 172,12 | 89,22 | 261,34 |
| $\Delta$ | 17,21 | 22,31 | 39,52 |
| $\Delta\%$ | 11% | 33% | 18% |

| Panel B | Firm A<br>Subsidiary<br>(1) | Firm B<br>Parent<br>(2) | Total<br><br>(3) |
|---|---|---|---|
| True value | 154,91 | 77,99 | 232,90 |
| Apparent value | 172,12 | 79,99 | 252,11 |
| $\Delta$ | 17,21 | 2,00 | 19,21 |
| $\Delta\%$ | 11% | 3% | 8% |

| Panel C | Firm A (1) | Firm B (2) | Total (3) |
|---|---|---|---|
| | Conglomerate | | |
| True value | | | 229,21 |
| Apparent value | 254,67 | | 254,67 |
| $\Delta$ | 25,47 | | 25,47 |
| $\Delta\%$ | 11% | | 11% |

Table 3: **Differences in percentages**
This table reports the differences across organizations between true and apparent values of units A and B, based on on the values in Table 2. Details of parameter estimation are in Appendix A.3. Column (1)reports the differences between true values between parent and standalone firms, conglomerates and standalone firms, and finally conglomerates and business groups. Similarly, column (2) reports the differences between apparent values between parent and standalone firms, conglomerates and standalone firms, and finally conglomerates and business groups.

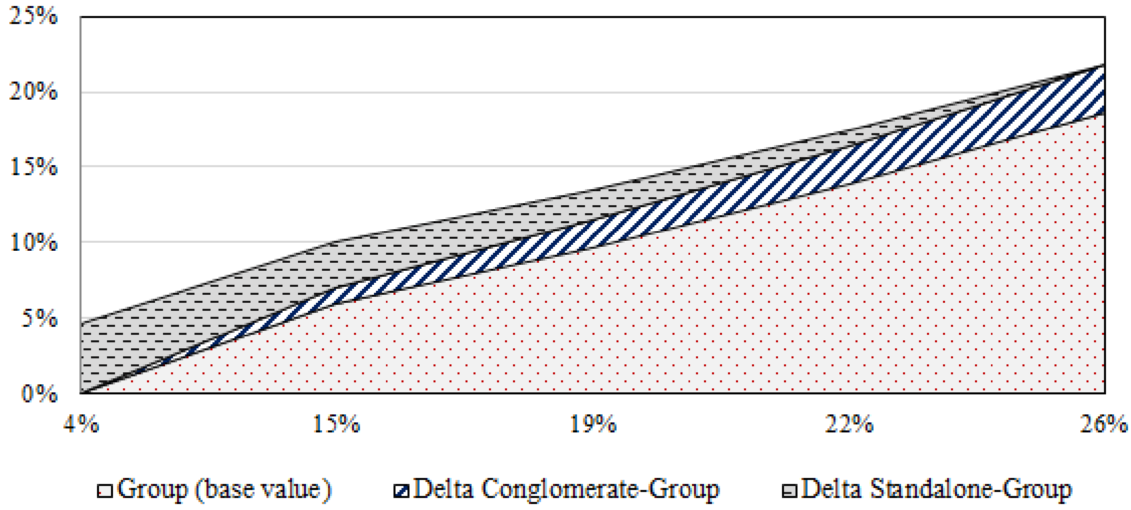
| | True value (1) | Apparent value (2) |
|---|---|---|
| Parent vs Stand-alone | 16.55% | -10.35% |
| Group vs Conglomerate | 1.61% | -1.01% |
| Conglomerate vs Stand-alone | 3.33% | -2.55% |

Table A1: **Notation**
This table reports the resume of the notation and the assumptions for the base parameters used in the present work (column (1)), compared to the one used in Leland (2007, column (2)).

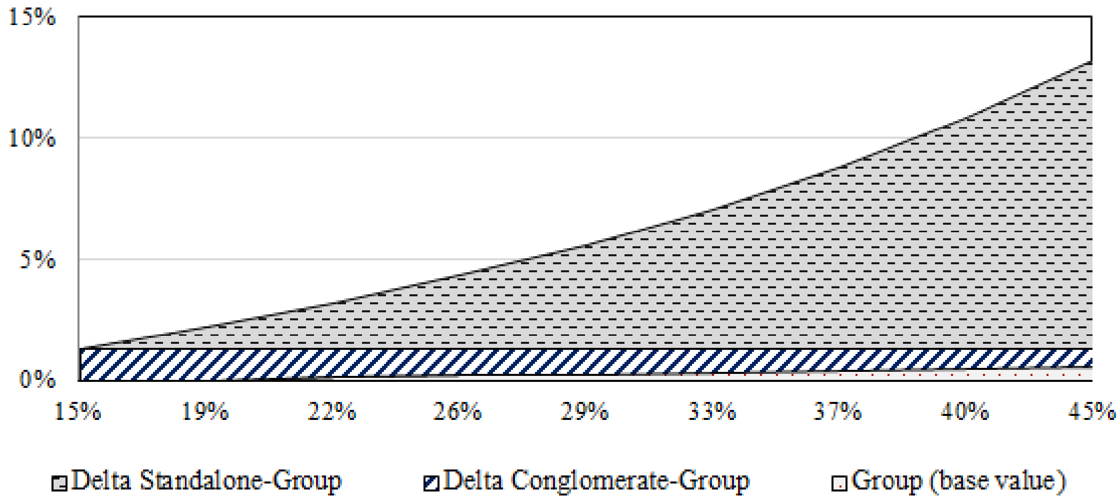
| From the present work (1) | From (Leland, 2007) (2) |
|---|---|
| • two periods, $t = 0, 1$, | • two periods, $t = 0, T$, |
| • two states of the world, $H$, $L$ | • $n \in [2, \infty]$ states of the world |
| • $X_i(.) \geq 0$ profit of firm $i$ in $t = 1$ with $X_i(L) = 0$ and $X_i(H) > 0$ | • $X_i \in \mathbb{R}$ profit of firm $i$ in $t = T$ |
| • probabilities $P_i(H) = p_i$ and $P_i(L) = (1 - p_i)$ | • with cdf $F(X)$ |
| • $D_i = 1$ debt of firm $i$ | • $D_{0,i}(P)$ debt of firm $i$ |
| • $R_i$ interest factor of firm $i$ | • $P$ interest factor of firm $i$ (ZCB) |
| • $r_T = 0$ nonannualazied risk-free interest rate | • $r_T \geq 0$ nonannualazied risk-free interest rate |
| • $K_i \geq 0$ future profit of firm $i$ in $t > 1$ | • $K_i \geq 0$ future profit of firm $i$ in $t > 1$ |

Table A2: **Mapping the Leland (2007) parameters into our state-space**
This table reports the list of all parameters used in the numerical example. In column (1), we report the values as in Leland (2007). In the second and third column, we report the values according to our model. Panel A reports the values of the base-case parameters, while Panel B reports the values of the parameters for the optimal capital structure of the firm in Leland and in our model for firms A and B. The definitions according our model are in parentheses.

| | | (Leland, 2007) | Firm A | Firm B |
|---|---|---|---|---|
| Panel A: Base-Case Parameters | | | | |
| Annual risk-free rate | $r$ | 5.00% | 0.00% | 0.00% |
| Time period/debt maturity (yrs) | $T$ | 5.00 | 5.00 | 5.00 |
| T-period risk-free rate | $r_T = (1+r)^T - 1$ | 0.28 | 0.00 | 0.00 |
| Capitalization factor | $Z = (1+rT)/rT$ | 4.62 | | |
| *Unlevered Firm Variables* | | | | |
| Expected future operational cash flow at T | $Mu$ | 127.63 | 230.00 | 130.00 |
| Expected operational cash flow value (PV) | $X_0 = Mu/(1+r)^T$ | 100.00 | 230.00 | 130.00 |
| Cash flow volatility at T | $Std$ | 49.19 | 75.00 | 75.00 |
| Annualized operational cash flow volatility | $Std/(X_0 T^{0.5})$ | 22.00% | 14.58% | 25.80% |
| Tax rate | $\tau$ | 20.00% | 0.00% | 0.00% |
| Value of unlevered firm w/limited liability | $V_0$ | 80.05 | 230.02 | 131.26 |
| Value of limited liability (after tax) | $(1-\tau)L_0$ | 0.05 | 0.02 | 1.26 |
| Panel B: Optimal Capital Structure | | | | |
| Default costs | $\alpha$ | 23.00% | 25.66% | 16.00% |
| Optimal zero-coupon bond principal | $P^*$ | 57.10 | 101.91 | 140.00 |
| Default value | $X^d$ | 67.65 | 101.91 | 140.00 |
| Breakeven profit level | $X^z$ | 14.90 | 1.91 | 40.00 |
| Value of optimal debt | $D_0^*$ | 42.20 | 100.00 | 100.00 |
| Value of optimal equity | $E_0^*$ ( $(X_i - R_i + K_i)p_i$) | 39.24 | 129.42 | 25.19 |
| Optimal levered firm value | $v_0^* = D_0^* + E_0^*$ | 81.44 | 229.42 | 125.19 |
| Optimal leverage ratio | $D_0^*/v_0^*$ | 51.81% | 43.59% | 79.88% |
| Annual yield spread of debt (%) | $(P_0^*/D_0^*)^{(1/T)} - 1 - r$ | 1.23% | 0.38% | 6.96% |
| Recovery rate | $R$ | 49.30% | 47.93% | 46.29% |
| Tax savings of leverage (PV) | $TS_0$ | 2.32 | 0.00 | 0.00 |
| Expected default costs (PV) | $DC_0$ | 0.72 | 0.61 | 6.66 |
| Value of optimal leveraging | $v_0^* - V_0$ or $TS_0 - DC_0$ | 1.60 | -0.61 | -6.66 |
| Capitalized value of optimal leverage | $Z(v_0^* - V_0)/V_0$ | 8.07% | 0.00% | 0.00% |




Panel A: Differences (%) in values across organizatations for changes (%) of unit A volatility




Panel B: Differences (%) in values across organizatations for changes (%) of unit B volatility

Figure 1: **Sensitivity of apparent values to a mean-preserving increase in units A and B's volatility**

This figure reports the percentage change in the apparent firm value when the cash flow volatility of firm A (panel A) and B (panel B) varies. We standardize the lowest apparent firm value for groups at 0% when the volatility equal is minimum, to report the percentage differences between groups and conglomerates, and between conglomerates and stand-alone firms, for incremental percentage differences in the cash flow volatilites. The dotted area at the bottom of the figures report the value of groups. The white-dashed area represents the excess value (in %) of conglomerates with respect to groups, and the grey-dashed area represents the excess value (in %) of standalones with respect to groups. The values of the volatility varies in an interval with respect to the value of the volatility reported in our base case in Table 1 (respectively, 15% for the volatility of unit A and 26% for the volatility of unit B). In panel A, we report the percentage differences in values between groups and conglomerates, and between conglomerates and stand-alone firms, for incremental percentage differences in the volatility of unit A. In panel B, we report the percentage differences between groups and conglomerates, and between conglomerates and stand-alone firms, for incremental percentage differences in the volatility of unit B.