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Filippo Taddei

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Collegio Carlo Alberto

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Abstract

Introducing heterogeneity of beliefs across different agents builds a link between wealth distribution and the equity premium. We demonstrate that an economy populated only by risk neutral agents may nonetheless display a strictly positive equity premium.

We then place our notion of belief heterogeneity within the popular representative agent construct. We show that any level of belief heterogeneity in the multi agent economy can be mapped into some specific degree of risk aversion of the representative agent economy that keeps equilibrium prices constant. A fully dynamic model follows.

Finally, we suggest an explanation for the recent behavior of the equity premium: a story of "heterogeneous optimism" versus "homogeneous pessimism" is presented.

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1 Introduction

This paper studies, in a general equilibrium framework, the role of belief heterogeneity in affecting financial asset pricing and its interaction with the distribution of wealth. This exercise not only contributes to the explanation of the "equity premium puzzle" (Mehra and Prescott (1985)) but also provides a novel interpretation of the temporal evolution of the premium in the last decade.

We will show how an economy populated only by risk neutral agents can nevertheless display a strictly positive equity premium, i.e. the difference between the rate of return of stocks and bonds. The extent of the premium will depend on the initial distribution of wealth, once we allow for heterogeneity of beliefs about the future among agents with otherwise identical preferences. It is worthy to highlight that, in general, belief heterogeneity affects the equity premium: even perfectly symmetric divergence of individual beliefs matter. In fact, in equilibrium, the equity premium is always determined as a weighted mean of individual beliefs: the weights are fundamentally determined by the relative share of wealth that each group sharing the same beliefs has. This is why the equity premium results as the interaction of belief heterogeneity and wealth distribution: without belief heterogeneity, wealth distribution does not matter[1]; with egalitarian distribution of wealth, only the beliefs of the average agent matters and thus heterogeneity, by itself, can not affect the equity premium. In general though, the only case in which a change in belief heterogeneity does *not* affect the equity premium is when this change exactly matches the distribution of wealth in the economy. But this clearly is a measure zero event so - we conclude - belief heterogeneity generically affects the equity premium[2]. These considerations will become transparent after section 3.

We will then reconcile heterogeneity of beliefs in a multi individual economy with the representative agent framework. We argue that, in the standard exercises, a representative agent's risk aversion simultaneously embodies two things: an economy wide measure of risk aversion *and* the level of belief heterogeneity present in the economy. We prove that any level of belief heterogeneity may be mapped into a particular degree of risk aversion for the corresponding representative agent economy.

We subsequently introduce a fully dynamic version of the model that connects our story to the standard consumption asset pricing model of Lucas [1978]. This infinite horizon economy delivers two interesting features. First, it endogenizes the duration of belief heterogeneity. Secondly, it allows the study of its cumulative effects along a possibly long (though finite) sequence of periods. Finally, we apply the theoretical interpretation provided above to analyze the temporal evolution of the equity premium in the last decade. We present some evidence supporting the view that a particularly high degree of belief heterogeneity was present in the economy at that time. Consistently with our theoretical results, we also provide a possible explanation of the recent evolution of the premium over time.

Within the wide range of models from which the "equity premium puzzle" emerged, it

[1] Assume throughout that individuals have identical utility function.

[2] Notice that our results do not require the assumption of short-sales constraints as in Miller [1977].

is recognized that stocks are not "sufficiently" risky to pay the premium that is actually observed. In other words: if the degree of risk aversion coming from micro data is reliable, the empirical difference in returns between stocks and bonds is too large compared to what can be explained through agent's risk aversion; equivalently, the consumption path of the representative agent is too smooth given the "low" return risk characteristic of the market portfolio of stocks.

Kocherlakota [1996] argues that there are three theoretically crucial assumptions on which the puzzle depends: (a) standard Constant Relative Risk Aversion (CRRA) preferences, (b) complete capital markets and (c) costless trading. The research relaxing the complete market assumption and the frictionless trade hypothesis [e.g. Aiyagari and Gertler, 1991; Telmer, 1993; Heaton and Lucas, 1996 and 1997)] has not seemed to provide substantial advancements in understanding the puzzle. We will argue that a simple application of the Arrow-Debreu framework can illustrate an alternative theoretical explanation of the "equity premium puzzle" that discusses only the underlying assumption of this class of models: the representative agent. The following exercise can thus be interpreted as an attempt to depart from the representative agent framework so as to isolate what heterogeneity in beliefs across agents really adds.

In order to keep the analysis as general as possible - but without useless complications - we start by employing a general equilibrium framework with two agents, two periods, two assets and one commodity. This choice is inspired by recent application of general equilibrium theory[3]. This setup has the benefit of clearly highlighting the completeness of the asset structure in a frictionless environment. In section 7, we extend the initial results to the infinite horizon economy and find that most results carry over despite minor complications.

A good deal of literature has focused, more or less directly, on the effects of various kinds of heterogeneity on asset prices. Using a closely similar framework to the one employed here, an important part of this literature has studied the effect of heterogeneous endowments on equilibrium pricing. They tipically assume that, with some positive probability, agents may face reduced (or zero) labor income. The consequences in terms of representative agent representation and aggregate fluctuations have been studied at least since Constantinides [1982], Constantinides and Duffie [1996] and Krusell and Smith [1995].

Although the studies abovementioned have provided an interesting branch of research, the empirical evidence for the US points out that only relatively few economic agents are "stockholders". Though this group has been steadily increasing since the Fifties, the 1998 "Survey of Consumer Finances" still reports that 95 % of stocks are owned by the top 10% of stock owners[4]. This evidence suggests what will be the central point of this paper: the study of equilibrium asset pricing should focus more on heterogeneity that is *not* income related. The recent shift in the literature toward preference heterogeneity - and belief heterogeneity in particular - as a determinant of equilibrium asset pricing may be interpreted to be

[3] In particular, Geanakoplos [2002].

[4] Guvenen [2002] reports that even if "the stockholding rate has reached almost 50 percent in 1999", "a significant fraction of households are holding very small amounts of stocks. In the 1984 PSID data, 24 percent of households declare themselves as stock owners whereas the fraction holding more than $10,000 worth of stocks is less than 10 percent (Mankiw and Zeldes 1991)". [p.10, ibid]

consequence of this evidence.

Various attempts employ a continuous time framework. Ziegler [2001] and Berrada [2003] focus on the statistical property of asset pricing when agents are assumed to have heterogeneous understanding about the dividend process underlying the economy. Wang [1995], Chan *et al* [2002], and Dumas [1987] consider specific examples with heterogeneous degrees of risk aversion. Their conclusions, though interesting to our purpose, are affected by the assumed functional forms. Moreover it is not immediate to reconcile them with the traditional discrete time framework so widely used in macroeconomics. Always in continuous time, Duffie *et al.* [2001] study the effect of belief heterogeneity on asset pricing when shorting of lended securities is possible. In their framework, as in ours, the rationale for trading is provided by heterogenous belief across different agents. They focus on the determination and evolution of prices when lending fees arise. One of the basic insights of our study, perhaps more macroeconomic oriented, is also to link heterogeneity in a multiagent economy with the representative agent framework.

Guvenen [2002] integrates preference heterogeneity in the standard dynamic programming framework to replicate the main financial and macroeconomic stylized facts. His results require a substantial heterogeneity in the elasticity of intertemporal substitution across agents and limited participation in the stock market. Unfortunately, although Guvenen's interesting results do not allow us to *isolate* the effect of preference heterogeneity and, in particular, belief heterogeneity. This is, as we argued, the important exercise that we will undertake in the present study.

An outline of the paper is as follows: section 2 introduces the general framework of analysis and analyzes the no arbitrage condition which must be satisfied in equilibrium. In section 3 we show that heterogeneity in beliefs establishes the relationship between equity premium and wealth distribution and we generalize it to a strictly concave economy. Section 4 reconciles our notion of heterogeneity of beliefs with the popular representative agent representation. Section 5 extends the static model to the traditional infinite horizon Lucas (1978) model and provides an interpretation about the recent behavior of the equity premium. Section 6 concludes.

2 A Simple General Equilibrium Model

2.1 Setup

Our initial framework of analysis is an Arrow-Debreu economy with production. For simplicity of exposition, we assume an economy composed of two agents (L, M) and two periods: today and tomorrow. Today $(t = 0)$ individual endowments are revealed and the agents choose which assets to buy in order to provide consumption for themselves in the future. Tomorrow $(t = 1)$ stochastic production takes place: in each of two contingengies $(s = 1, 2)$ the (contingent) total production is distributed to each individual according to the share of property rights (stocks) he holds. There is also available a riskless asset, in zero net supply, that pays one unit of the consumption good in *both* contingencies.

At $t = 0$ agents (L, M) are endowed only with fractions of the perfectly divisible claim to total output, thus this asset entitles the owner to a share of total production tomorrow. We will denote this endowment as (z_L^0, z_M^0) where $\sum_{i=L,M} z_i^0 = 1$. Individuals then decide how to allocate their wealth for future consumption: they can exchange property rights over the production technology (i.e. z_i) and/or trade the riskless asset b_i. Asset payoffs are the only source of income in both contingencies $s = 1, 2$ at $t = 1$.

To summarize, only two kinds of asset can be traded between at $t = 0$, a stock z_i, $i = L, M$ where $\sum_{i=L,M} z_i = 1$, which entitles the holder to a share of production at $t = 1$ and a bond b_i, $\sum_{i=L,M} b_i = 0$ that guarantees 1 unit of the consumption good in both contingencies. Choosing a portfolio at $t = 0$ implies therefore some uncertainty. The asset payoff are as follows:

$$\begin{bmatrix} & \text{Boom } (s=1) & \text{Recession } (s=2) \\ \text{bond } b & 1 & 1 \\ \text{stock } z & d_B = B & d_R = R \end{bmatrix}; B > R > 1$$

M and L differ in their behavior in only one way: they have different priors used to maximize expected utility. Following Savage [1954] interpretation, we will view priors as an assessment about the future that can be "extracted" from the individual's preference order. In particular, we will asssume that L and M have, respectively, priors $P_L(s)$ and $P_M(s)$:

$$\begin{array}{l} P_L(s) = \begin{cases} \pi \; if \; s = 1 \\ (1-\pi) \; if \; s = 2 \end{cases} \\ P_M(s) = \begin{cases} \rho \; if \; s = 1 \\ (1-\rho) \; if \; s = 2 \end{cases} \end{array} ; \pi > \rho; \pi > (1/2)$$

Therefore, the expected returns of z and b must bear the following relationship to one another[5]:

$E_{P_M}(b) = E_{P_L}(b) = 1 < E_{P_M}(z) = \rho B + (1-\rho)R < E_{P_L}(z) = \pi B + (1-\pi)R$.

The problem Υ^i, $i \in \{L, M\}$, becomes[6]:

$$\begin{array}{lcc} \max_{z_i, b_i} \sum_{s=1,2} U^i(c_s) P_i(s); \text{ s.t.:} & & \\ p_z z_i + p_b b_i \leq p_z z_i^0 & at & t = 0 \\ c_L^s \leq d_s \cdot z_i + b_i & at & s = 1, 2 \\ d_s = \text{payoff of z}, \; c_i^s \geq 0, \; i \in (M, L) & & \end{array} \tag{1}$$

We seek an explanation of the equity premium that *abstracts* from risk aversion. Therefore we assume a linear utility function, i.e. $U^i(c^s) = c_i^s$, $i \in (M, L)$, for both agents in order to focus on belief heterogeneity alone as a determinant of the equity premium.

2.2 The No Arbitrage Condition: Asset Prices and Technology

Since equilibrium prices must not create arbitrage opportunities for any agent, the no arbitrage condition can be used to determine the interval in which equilibrium asset prices

[5] The second strict inequality is the result of our assumption on (P_L, P_M).

[6] We abstract from consumption at $t = 0$ to avoid complications.

lie. No Arbitrage implies that there is no portfolio giving strictly positive payoff in one state (and at least zero in all the others) at *zero cost*. Formally:

[No Arbitrage] In equilibrium, there is no portfolio (z, b) available to any individual such that: $p_z z + p_b b = 0$ and $d_s \cdot z + b \geq 0$,*with strict inequality for some s.*

Before stating the consequences of the condition above, we must make a crucial observation. Since L, M have different beliefs about the future, the final portfolio allocation depends on the difference between the prior of M and the prior of L. In fact, by assumption, $E_{P_M}(z) < E_{P_L}(z)$. Thus, since utility functions are linear, the highest price that L and M, respectively, are willing to pay for purchasing the risky asset is different. In particular, the subjective limit price ($\bar{p}_z^i = E_{P_i}(z)$, $i \in (L, M)$) are such that $\bar{p}_z^L > \bar{p}_z^M$. Therefore:

Proposition 1 *If beliefs are heterogeneous across agents, in equilibrium, M (L) goes short (long) in stocks and long (short) in bonds, i.e.* $z^M < 0$ *and* $z^L > 0$ *(*$b^M > 0$ *and* $b^L < 0$*).*

The proof is immediate. It is only worthwhile to highlight that going short on the risky asset implies going long on the bond because of the non negativity constraint on $t = 1$ consumption. To determine the interval in which equilibrium asset prices must lie, we argue by contradiction. If an arbitrage portfolio (z^A, b^A) were to exist, the following must simultaneously hold:

$d_s \cdot z^A + b^A \geq 0 \Longleftrightarrow b^A \geq -d_s \cdot z^A$ with strict inequality for some s;

$p_z z^A + p_b b^A = 0$.

But then:

$p_z z^A + p_b \left(-d_s \cdot z^A\right) \leq p_z z^A + p_b b^A = 0$ with strict inequality for some s

Since no arbitrage has to hold for both individuals we have:

- $z_M < 0 \Rightarrow p_z z^A + p_b \left(-d_s \cdot z^A\right) \leq 0 \Rightarrow$

$\Longrightarrow \frac{p_z}{p_b} \geq d_s, \forall s \in \{1, 2\} \Longrightarrow \frac{p_z}{p_b} \geq \max_s d_s = B$

- $z_L > 0 \Rightarrow p_z z^A + p_b \left(-d_s \cdot z^A\right) \leq 0 \Rightarrow$

$\Longrightarrow \frac{p_z}{p_b} \leq d_s, \forall s \in \{1, 2\} \Longrightarrow \frac{p_z}{p_b} \leq \min_s d_s = R$

Therefore the equilibrium (relative) price is bounded by the payoffs of the production technology:

$R = \min_s d_s < \frac{p_z}{p_b} < \max_s d_s = B$

3 The Relation between the Equity Premium and the Distribution of Wealth in Equilibrium

3.1 The Linear Economy

We will now compute the equilibrium. This computation will deliver the relation between the equity premium and the distribution of wealth when belief heterogeneity is present. An equilibrium $(\Upsilon^i, z_i^0, i \in (M, L))$ is defined by $(b_i, z_i, p_z, p_b, i \in (M, L))$ such that:

$$\sum_{i=L,M} b_i = 0 \tag{2}$$

$$\sum_{i=L,M} z_i = 1 \tag{3}$$

$$d_s = \sum_{i=L,M} c_s^i ; s = 1, 2 \tag{4}$$

$$(b_i, z_i) \in \arg\max \Upsilon^i, i \in (M, L) \tag{5}$$

At given equilibrium price, agents maximize utility under their budget constraints - in particular, consumptions tomorrow must be non negative for both L and M - and all markets clear.

Since what matters is the relative price of stocks and bonds, it does no harm to normalize the price of the bond, i.e. $p_b = E^M(b) = E^L(b) = 1$. With this normalization, the equilibrium (relative) price of the risky asset lies between the two subjective limit prices, $\bar{p}_z^i$[7], $i \in (M, L)$, $\bar{p}_z^M < \bar{p}_z^L$. Otherwise, if $p_z < \bar{p}_z^M$, both agents would purchase stocks and an excess demand for the risky asset z would appear. If, instead, $p_z > \bar{p}_z^L$, all the individuals would sell stocks and an excess supply would appear. One should keep in mind that the subjective limit price is the price that makes one agent indifferent between holding or selling an asset.

With the equilibrium price $p_z \in \left[\bar{p}_z^M, \bar{p}_z^L\right]$, M finds rewarding to sell z (at least he is indifferent). p_z is so high from M's point of view that he sells all his financial endowment, z_M^0, going even short on z and he purchases as many bonds as possible with the proceeds. An opposite reasoning holds for L. Since M goes short on the risky asset and default is not allowed in any contingency, M must purchase bonds, b, from L. At the same time and for opposite reasons, L is willing to buy as much z as he can, selling bonds. But L also has to be sure that he will not default in any state, in particular in the recession ($s = 2$) when what he gets from the risky asset is low and what he has to pay to the bond holder is relatively high.

Therefore, since M goes short on the risky asset ($z^M < 0$) and long on the bond $\left(b^M > 0\right)$ as much as he can, he will eventually hit the nonnegativity constraint on consumption in the boom ($s = 1$). M's optimization will thus deliver $c_M^B = 0$, i.e. M's consumption at $s = 1$ is null. By an opposite reasoning L will hit the constraint at $s = 2$ and so $c_L^R = 0$:

$$c_L^2 = 0, (RECESSION) \Leftrightarrow R \cdot z_L + b_L = 0 \Leftrightarrow b_L = -R \cdot z_L < 0 \tag{6}$$

$$c_M^1 = 0, (BOOM) \Leftrightarrow B \cdot z_M + b_M = 0 \Leftrightarrow b_M = -B \cdot z_M > 0 \tag{7}$$

Equations (2), (3), (6), (7) are a linear system of four equations in four unknowns whose solution is:

$$b_M = \tfrac{RB}{B-R} > 0; \quad b_L = \tfrac{RB}{R-B} < 0 \tag{8}$$

[7] Defined in the previous section.

$$z_M = \frac{R}{R-B} < 0; \quad z_L = \frac{B}{B-R} > 0 \tag{9}$$

Plugging these results in L's budget constraint at $t = 0$ and recalling that the bond price is $p_b = 1$, we get:

$$\left(\frac{B}{B-R}\right) - \left(\frac{RB}{B-R}\right)\left(\frac{1}{p_z}\right) = z_0^L \tag{10}$$

(10) shows the relation between wealth distribution and asset pricing. It is immediate to observe that an higher share of wealth of the optimistic individual L translates into higher stock price p_z.

In Proposition 2 we argued that, at equilibrium prices, M goes short and L goes long on the risky asset. This is the case because the price of the risky asset, p_z, lies in the interval $\left[\bar{p}_z^M = E_{P_M}(z), \bar{p}_z^L = E_{P_L}(z)\right]$, where $\bar{p}_z^i$, $i \in (M, L)$, are the subjective limit prices. At any price in the open interval, M goes short in stocks and long in bonds while L does exactly the opposite. At $\bar{p}_z^M$, M is indifferent between buying or selling the risky asset but L strictly prefers to buy it. At $\bar{p}_z^L$ the reverse is true: L is indifferent but M wants to sell z as much as he can. This argument shows that any $p_z \in \left[\bar{p}_z^M, \bar{p}_z^L\right]$ can support our equilibrium allocations [equations (8), (9)].

To summarize, wealth distribution maps into equilibrium price as follows (Figure 1):

$$\begin{bmatrix} \text{No Equilibrium} & p_z > \bar{p}_z^L \\ z_L^0 \in \left[\overline{z_L^0}, 1\right] & p_z = \bar{p}_z^L \\ \left(\frac{B}{B-R}\right) - \left(\frac{RB}{B-R}\right)\left(\frac{1}{p_z}\right) = z_L^0 & p_z \in \left(\bar{p}_z^M, \bar{p}_z^L\right) \\ z_L^0 \in \left[0, \underline{z_L^0}\right] & p_z = \bar{p}_z^M \\ \text{No Equilibrium} & p_z < \bar{p}_z^M \end{bmatrix}$$

$$\begin{aligned} \text{where } \underline{z_L^0} &= z_L^0\left(\bar{p}_z^M\right) = \left(\frac{B}{B-R}\right) - \left(\frac{RB}{B-R}\right)\left(\frac{1}{\bar{p}_z^M}\right); \\ \overline{z_L^0} &= z_L^0\left(\bar{p}_z^L\right) = \left(\frac{B}{B-R}\right) - \left(\frac{RB}{B-R}\right)\left(\frac{1}{\bar{p}_z^L}\right) \end{aligned}$$

If the wealth distribution is such that $z_0^L \in \left[\underline{z_0^L}, \overline{z_0^L}\right]$, the equilibrium price $p_z \in \left[\bar{p}_z^M, \bar{p}_z^L\right]$ and is increasing the richer L gets. The discussion above applies: L goes long in stocks and short in bonds and M does the opposite since the equilibrium price is determined according to (10).

If the share of wealth of L is low ($z_0^L \in \left[0, \underline{z_0^L}\right]$) the price is constrained at the lower bound of the interval, $\bar{p}_z^M$. Even though the lack of resources of L, the stock buyer, tends to push the price of the risky asset down, market equilibrium considerations prevent the price from falling further. L will then adjust his portfolio since the amount of risky asset he can afford has now decreased. Notice that, because of the change in L's portfolio when his wealth is particularly low, L's and M's consumption will adjust (in particular L will decrease consumption in contingency 1 and M will have positive consumption in both contingencies). Since at $\bar{p}_z^M$ M is indifferent between selling or holding z, any portfolio allocation that satisfies his budget constraints with equality is optimal. If, finally, $z_0^L \in \left[\overline{z_0^L}, 1\right]$, L's wealth would now push up p_z but the market equilibrium locks the price at $\bar{p}_z^L$. The optimal portfolio allocation would change in an opposite fashion to the previous case and so we will not repeat the reasoning. We therefore conclude that any level of wealth maps into some asset prices.

Defining the equity premium as the difference between the rates of return of the risky and the riskless asset, we have:

$$R_z - R_b = \frac{E_{P_L}(z)}{p_z} - \frac{E(b)}{p_b} = \frac{E_{P_L}(z)}{p_z} - 1$$

We already observed, by (10), that the price of stocks, p_z, is monotonically increasing in L's share of financial wealth, z_0^L. Since the price of bond is normalized to 1, we can now relate the distribution of wealth to the risk premium: the premium depends inversely on the price of the risky asset which, in turn, is positively correlated with the L's initial share of financial wealth.

Since different agents have different beliefs about the future, the expected value of asset payoffs is subjective. When we wrote the expression for the equity premium, we used the expected payoff of stocks according to the beliefs of L, the only stockholder in equilibrium. However, we could have assumed that the expected payoff of stock had to be computed with respect to the probability assessment of M, the pessimistic agent holding zero stock in equilibrium. In that case, a strictly negative equity premium would have resulted, though still depending on the wealth distribution. Our assumption is due to the desire of highlighting the relevance of belief heterogeneity in addressing the equity premium puzzle. But the central point always holds: with belief heterogeneity the wealth distribution affects the equity premium.

In general, two interpretations of the equity premium can be raised to justify the way we describe it. Firstly, we can view it as the *subjective* equity premium, i.e. the equity premium computed respect to the beliefs of the stockholder. This agent is, needless to say, the most optimistic individual: he assigns the highest expected value to the risky asset. Since it is certainly difficult to assume that each agent knows the actual distribution of relevant stochastic events, this interpretation could be seen as a way to convey some realism in the model. The main result here is the relation between asset pricing and distribution: agents' disagreement about the future is the linking mechanism. The drawback of this interpretation is the difficulty in assessing our story through empirical evidence.

An alternative interpretation is that we have described an economy in which some agents in the economy are pessimisticly wrong about the future. If this fraction of the population has positive wealth, stock prices will move down accordingly. Then the equity premium will tend to increase. More importantly, even if the large majority of agents knows the actual distribution of dividends in the future and is never wrong about its structure, mistakes by a small part of the population can still deliver a strictly positive equity premium in risk neutral environment. One could identify this kind of equity premium as an "heterogeneity premium"[8].

In either of these cases, however, a complete mapping between wealth distribution and the equity premium is established. To summarize:

Proposition 2 (Wealth and Asset Pricing) *Given the level of belief heterogeneity in a risk neutral economy, the higher is the financial wealth of the most optimistic individual, z_0^L, the higher is the relative price of stock and the lower the equity premium. Moreover this relationship is monotonic.*

The intuition is straightforward: if everyone is optimistic in the economy (L type), there is nothing to be rewarded for buying the risky asset. Vice versa, the higher the proportion of pessimistic investors (M type) the higher the risk premium guaranteed to the (optimistic) buyer of the risky asset will be. Notice finally that there is only one price at which the equity premium is zero: $\bar{p}_z^L$.

3.2 The Strictly Concave Economy[9]

The relationship between financial wealth and the equity premium is by no means dependent on the assumption of linear utility. In this section we will restate our theory in an environment where agents possess differentiable, strictly concave ($U_c^i() > 0, U_{cc}^i() < 0, i = L, M$) utilities which satisfy the following:

[Weak Inada] Let x and y be two consumption vectors, $x >> 0$ and $y_c = 0$ for some consumption good c, then $U^i(x) > U^i(y)$, $i = L, M$.

In every other respect we consider an economy equivalent to the original. Since in equilibrium all agents optimize, we start from the individual maximization problems Υ^L and Υ^M - defined in (1) - with strictly concave utility and Condition *4* holding. Equivalently the individual problem becomes:

$$\max_{z_i, b_i} \xi^i = \sum_{s=1,2} U^i(c_s)P_i(s) + \mu_0^i(p_z z_i^0 - p_z z_i - p_b b_i) + \sum_{s=1,2} \mu_s^i c_s{}^{10}; i = L, M$$

where μ_1^i, μ_2^i are the lagrange multipliers on the nonnegativity constraints on future consumption. Therefore:

$$\frac{\partial \xi^i}{\partial z_i} = 0 \Leftrightarrow p_z \mu_0^i - B\mu_1^i - R\mu_2^i = \sum_{t=1,2} U_c^i\left(c_s\right) d_s P_i(s) \tag{11}$$

[8] Abel [2000] already shows that "uniform pessimism and doubt both increase the average equity premium" (p.2) in a representativa agent economy. Here we point out that the discussion should include another factor. The pessimism of a small fraction of the population is necessary but not sufficient to generate a strictly positive equity premium. In fact, given belief heterogeneity, the premium is determined by wealth distribution, as the proposition states.

[9] See appendix for details regarding this section.

[10] We do not consider explicitly tomorrow budget constraints because, by monotonicity, all that is received tomorrow is consumed.

$$\frac{\partial \xi^i}{\partial b_i} = 0 \Leftrightarrow p_b \mu_0^i - \mu_1^i - \mu_2^i = \sum_{s=1,2} U_c^i\left(c_s\right) P_i(s) \tag{12}$$

By Condition *4*, consumption is strictly positive in both periods and so the lagrange multipliers on the nonegativity constraints are equal to zero. Therefore, in equilibrium, each individual will choose his portfolio so that:

$$MRS_i = \frac{\sum\limits_{s=1,2} U_c^i\left(c_s\right) d_s P_i(s)}{\sum\limits_{s=1,2} U_c^i\left(c_s\right) P_i(s)} = \frac{p_z}{p_b}, i = L, M \tag{13}$$

Equating the marginal rate of substitution for both agents we have:

$$\Theta = MRS_L - MRS_M =$$

$$= \frac{\pi B U_c^L\left(c_B^L\right) + (1-\pi) R U_c^L\left(c_R^L\right)}{\pi U_c^L\left(c_B^L\right) + (1-\pi) U_c^L\left(c_R^L\right)} - \frac{\rho B U_c^M\left(c_B^M\right) + (1-\rho) R U_c^M\left(c_R^M\right)}{\rho U_c^M\left(c_B^M\right) + (1-\rho) U_c^M\left(c_R^M\right)} = 0 \tag{14}$$

Using the budget constraint at $t = 0$ to express z as a function of b and equilibrium condition on the bond and stock market, $\Theta\left(z_L, p_z/p_b; z_0^L\right)$ is an implicit expression linking wealth distribution $\left(z_0^L\right)$ to the price of stock (p_Z). Since our ultimate objective is to get a relation between the two quantities, we apply the Implicit Function Theorem. In order to do so we can not rely on $\Theta\left(z_L, p_z/p_b; z_0^L\right)$ only because there are two endogeneous variables in it. Equations (13) and (14) define a system of equations that characterize the equilibrium:

$$\begin{cases} \Theta\left(z_L, p_z/p_b; z_0^L\right) = 0 \\ MRS_L\left(z_L, p_z/p_b; z_0^L\right) - \left(p_z/p_b\right) = 0 \end{cases}$$

Defining $\left[z_L, \left(p_z/p_b = \Pi\right)\right]$ as endogenous variables and $\left(z_0^L\right)$ as exogenous, we have the following smooth functional:

$$F\left[z^L, \left(p_z/p_b = \Pi\right); z_0^L\right] = \left[\begin{array}{c} \Theta\left(z_L, \Pi; z_0^L\right) \\ MRS_L\left(z_L, \Pi; z_0^L\right) - (\Pi) \end{array}\right] : R^2 \times R \longrightarrow R^2$$

After linearizing the system around the equilibrium, some algebra and some parametric assumptions, we get:

$$\frac{\partial\left(p_z/p_b\right)}{\partial z_0^L} = \begin{cases} < 0 \text{ if is } z_0^L \text{ "high", i.e. } z_0^L > z_L \\ > 0 \text{ otherwise} \end{cases} \tag{15}$$

(15) confirms the existence of a relation between the share of financial wealth of L and the stock price, i.e. between wealth distribution and asset pricing. To summarize:

Proposition 3 (Wealth and Asset Pricing, Strictly Concave Economy) *In strictly concave economies the relation between wealth distribution and asset pricing is not monotonic. Under our restrictions: when the share of financial wealth,* z_0^L*, of the most optimistic individual is relatively low* $\left(z_0^L < z_L\right)$*,then the income effect dominates the substitution effect and the relative price of stocks is increasing in his wealth; when the share of financial wealth,* z_0^L*, of the most optimistic individual is relatively high* $\left(z_0^L > z_L\right)$*, then the substitution effect dominates and the relative price of stocks is decreasing in his wealth.*

This result has an intuitive explanation: for a relatively low share of wealth for L, the richer he becomes, the more the relative price of stock will tend to increase. Then, once L's wealth increases above a certain threshold, the substitution effect between consumption in the boom and recession will start to dominate the income effect. This will make L willing to get a more balanced consumption across contingencies and he will then want to sell some of his endoment of stock to purchase bonds. Therefore, in order for the market equilibrium to emerge, $\frac{p_z}{p_b}$ will have to fall.

The considerations above - though subject to some restrictions regarding the size of some second derivatives - suggest that, in the case of an economy where agents have some degree of risk aversion, there will be a particular wealth distribution that, maximizing the relative price of stock over bond, will minimize the equity premium. This result, achieved under fairly general condition, probably deserves some empirical attention. In conclusion, if the assumption of belief heterogeneity is plausible[11], then studying the equity premium should not abstract from distributional consideration in both linear and strict concave economies.

4 Mapping Belief Heterogeneity into Risk Aversion

In this section we relate the previous discussion to the representative agent framework. We discuss the way its risk aversion is interpreted in the standard exercises and argue that this interpretation implicitly embodies belief heterogeneity.

We define two strictly concave economies: a multi agent economy E_1 and a representative agent economy E_2. There are 2 periods (today ($t = 0$) and tomorrow ($t = 1$)) with S contingencies ($s = \{1, ..., S\}$), resolved between the first and the second period. We will study the conditions under which these economies deliver market equilibria that are equivalent in terms of pricing.

E_1 is the multi agent economy. Agent ($h \in \{1, ..., H\}$) maximes expected utility:

$$U^h = \sum_s \pi_h(s) u(c_s^h)$$
$$\text{subject to:} \sum_s p_s c_s^h \leq \sum_s p_s e_s^h \qquad at \qquad t = 0$$
$$u : R^S \to R, \text{twice differentiable}$$
$$u_c(.) > 0, u_{cc}(.) < 0, \lim_{c \to 0} u_c(.) = +\infty, h \in \{1, ..., H\}$$
$$\pi_h(s) > 0, \forall s, \sum_s \pi_h(s) = 1, \text{ a probability measure.}$$

[11] Maybe option prices (e.g. the call premium on the S&P 500 portfolio) may provide an instrument to measure belief heterogeneity in the equity market.

We will denote this economy as $E_1 = [(u^h = u, e^h)_{h\in\{1,...,H\}}]$ where $e^h, c^h \in R^S$.

E_2 is an economy in which the representative agent maximizes:

$$U^{RA} = \sum_s \pi_{RA}(s) u(c_s^{RA})$$
$$\text{subject to: } \sum_s p_s c_s^{RA} \leq \sum_s p_s e_s^{RA} \qquad at \qquad t=0$$
$$u(.) \text{ as above; } e_s^{RA} = \sum_{h\in\{1,...,H\}} e_s^h, \forall s$$

Aggregate endowments is thus the same in both economies, contingency per contingency. For brevity, we will denote the second economy with $E_2 = [(u^{RA} = u, e^{RA})]$.

In both economies agents purchase at $t = 0$ the contingent commodity for every contingency $s = \{1, ..., S\}$. Thus, the number of contingent commodities is S.

Definition 1 *A Market Equilibrium (ME) for our economies is defined by a price vector* $(p_1, ..., p_S) \in R^S$ *and an allocation* $\left(c_1^h, ..., c_S^h\right)_{h\in\{1,...,H\}}$ *for* E_1 *and* $\left(c_1^{RA}, ..., c_S^{RA}\right)$ *for* E_2 *such that:*

$$\sum_{h\in\{1,...,H\}} e_s^h = \sum_{h\in\{1,...,H\}} c_s^h = e_s^{RA}, \forall s$$

$$(c_s^i) \in \arg\max U^i, \ i \in (h \in \{1, ..., H\} \ \ or\ RA)$$

Since E_1 and E_2 are differentiable economies, invidual's optimization delivers equilibrium allocations satisfying:

$$MRS_h = \frac{u_c(c_s^h)\pi_h(s)}{u_c\left(c_{s'}^h\right)\pi_h(s')} = \frac{p_s}{p_{s'}}, h \in \{1, ..., H\} \tag{16}$$

The condition above is the standard equality between the marginal rate of substitution (MRS) of consumption across different states and its relative price. (16) implies:

$$MRS_h = \frac{u_c(c_s^h)\pi_h(s)}{u_c\left(c_{s'}^h\right)\pi_h(s')} = \frac{u_c(c_s^{h^*})\pi_{h^*}(s)}{u_c\left(c_{s'}^{h^*}\right)\pi_{h^*}(s')} = MRS_{h^*} = \frac{p_s}{p_{s'}}$$
$$\forall h, h^* \in \{1, ..., H\}$$

An equilibrium always exists and, by our assumptions on preferences (strict monotonicity is implied), every agent consumes all his endowment (in monetary terms).

We consider a simple exercise. We compare equilibria in E_1 (the multi agent economy) and equilibria in E_2 (the representative agent economy) in an attempt to map belief heterogeneity of the first economy into risk aversion of the representative agent in the second one. In particular, we study what happens to the representative agent risk aversion when belief heterogeneity arises in the multi agent economy *if* we want the two economies to be equivalent in pricing terms. Thus:

Definition 2 *We define two economies to be price-equivalent (PE) if they have the same ME price.*

Firstly, we observe:

Proposition 4 *When one individual, h, changes the prior relevant for his consumption decisions, ME prices adjust, given the initial allocation of endowment.*

Proof. [Proof of Proposition 8] Assume, by contradiction, that ME prices do not move when h changes her probabilty assessment about the future s.t. $\pi_h(s)$ increases and $\pi_h(s')$ falls, i.e. contingency s becomes subjectively more likely. So:

$$MRS_h = \frac{u_c\left(c_s^h\right) \cdot \pi_h(s) \uparrow}{u_c\left(c_{s'}^h\right) \cdot \pi_h(s') \downarrow} \neq \frac{p_s}{p_{s'}}$$

which breaks the optimality of c^h for h. Therefore h's equilibrium consumption must move. But since, at the old prices, $\sum\limits_{h \in \{1,\dots,H\}} e_s^h = \sum\limits_{h \in \{1,\dots,H\}} c_s^h$ the change in c^h implies the change in $c^{h'}$ for some $h' \neq h$. But this implies that ME prices must move: contradiction. ■

Secondly, we consider that equilibrium in E_2 implies the following:

Remark 1 *In E_2 economies, ME consumption for the representative agent must match the aggregate endowment in each contingency, i.e.* $c_s^{RA} = e_s^{RA}$, $\forall s$.

We can discuss the relation between belief heterogeneity in the multiagent economy and the risk aversion of the representative agent. In order to make our case clearer, we consider economies with only two contingencies ($S = 2$) and only two agents in E_1. Our reasoning can be easily extended to finite contingencies ($S > 2$) and an arbitrary number of households ($H > 2$).

The exercise runs as follows. Start from the situation in which E_1 and E_2 are PE (they have same equilibrium prices). Assume that, for a fair comparison, the E_1 and E_2 are *structurally* equivalent, i.e. they have the same underlying stochastic process determining which contingency realizes tomorrow. Then *at least* one individual in the multi agent economy changes his probability assessment about the future, diverging from the others. We study what happens to ME prices in E_1. Finally, we will consider how the representative agent's risk aversion must change so that E_2 remains PE to E_1.

For the sake of simplicity, we will think about contingencies tomorrow as simply Boom ($s = 1$) and Recession ($s = 2$): aggregate endowment is bigger in the first than in the second contingency, i.e. $e_1^{RA} > e_2^{RA}$. For simplicity, we begin by assuming an economy where the two individuals are identical in everything *but* endowments. Then we will consider the case where their priors satisfy $\pi_1(1) > \pi_2(1)$; i.e., individual 1 believes that booms become more likely than individual 2. In this setting it is natural to measure the degree of belief heterogeneity in the economy by the ratio $\left(\frac{\pi_1(1)}{\pi_2(1)}\right)$ (e.g. a value equal to one would correspond to perfect homogeneity). Conducting this exercise for all possible prior movements delivers the following proposition:

Proposition 5 *Assume the initial distribution of endowments is non trivial and E_1 and E_2 are initially PE. The two economies remain PE if an increase (decrease) in $\left(\frac{\pi_1(1)}{\pi_2(1)}\right)$ is* *accompanied by an increase (decrease) in the representative agent's Risk Aversion in* E_2.[12]

Proof. *[Proof of Proposition 10] The proof is immediate and we will illustrate only the case when* $\left(\frac{\pi_1(1)}{\pi_2(1)}\right)$ *increases. By Proposition 8 we know that if the prior of one agent changes so do equilibrium prices. Assume, in particular, that individual* 1 *changes his prior while* 2 *stays put:* $\frac{\pi_1(1)}{\pi_2(1)}$ *increases if* $\pi_1(1)$ *raises. Then, at the original consumption bundles,* $MRS_1 > MRS_2 = \frac{p_1}{p_2}$. *Therefore individual* 1 *starts moving consumption from the second to the first contingency. Thus, the relative price in* E_1 *have to increase to cope with excess demand for good 1 and excess supply for good 2. In* E_2, *representative agent consumption can not adjust (by Remark 9) and his prior is locked by the true stochastic process. Therefore,* E_1 *and* E_2 *remain PE only if representative agent risk aversion is increased (i.e. increase the curvature of his indifference curves*$\frac{u_c(e_1^{RA})}{u_c(e_2^{RA})}$*).* ■

Example 1 (CRRA Utility) *One of the most widely used utility functions is the Constant Relative Risk Aversion type:*

$$U(c) = \begin{cases} \ln c \text{ if } \gamma = 1 \\ \frac{c^{1-\gamma}}{1-\gamma} \text{ otherwise} \end{cases} \quad \gamma > 0$$

where γ *is the measure of individual relative risk aversion. We use the same argument in the proof above and apply it to this particular case. In fact, for the representative agent in* E_2,*the following must hold in equilibrium:*

$$MRS_{RA} = \frac{u_c\left(e_1^{RA}\right)}{u_c\left(e_2^{RA}\right)} = \left(\frac{\pi_{RA}(1)}{\pi_{RA}(2)}\right)\left(\frac{e_1^{RA}}{e_2^{RA}}\right)^{-\gamma} = \left(\frac{p_1}{p_2}\right)^*$$

We want the multiagent economy and the representative agent economy, E_1 *and* E_2 *respectively, to remain PE. Can a change in the risk aversion coefficient* (γ) *of the representative agent deliver the desired result? This is in fact the case: given any equilibrium price* $\frac{p_1}{p_2}$ *in* E_1, *we may choose* γ *such that* E_2 *is PE to* E_1. *The following:*

$$\gamma = \log_{\left(\frac{e_2^{RA}}{e_1^{RA}}\right)}\left[\left(\frac{p_1}{p_2}\right)^*\left(\frac{\pi_{RA}(2)}{\pi_{RA}(1)}\right)\right]$$

provides the coeffificeint of relative risk aversion that guarantees the desired result. Since the logarithm is a monotonic function, an increase in equilibrium prices in the multiagent economy must correspond to a higher risk aversion of an "equivalent" representative agent economy.

The last proposition highlights the relation between belief heterogeneity in the multi agent economy and risk aversion of the representative agent. If we want the two economies,

[12] The level of contingent production is fixed in this economy and so belief heterogeneity affects prices only. In an economy where the level of production in each contingency is endogenous, also quantities would be affected by belief heterogeneity and the effect on prices would be quantitatively smaller, though qualitatively would the same.

E_1 and E_2 to remain PE, an increase in belief heterogeneity fostered by optimistic (pessimistic) beliefs [$\pi_h(1)$ increases] implies an increase (decrease) in the risk aversion of the corresponding representative agent.

In order to extend our analysis to asset pricing, we only need to observe:

Remark 2 *By the Isomorphism between Debreu Economies (i.e. with markets for every contingent commodity) and Complete Asset Market Economies, every set of equilibrium prices in the former can be mapped into a set of equilibrium asset prices in the latter that preserves the equilibrium allocation and viceversa.*

Our argument provides therefore a novel interpretation of the risk aversion of the representative agent. Employing the metaphor of a single individual for an entire economy implicitly hides the role of belief heterogeneity[13]. In fact, in the standard simulation exercise, the probabilistic assessment of the representative agent is pinned down by the empirical time series and so can not reflect any disagreement between agents. Consistently with the simulation exercise, we have compared, in terms of equilibrium pricing, agents with shifting heterogenous beliefs with a representative agent that never changes its probability assessment. But if this is right, this section suggests that the simulation of representative agent models nests belief heterogeneity into the curvature of the utility function, a measure of risk aversion. This may explain the poor performance of these models in replicating observed asset pricing. To summarize:

Remark 3 *Our analysis suggests the difficulty in assigning a measure of individual risk aversion to the representative agent: using the representative agent as a modelling device implies that the agent's risk aversion captures not only individuals pure risk aversion but also the level of belief heterogeneity in the underlying multi agent economy.*

In conclusion: since the basic problem of the "equity premium puzzle" is that the risk aversion required by standard models to match observed premia is unreasonably high, our discussion suggests that theory may be reconciled with evidence once we can take belief heterogeneity in full account.

5 The Dynamic Analysis

5.1 Dynamic Setup

The discussion above was intended to highlight some general matters conceptual in nature. It remains to place them in a fully dynamic setting. This section presents a framework that allows the study of the effects of belief heterogeneity on asset pricing in an infinite horizon economy. Here we will not focus on the crucial question regarding how probability assessments start diverging and belief heterogeneity arises.

[13]In general, the aggregation through the representative agent is derived assuming a continuum of agents in the underlying economy. Our argument goes naturally through in this case: if a group of agents of non zero measure has different beliefs from the rest of the population, equilibrium prices take this heterogeneity into account in the same way - qualitatively speaking - analyzed in the two agents argument.

We start from an economy in which all agents are perfectly homogeneous in terms of utility functions and probability assessments. We then assume that, at some point in time, an extra contingency that was not available before arises. For simplicity, we maintain the assumption that there are two agents in this economy. One agent is aware that the set of possible contingencies is now enlarged while the other continues to believe that nothing changed. The best way to picture such scenario is to think about a technological breakthrough in the economy (e.g. aspirin, telephone, computer).

This is not an heroic assumption: in many relevant historical instances in which an innovation was proposed, potential investors have disagreed about its profitability. There is a long record of successful innovations that were initially dismissed by the most important industrial leaders[14]. It is the *ex ante* inability to detect successful innovations that determines belief heterogeneity across investors: our modelling choice is to introduce an additional "good" contingency about which agents are asymmetrically informed.

The structure is very similar to previous sections. There are two agents (L and M) in the economy who, in any period t, receive the payoffs of their portfolios, consume the only commodity present in the economy and trade the two available assets, stock z and bond b, to transfer part of their wealth to the future. In each period t one contingency $s \in \{1, ..., S\}$ is realized. The assets payoffs depend upon the particular contingency that is realized and the matrix of asset payoffs is defined as follows:

$$\begin{bmatrix} & s=1 & ... & s=S \\ \text{bond } b & 1 & ... & 1 \\ \text{stock } z & d_1 & ... & d_S \end{bmatrix}$$

Aggregate endowment is increasing in the label of the contingency[15], i.e. $s = 1$ is the case in which aggregate endowment - and dividend to stocks - is the lowest and $s = S$ when it is the highest. Since, as in Lucas (1978), we interpret the stock as a claim on aggregate endowment, its payoff is (weakly) increasing in the label of the contingency.

Agents maximixe the following infinite horizon problems:

$$\max \sum_{t=0}^{\infty} \beta^t \left[\sum_{s=1}^{S} P_i(s) U^i(c_s) \right], \text{ s.t.:}$$
$$p_z^t z_i^{t+1} + p_b^t b_i^{t+1} + c_i^t \leq (p_z^t + d^t) z_i^t + p_b^t b_i^t; \sum_s P_i(s) = 1$$
$$U_c^i() > 0, U_{cc}^i() < 0, c_i^t \geq 0,\ i \in (M, L); \lim_{c \longrightarrow 0} U_c^i(c) = +\infty$$

Therefore we can cast the problem in the standard dynamic programming way:

$$V_t^i(z_t, b_t; d^t, p_z^t, p_b^t) = \max_{c_i^t, z_i^{t+1}, b_i^{t+1}} \left\{ U_t(c_t) + \beta \left[E_t V_{t+1}^i (z_{t+1}, b_{t+1}; d_s^{t+1}) \right] \right\} \tag{17}$$

[14]Few examples: "640K ought to be enough for anybody." – Bill Gates, 1981, rejecting a proposal for larger computer memory; "Who the hell wants to copy a document on plain paper?", 1940 Rejection Letter to Chester Carlson, inventor of the Xerox machine. In fact, over 20 companies rejected the invention between 1939 and 1944. A list presenting other striking examples can be found at http://www.columbia.edu/~xs23/reject.htm

[15]Note that this is a different convention from previous sections.

(17) delivers the necessary and sufficient FOC's (the variable we are deriving with respect to is subscripted)[16]:

$$z^{t+1} : \beta E_t \left[V^i_{t+1,z_i^{t+1}} \right] = \left[U_c^{i,t} \right] p_z^t$$

$$b^{t+1} : \beta E_t \left[V^i_{t+1,b_i^{t+1}} \right] = \left[U_c^{i,t} \right] p_b^t$$

and the following Envelope Conditions:

$$z^{t+1} : \left[U_c^{i,t} \right] (p_z^t + d^t) = \left[V^i_{t,z_i^t} \right]$$

$$b^{t+1} : \left[U_c^{i,t} \right] = \left[V^i_{t,b_i^t} \right]$$

which turn out to be the well known Euler conditions:

$$\frac{\beta E_t \left[U_c^{i,t+1} (p_z^{t+1} + d^{t+1}) \right]}{\left[U_c^{i,t} \right]} = p_z^t$$

$$\frac{\beta E_t \left[U_c^{i,t+1} \right]}{\left[U_c^{i,t} \right]} = p_b^t$$

One must keep in mind that the Euler conditions above must hold for both individuals at the same time, *given* equilibrium prices. We define:

Definition 3 *A Market Equilibrium is the sequence of prices* $(\{p_z^t\}_{t=1}^{\infty}, \{p_b^t\}_{t=1}^{\infty})$ *and consumptions and portfolios* $(\{c_i^t\}_{t=1}^{\infty} \{z_i^t\}_{t=1}^{\infty}, \{b_i^t\}_{t=1}^{\infty}, i \in (M, L))$ *such that:*

$$\begin{gathered} (c_i^t, z_i^t, b_i^t) \in \arg\max V_t^i(z_t, b_t; d^t), \forall t \\ \textstyle\sum_{i=L,M} b_i^t = 0, \forall t \\ \textstyle\sum_{i=L,M} z_i^t = 1, \forall t \end{gathered}$$

For the sake of clarity, we start from the case where both individuals have the same probability assessment, i.e. $P_L(s) = P_M(s)$ $\forall s$,and their utility functions are identical and homothetic[17]. It is then obvious to observe that asset pricing is wealth distribution independent. Our economy is no different, in terms of equilibrium asset pricing, from the representative agent one.

We then study the case in which an innovation is introduced and agents are asymmetrically informed about its consequences. In order to make matters consistent with the discussion throughout the paper, we assume that markets are complete and remain such.

[16] By the usual Blackwell contraction mapping argument, $V_t^i(.;.)$ exists since the utility function lies in the complete space of continuous and bounded functions. Moreover $V_t^i(.;.)$ is differentiable by Theorem 4.11 [p.85, Stokey et al., 1989].

[17] Notice that the CRRA utility function usually assumed in this class of consumption based asset pricing model is homothetic.

Since we have only two assets, this implies that the economy before innovation is made of two contingencies, i.e. $S = 2$[18].

When an innovation takes place, an extra contingency that was not available before ($s^* = S + 1$) becomes then possible. This has non zero probability and, if realized, it pays off as in the best possible case, i.e. $d_S = d_{S^*}$. One can think about s^* as a sunspot. This last assumption allows us to maintain the complete market setting after the innovation is introduced. This is important: otherwise it would be impossible to disentagle the effect on asset pricing due to the switch to incomplete markets on the one hand and the effect due to belief heterogeneity on the other.

As assumed before, agents are asymmetrically informed about the new contingency and, in particular, we assume only L is aware of it. M still sticks to the previous probability assessment, the one that was shared by both individuals before the innovation took place. Naturally belief heterogeneity can not last forever. Instead of putting on M's rationality the strict requirement of continuous updating, we will assume that M "catches" the true probability distribution of the economy only when the extra contingency s^* happens. In a sense he believes only what he sees.

We argue that this modelling choice has two major advantages:

- it renders the persistence of belief heterogeneity endogeneous. It depends on how much time it will take for the contingency on which individuals are asymmetrically informed to happen;
- it does not allow the agents to "agree to disagree forever". There is a transition in which belief heterogeneity plays a role in equilibrium asset pricing but this effect eventually disappear (with probability one) and the risk motive will be the only to be rewarded by the equity premium.

We now study the behavior of asset pricing since an innovation is introduced in the economy. After its introduction, L's probability assessment changes from $P_L(.)$ to $P_L^{INN}(.)$, which assign more weight to the highest dividend that stocks pay:

$$P_L^{INN}(d_S = d_{S^*}) = P_L(d_S) + P_L^{INN}(d_{S^*})$$
$$\text{and } P_L(s_1) > P_L^{INN}(s_1)$$

Given his previous optimal path of consumption the prices that L is willing to pay to purchase stocks increases suddenly. In particular it becomes higher than the old equilibrium prices p_z^*. Formally:

$$\frac{\beta E_t\left[U_c^{i,t}+1(p_z^{t+1}+d^{t+1})\right]}{\left[U_c^{i,t}\right]} = p_z^* < \left(p_z^t\right)^L = \frac{\beta E_t^{INN}\left[U_c^{i,t}+1(p_z^{t+1}+d^{t+1})\right]}{\left[U_c^{i,t}\right]}$$ [19]

[18] Notice that different stocks could be assumed so to allow a baseline economy facing more than two possible contingencies in each period.

[19] Implicit is the assumption: $\left[U_c^{i,t+1}(p_z^{t+1}+d_S^{t+1})\right]_{s=S} > \left[U_c^{i,t+1}(p_z^{t+1}+d^{t+1})\right]_{s=1}$, i.e. consumption is not too unbalanced across contingencies $s = 1$ and $s = S$.

where E_t^{INN} is the expectation taken with respect to the new probability assessment, $\left(p_z^t\right)^L$represents the highest prices that L is willing to pay for stock, given the optimal consumption stream before the innovation's introduction.

Therefore old prices can no longer maintain the equilibrium. On the one hand, since L now perceives the highest dividend to be more likely, stocks are particularly valuable to him. On the other side, the limit price of bonds for L, given his previous optimal consumption satisfies the following:

$$\frac{\beta E_t\left[U_c^{i,t}+1\right]}{\left[U_c^{i,t}\right]} = p_b^* > \left(p_b^t\right)^L = \frac{\beta E_t^{INN}\left[U_c^{i,t}+1\right]}{\left[U_c^{i,t}\right]}$$

Since L will adjust his consumption demanding more stocks and selling bonds, the stock price p_z^t will tend to increase while the bond price p_b^t will tend to decrease. Therefore the realized equity premium *unambiguously* increase at time t. In fact:

$$\frac{p_z^t + d^t}{p_z^{t-1}} - \frac{1}{p_b^{t-1}} = (R_{Stock})_t - (R_{Bond})_t$$

How belief heterogeneity affects the equity premium in subsequent periods will depend upon the particular sequence of contingencies, and thus stock payoffs, that will be realized. For intance, if the initial distribution of wealth is egualitarian and a contingency with high aggregate endowment is realized, L will be relatively more wealthy and stock prices will tend to be higher. The intuition is clear nonetheless and is in line with the argument of the previous section. Belief heterogeneity is boosted by optimistic beliefs: this was modeled here by the fact that one agent is unaware of the actual potential of available technology and so an higher equity premium results. Naturally the effect of belief heterogeneity will disappear once the new contingency is realized and homogeneous beliefs reappear. Eventually, the multi agent economy, isolated from innovations, will be equivalent, in terms of asset pricing, to the representative agent economy.

5.2 Recent Behavior of the Equity Premium: Heterogeneous Optimism versus Homogeneous Pessimism

Building on the previous discussion, we may employ our model to suggest a plausible interpretation of recent stock market behavior. We have argued that, if some agents with positive wealth are unaware of the true potential of the available technology, equity premia will tend to increase. This upward pressure on the equity premium takes place even if the large majority of agents know the actual distribution of dividends in the future and is never wrong about it. The dialectic between optimism and pessimism - implicitly hidden in a standard representative agent framework - could be a contributing force, though not the only one, to explain the temporal evolution of the equity premium.

While a great deal of research has attempted to provide explanations for the "unexpectedly", in terms of incorporated risk, high level of the equity premium[20], other empirical

[20] Mehra [2002] offers an updated review of this literature.

work has focused on the evolution of the equity premium over time (e.g. Blanchard [1993] and Jagannathan et al. [2001]). We will employ our arguments to address the latter branch of research and we will thus try to interpret the evolution of the equity premium in the last business cycle . We will consider the period that ranges from the high equity premium of the last economic expansion (between 1991 and 2001 [I quarter]) to the years 2001 [II quarter]-2002 when the economy slowed down and the equity premium was substantially reduced[21].

It is a well documented fact that, historically, the *realized* equity premium is procyclical in the period 1890-1991[22]. If we restrict our attention to the last twenty years, we observe a slightly negative correlation with the business cycle. This correlation increases sharply if we restrict ourselves to, roughly, the last decade[23]. In particular, it may be observed that the equity premium was historically high in the decade '91-'01 and dropped in more recent years (see Figure 2).

We focus on the following question: what could have contributed initially to such high premium and to its subsequent drop? We will argue that the shifts in "belief heterogeneity" may have played an important role in determining the temporal evolution of the premium.

Some casual evidence supporting our claim can be found in the IBES[24] data. This database - collecting forecasts by institutional investors about the stock market - reports, for the Standard & Poor 500 index[25], the forecasts' standard deviation. Normalizing by the mean of forecasts in each month, this standard deviation went up to almost 21%, in the period '91-'01 suggesting a significant rise in belief heterogeneity. Though data are lacking and we can not make comparisons with previous cycles until better instruments or richer datasets become available, this evidence suggests that investors were, at the time, in sizable disagreement about the stock market. It thus appears that belief heterogeneity characterized the economy between '91-'01. In fact, this disagreement may be considered a specific feature of that decade, as it is argued below.

We now propose an interpretation of the shift from high equity premium in the boom to low premium in the recession that stems from our theoretical framework. Our explanation is based on a simple observation: the decade '91- '01 with historically high equity premium was characterized by a specific kind of optimism, an heterogeneous one. Investors agreed on the good prospects of the market but disagreed on the extent of the boom. This expectational heterogeneity is not surprising if one thinks about the specific conditions of this lasting economic upturn. Firstly, the introduction of new technologies fostered asymmetric information giving room to different beliefs about their profitability. Secondly, the inflow of foreign capital in the american financial market has been particularly intense in the last

[21] These periods are selected on the basis of the NBER announcements to establish the duration of expansion and contraction in the current business cycle. The beginning of this cycle is recognized in the second quarter of 1991 and the peak of it has been identified in the first quarter of 2001.

[22] Guvenen [pg.18, 2002] is a useful reference.

[23] Using quarterly data for GDP real growth from NIPA and the Fama French Factors we get a correlation coefficient between GDP growth and realized quarterly equity premium of -0.032 for the period that ranges from 1980 (I) to 2002 (IV). A positive correlation of 0.36 appears if we restrict ourselves to the period from 1993 (I) to 2002 (IV).

[24] Institute for Business and Economic Survey

[25] A reasonable proxy for the market portfolio.

decade: this may have added an extra source of expectational heterogeneity about the US market. As a result of these factors, it may be reasonable to consider that intense belief heterogeneity was a prevailing factor of the market at the time. In the spirit of our model, this "optimistic" heterogeneity has to be viewed as a factor responsible in increasing the equity premium.

By a similar but opposite reasoning we may explain the '01-'02 market premium drop. One could argue that the peculiarity of the recent economic slowdown has been the high degree of homogeneity in expectations. This was the result of different contingent factors: a sluggish economy after a long period of outstanding economic performance and unrelated financial scandals have certainly been major causes of the abrupt fall in trust toward the new economy. But this fact is particularly important for the sake of our argument: these factors were, contrary to the past, sufficiently persuasive and evident to induce homogeneous perceptions and, thus, expectations. Technological innovation with uncertain potential became therefore a less important determinant of agents expectations. The heterogeneity connected to it did, in fact, drop.

In conclusion, our interpretation is that the recent fall in the equity premium was enhanced by the shift toward expectational homogeneity. It was not the simple fact that agents beliefs had turned pessimistic. Crucially, agents tend to agree on the perspective of the market now, exactly as they previously disagreed. They are certainly pessimistic but, more importantly, they agree in their pessimism, i.e. they are homogeneously pessimistic. This homogeneity may have been a further reason to undercut, under the logic of our model, the equity premium and so explain the empirical evidence we observe.

In assessing this interpretation, one should not forget two issues. First, this perspective focuses only on the last decade, and can not be casually extrapolated. In general, we do not claim that all the economic slowdowns are characterized by expectational homogeneity and all the booms feature heterogeneity. We argued that this was the peculiarity of the last ten years or so. Secondly, this reasoning does not disregard the classical risk motive as a fundamental factor in determining asset pricing. It simply adds another explanatory variable, namely heterogeneity of beliefs, in the interpretation of the temporal evolution and amplitude of the equity premium. Further empirical research would be necessary when better data about belief heterogeneity will become available.

6 Conclusions

Heterogeneity in beliefs emerges as a key factor to drive the equity premium. This discussion does not analyze the fundamental economic question of how belief heterogeneity arises. Assuming that belief heterogeneity exists in the real world, our focus is to study its effects on equilibrium asset pricing: how long does it last, how it interacts with wealth distribution and propagate, what kind of transition it generates. These are the questions we have been answered here.

The paper starts discussing, in an simplified framework, the direct implications of belief heterogeneity over asset pricing. In this theoretical enquiry we point out some general considerations. Firstly, belief heterogeneity plays a role, no matter how different agents' expectations are. As far as they differ by some amount an "heterogeneity premium" appears. Even in a risk neutral economy such premia may be strictly positive. Naturally, the more disparate are agents' priors, the wider, in general, is the effect. The actual number of agents and risky assets in the economy is irrelevant for the sake of our qualitative results.

Secondly, our discussion provides a case against the representative agent framework[26]. It shows that the market interaction between "optimism" and "pessimism" - naturally hidden in a representative agent framework - is crucial to deliver a link between wealth distribution and equity premium. In section 4 we argue that, in line with the literature[27] but adopting a more general standpoint, anytime we want to use the representative agent paradigm, we have to be aware that his risk aversion will incorporate a measure of belief heterogeneity in the underlying multi agent economy. Therefore, it turns out that attaching to the representative agent a degree of risk aversion arising from micro study in general biases the exercise.

Thirdly, and more technically, our discussion was quite general: introducing consumption at $t = 0$, an inter-temporal discount factor, individual risk aversion and setting up a fully dynamic model as the one presentes in section 5 does not contradict our initial intuition. Belief heterogenity still remains a crucial factor to determine equilibrium asset pricing and, in general, interacts with the distribution of wealth.

Fourthly, we propose a possible application of our arguments. We do not want to push our interpretation of the premium's temporal evolution too far and, in absence of further empirical study, our point is not conclusive. It is nonetheless instructive and provides an example of how our point of view could be put to work in interpreting the data: the financial market is a place that responds to technological innovation in particular and information in general. Different beliefs continuously confront each other in the financial markets. Even if agents update their assessments, it would be surprising if this updating delivered complete homogeneity at any point in time. Thus the interaction between different beliefs should be studied: this paper does so and highlights that limited distributional issues may become important when this interaction does not result in homeogeneous views.

Finally, a consideration about the general perspective in this work. One could argue that the framework presented is unrealistic: the "big players" in financial markets - the 10 % of agents that have 95% of stocks in the US economy - tend to have homogeneous beliefs about the future; they share information and are well informed. So they agree on the likelihood of major events like the ones portrayed in our arguments. Moreover, even if they disagree, the price would signal the heterogeneity in beliefs across agents and individuals would update their probability assessment until some agreement is reached. This is basically the argument that economic agents can *not* "agree to disagree"[28]. Since these people are the ones that drive financial markets - the argument says - the model is empirically irrelevant.

We can think about at least two reasons why this is not the case. The first is that, as

[26] Constantinides [1982] seems at odds with our findings. It is probably worthy to notice that he points out how heterogeneity in the endowment of zero net supply assets turns out to deliver an observationally equivalent economy to the one with the representative agent. Notice that we assume heterogeneity in beliefs *and* in the endowment of z, of which net supply is equal to one.

[27] Wang [1995], Berrada [2002], Dumas [1987], Guvenen [2002], Ziegler [2002] all recognize, sometimes in different frameworks, the importance of heterogeneous preferences.

[28] Milgrom and Stokey [1982].

discussed in section 5, even major investors disagree. They just do *not* do it forever[29]. The second is that we are not modelling the rationality of professional investors only but more generally the investing attitude of individual savers. Therefore, one must be very careful in interpreting our priors as objective probabilities and then considering the likelihood of updating: in our model these are subjective probability assessments which, in the spirit of Savage [1954], one can extract from agents' choices. Since it does not seem unrealistic to assume that different people have different preference orders, different beliefs about the future may easily emerge. Belief heterogeneity are thus derived by preference heterogeneity. But if the individuals in the model - the stockholders in developed economies - face the choice of investing in those mutual funds that hold most of their portfolio in stocks or those that are bond based, belief heterogeneity cannot be ignored in the study of asset pricing.

7 Appendix

This appendix reports the calculations behind the results in Section 3.2. Using the budget constraint at $t = 0$ to express z as a function of b and equilibrium condition on the bond and stock market, we can rewrite (14) and (13) as:

$$\Theta\left(z_L, p_z/p_b; z_0^L\right) = 0 =$$

$$= \frac{\pi B U_c^L\left[B\cdot z_L + \frac{p_z}{p_b}\left(z_0^L - z_L\right)\right] + (1-\pi) R U_c^L\left[R\cdot z_L + \frac{p_z}{p_b}\left(z_0^L - z_L\right)\right]}{\pi U_c^L\left[B\cdot z_L + \frac{p_z}{p_b}\left(z_0^L - z_L\right)\right] + (1-\pi) U_c^L\left[R\cdot z_L + \frac{p_z}{p_b}\left(z_0^L - z_L\right)\right]} +$$

$$-\frac{\rho B U_c^M\left[B\cdot(1-z_L) + \frac{p_z}{p_b}\left(z_L - z_0^L\right)\right] + (1-\rho) R U_c^M\left[R\cdot(1-z_L) + \frac{p_z}{p_b}\left(z_L - z_0^L\right)\right]}{\rho U_c^M\left[B\cdot(1-z_L) + \frac{p_z}{p_b}\left(z_L - z_0^L\right)\right] + (1-\rho) U_c^M\left[R\cdot(1-z_L) + \frac{p_z}{p_b}\left(z_L - z_0^L\right)\right]} \tag{18}$$

and

$$MRS_L\left(z_L, p_z/p_b; z_0^L\right) - (p_z/p_b) = 0 =$$

$$= \frac{\pi B U_c^L\left[B\cdot z_L + \frac{p_z}{p_b}\left(z_0^L - z_L\right)\right] + (1-\pi) R U_c^L\left[R\cdot z_L + \frac{p_z}{p_b}\left(z_0^L - z_L\right)\right]}{\pi U_c^L\left[B\cdot z_L + \frac{p_z}{p_b}\left(z_0^L - z_L\right)\right] + (1-\pi) U_c^L\left[R\cdot z_L + \frac{p_z}{p_b}\left(z_0^L - z_L\right)\right]} - p_z/p_b \tag{19}$$

$\Theta\left(z_L, p_z/p_b; z_0^L\right)$ is an implicit expression that links, between other things, wealth distribution $\left(z_0^L\right)$ to the stock price (p_Z). In order to apply the Implicit Function Theorem we can not rely on $\Theta\left(z_L, p_z/p_b; z_0^L\right)$only because there are two endogenous variables in it. (19) and (18) define a system of equations that fully characterize the equilibrium of our economy:

[29] Geanakoplos and Polemarchakis [1982] proves that, if agents have different information, they will agree on a common prior in finitely many steps.

$$\begin{cases} \Theta\left(z_L, p_z/p_b; z_0^L\right) = 0 \\ MRS_L\left(z_L, p_z/p_b; z_0^L\right) - (p_z/p_b) = 0 \end{cases}$$

Since $[z_L, (p_z/p_b = \Pi)]$ are endogenous variables and $\left(z_0^L\right)$ is exogenous form the point of view of the economy, we can write the following smooth functional:

$$F\left[z_L, (p_z/p_b = \Pi); z_0^L\right] = \begin{bmatrix} \Theta\left(z_L, \Pi; z_0^L\right) \\ MRS_L\left(z_L, \Pi; z_0^L\right) - (\Pi) \end{bmatrix} : R^2 \times R \longrightarrow R^2$$

Linearizing the system around its equilibrium and omitting the argument of Θ and MRS_L for simplicity, we get:

$$\begin{bmatrix} \partial\Theta/\partial z^L & \partial\Theta/\partial\Pi & \partial\Theta/\partial z_0^L \\ \partial\left[MRS_L - \Pi\right]/\partial z^L & \partial\left[MRS_L - \Pi\right]/\partial\Pi & \partial\left[MRS_L - \Pi\right]/\partial z_0^L \end{bmatrix} \begin{bmatrix} \partial z^L \\ \partial\Pi \\ \partial z_0^L \end{bmatrix} = [0]$$

Hence:

$$[J]_{2\times 2} \begin{bmatrix} \partial z^L \\ \partial\Pi \end{bmatrix}_{2\times 1} = -\begin{bmatrix} \partial\Theta/\partial z_0^L \\ \frac{\partial[MRS_L - \Pi]}{\partial z_0^L} \end{bmatrix}_{2\times 1} \partial z_0^L$$

where $J = \begin{bmatrix} \partial\Theta/\partial z^L & \partial\Theta/\partial\Pi \\ \partial\left[MRS_L - \Pi\right]/\partial z^L & \partial\left[MRS_L - \Pi\right]/\partial\Pi \end{bmatrix}$

After controlling that the appropriate condition applies[30], we have:

$$\begin{bmatrix} \partial z^L/\partial z_0^L \\ \partial\Pi/\partial z_0^L \end{bmatrix} = -[J]^{-1} \begin{bmatrix} \frac{\partial\Theta}{\partial z_0^L} \\ \frac{\partial[MRS_L - \Pi]}{\partial z_0^L} \end{bmatrix}$$

where:

$$[J]^{-1} = \frac{1}{\det J} \begin{bmatrix} \partial\left[MRS_L - \Pi\right]/\partial\Pi & -\partial\Theta/\partial\Pi \\ -\partial\left[MRS_L - \Pi\right]/\partial z^L & \partial\Theta/\partial z^L \end{bmatrix}$$

Therefore we can focus on

$$\frac{\partial\left(p_z/p_b\right)}{\partial z_0^L} = \frac{\partial\Pi}{\partial z_0^L} = -\overset{(+\text{ or }-)}{\frac{1}{\det J}} \left[-\overset{(-)}{\frac{\partial\left[MRS_L - \Pi\right]}{\partial z^L}} \cdot \overset{(-)}{\frac{\partial\Theta}{\partial z_0^L}} + \overset{(-)}{\frac{\partial\Theta}{\partial z^L}} \cdot \overset{(-)}{\frac{\partial\left[MRS_L - \Pi\right]}{\partial z_0^L}} \right] = \qquad (20)$$

[30] We must check that the matrix $\begin{bmatrix} \frac{\partial\Theta}{\partial z^L} & \frac{\partial\Theta}{\partial p} \\ \frac{\partial[MRS_L - p]}{\partial z^L} & \frac{\partial[MRS_L - p]}{\partial p} \end{bmatrix}$ is invertible. We will here appeal to the regularity argument that almost all matrix have maximal rank. One should keep in mind that there could be some values of the parameters for which the matrix is not invertible but these cases should be considered an exception.

which is clearly indeterminate. In order to draw conslusions about the linkage between wealth and asset pricing in this framework we will assume:

$$\overset{(-)}{\frac{\partial\left[MRS_L-\Pi\right]}{\partial z^L}}\cdot\overset{(-)}{\frac{\partial\Theta}{\partial z_0^L}}>\overset{(-)}{\frac{\partial\Theta}{\partial z^L}}\cdot\overset{(-)}{\frac{\partial\left[MRS_L-\Pi\right]}{\partial z_0^L}} \tag{21}$$

$$\overset{(-)}{\frac{\partial\Theta}{\partial z^L}}\cdot\overset{(-\text{ or }+)}{\frac{\partial\left[MRS_L-\Pi\right]}{\partial\Pi}}<\overset{(-)}{\frac{\partial\left[MRS_L-\Pi\right]}{\partial z^L}}\cdot\overset{(-\text{ or }+)}{\frac{\partial\Theta}{\partial\Pi}} \tag{22}$$

Moreover, to avoid considerations on the sign of the third derivative of utility, we assume that $U_{cc}$is negative and constant. Unfortunately it is hard to attach an economic interpretation to the inequlities 21 and 22 but this is the price we have to pay if we want to make distributional considerations in a strictly concave environment as general as possible. Therefore we can conlude:

$$\frac{\partial\left(p_z/p_b\right)}{\partial z_0^L}=\begin{cases}<0\text{ if is }z_0^L\text{ “high”, i.e. }z_0^L>z_L\\ >0\text{ otherwise}\end{cases} \tag{23}$$

The signs of all the terms in 20 are determined below:

$$\frac{\partial\left[MRS_L-\Pi\right]}{\partial z_0^L}=$$

$$=\partial\left[\frac{\pi BU_c^L\left[B\cdot z_L+\Pi\left(z_0^L-z_L\right)\right]+(1-\pi)RU_c^L\left[R\cdot z_L+\Pi\left(z_0^L-z_L\right)\right]}{\pi U_c^L\left[B\cdot z_L+\Pi\left(z_0^L-z_L\right)\right]+(1-\pi)U_c^L\left[R\cdot z_L+\Pi\left(z_0^L-z_L\right)\right]}\right]/\partial z_0^L=$$

$$=\frac{U_{cc}^L\left[c_B\right]U_c^L\left[c_R\right]-U_{cc}^L\left[c_R\right]U_c^L\left[c_B\right]}{\left[\pi U_c^L\left[B\cdot z_L+\frac{p_z}{p_b}\left(z_0^L-z_L\right)\right]+(1-\pi)U_c^L\left[R\cdot z_L+\frac{p_z}{p_b}\left(z_0^L-z_L\right)\right]\right]^2}\pi(1-\pi)\Pi\left(B-R\right)<\mathbf{0}$$

and

$$\frac{\partial\left[MRS_L-\Pi\right]}{\partial z^L}=$$

$$=\partial\left[\frac{\pi BU_c^L\left[B\cdot z_L+\Pi\left(z_0^L-z_L\right)\right]+(1-\pi)RU_c^L\left[R\cdot z_L+\Pi\left(z_0^L-z_L\right)\right]}{\pi U_c^L\left[B\cdot z_L+\Pi\left(z_0^L-z_L\right)\right]+(1-\pi)U_c^L\left[R\cdot z_L+\Pi\left(z_0^L-z_L\right)\right]}\right]/\partial z^L=$$

$$=\frac{(B-\Pi)(U_{cc}^L\left[c_B\right]U_c^L\left[c_R\right]-)-(R-\Pi)\left(U_{cc}^L\left[c_R\right]U_c^L\left[c_B\right]\right)}{\left[\pi U_c^L\left[B\cdot z_L+\frac{p_z}{p_b}\left(z_0^L-z_L\right)\right]+(1-\pi)U_c^L\left[R\cdot z_L+\frac{p_z}{p_b}\left(z_0^L-z_L\right)\right]\right]^2}\pi(1-\pi)(B-R)<\mathbf{0}$$

hence, since $z^L$and z_0^L enter with negative sign in MRS_M, the followings hold:

$$\left.\begin{array}{l}\frac{\partial\Theta}{\partial z_0^L}=\left[\overset{(-)}{\frac{\partial(MRS_L)}{\partial z_0^L}}-\overset{(+)}{\frac{\partial(MRS_M)}{\partial z_0^L}}\right]\\ \frac{\partial\Theta}{\partial z^L}=\left[\overset{(-)}{\frac{\partial(MRS_L)}{\partial z^L}}-\overset{(+)}{\frac{\partial(MRS_M)}{\partial z^L}}\right]\end{array}\right] < 0 \text{ since } U_{cc}^i < 0^{31}$$

And finally the sign of $\det J$:

$$\det J = \left(\overset{(-)}{\frac{\partial\Theta}{\partial z^L}}\cdot\overset{(-\text{ or }+)}{\frac{\partial\left[MRS_L-\Pi\right]}{\partial\Pi}}\right)-\left(\overset{(-)}{\frac{\partial\left[MRS_L-\Pi\right]}{\partial z^L}}\cdot\overset{(-\text{ or }+)}{\frac{\partial\Theta}{\partial\Pi}}\right)=$$

$$=\begin{cases} >0 \text{ if is } z_0^L \text{ ``high'', i.e. } z_0^L > z_L \\ <0 \text{ otherwise}\end{cases}$$

Since:

$$\frac{\partial\left[MRS_L-\Pi\right]}{\partial\Pi}=$$

$$=\partial\left[\frac{\pi BU_c^L\left[B\cdot z_L+\Pi\left(z_0^L-z_L\right)\right]+(1-\pi)RU_c^L\left[R\cdot z_L+\Pi\left(z_0^L-z_L\right)\right]}{\pi U_c^L\left[B\cdot z_L+\Pi\left(z_0^L-z_L\right)\right]+(1-\pi)U_c^L\left[R\cdot z_L+\Pi\left(z_0^L-z_L\right)\right]}-\Pi\right]/\partial\Pi=$$

$$=\left(z_0^L-z_L\right)\pi(1-\pi)(B-R)\frac{\left(U_{cc}^L\left[c_B\right]U_c^L\left[c_R\right]-U_{cc}^L\left[c_R\right]U_c^L\left[c_B\right]\right)}{\left[\pi U_c^L\left[c_B\right]+(1-\pi)U_c^L\left[c_R\right]\right]^2}=$$

$$=\begin{cases} <0 \text{ if is } z_0^L \text{ ``high'', i.e. } z_0^L > z_L \\ >0 \text{ otherwise}\end{cases}$$

and, keeping in mind that $z^L$and z_0^L enter with negative sign in MRS_M:

$$\frac{\partial\Theta}{\partial\Pi}=\frac{\partial MRS_L}{\partial\Pi}-\frac{\partial MRS_M}{\partial\Pi}=\begin{cases} <0 \text{ if is } z_0^L \text{ ``high'', i.e. } z_0^L > z_L \\ >0 \text{ otherwise}\end{cases}$$

7.1 Data Source

Equity premium data are U.S. Research Returns Data, Fama-French Factors: http://mba.tuck.dartmouth.ed

The data about heterogeneity in forecasts are taken by IBES database.

[31] Ignore the case at which $z_L^0 = \{0, 1\}$ because there is no trade at those initial endowments.

References

[1] Abel, Andrew, “An Exploration of the Effects of Pessimism and Doubt on Asset Returns”, Wharton Working Paper (2000)

[2] Aiyagari, Rao, and Mark Gertler, ”Asset Return with Transaction Costs and Uninsurable Individual Risk”, Journal of Monetary Economics, XXVII (1991), 311-331

[3] K. J. Arrow, The Role of Securities in the Optimal Allocation of Risk-bearing, The Review of Economic Studies, XXXI, No. 2. (Apr., 1964)

[4] Barberis, Nicholas, and Richard Thaler, ”A Survey of Behavioral Finance”, Handbook of the Economics of Finance, ed. by G. M. Constantinides, M. Harris and R. Stulz, North Holland, Amsterdam, (2003)

[5] Berrada, Tony, "Incomplete Information, Heterogeneous Beliefs and the Statistical Properties of Asset Prices", Working Paper HEC, (2003)

[6] Blanchard, Olivier, "The Vanishing Equity Premium", MIT mimeo, (1993)

[7] Chan, Yeung, and Leonid Kogan, ”Catching Up with the Joneses: Heterogeneous Preferences and the Dynamics of Asset Prices”, Journal of Political Economy , CX (2002), 1255-1285

[8] Constantinides, George, "Intertemporal Asset Pricing with Heterogeneous Consumers and without Demand Aggregation", Journal of Business, 55 (1982), 253-67

[9] Constantinides, George, and Darrell Duffie, ”Asset Pricing with Heterogeneous Consumers”, Journal of Political Economy, CIV(1996), 219-240

[10] Duffie, Darrel, Nicolae Garleanu and Lasse Heje Pedersen, “Securities Lending, Shorting, and Pricing”, Stanford University Working Paper (2001)

[11] Dumas, Bernard, “Two-Person Dynamic Equilibrium in the Capital Market”, Review of Financial Studies, II (1989), 157-188

[12] Geanakoplos, John, "Liquidity, Default and Crashes: Endogenous contracts in general Equilibrium", Cowles Foundation Paper 1316RR, (2002)

[13] ———, and Heraklis Polemarchakis, ”We can’t disagree forever”, Journal of Economic Theory”, 28 (1982), 194-201

[14] Guvenen, Fatih, "A Parsimonious Macroeconomic Model for Asset Pricing: Habit Formation or Cross-sectional Heterogeneity?", Working Paper, University of Rochester, (2002)

[15] Jagannathan, Ravi, Ellen McGrattan and Anna Scherbina, ”The Declining U.S. Equity Premium”, NBER Working Paper w8172, (2001)

[16] Knight, Frank, "Risk, Uncertainty and Profit", (Boston, MA, Houghton Mifflin, 1921)

[17] Kocherlakota, Narayana, "The Equity Premium: It's still a Puzzle", Journal of Economic Literature, XXXIV (1996), 42-71

[18] Krusell, Per, and Anthony Smith, "Income and Wealth Heterogeneity in the Macroeconomy", Journal of Political Economy, CVI (1996), 867-896

[19] Heaton, John, and Deborah Lucas, "Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing", The Journal of Political Economy, CIV (1996): 443-487

[20] Heaton, John, and Deborah Lucas, "Market Frictions, Savings Behavior and Portfolio Choice", Journal of Macroeconomic Dynamics, (1997)

[21] Lucas, Robert, "Asset Pricing in an Exchange Economy", Econometrica, XLVI (1978), 1429-1445

[22] Mehra, Rajnish "The Equity Premium: Why is it a Puzzle", Financial Analyst Journal, (2003), 59 - 64.

[23] Mehra, Rajnish, and Edward Prescott, "The Equity Premium: A Puzzle", Journal of Monetary Economics, XIV (1985)

[24] Milgrom, Paul, and Nancy Stokey, "Information, Trade and Common Knowledge", Journal of Economic Theory, XXVI (1982), 17-27

[25] Miller,Edward, "Risk, Uncertainty and Divergence of Opinion", Journal of Finance, XXXII (1977), 1151-1168

[26] Savage, Leonard, "Foundations of Statistics", (New York, NY, Dover Publications, 1954)

[27] Telmer, "Asset Pricing Puzzles and Incomplete Markets", Journal of Finance (1993)

[28] Wang, Jiang, "The Term Structure of Interest Rates in a Pure Exchange Economy with Heterogeneous Investors", Working paper No. 5172, NBER (1995)

[29] Ziegler, Alexandre, "State-price Densities Under Heterogeneous Beliefs, the Smile Effect and the Implied Risk Aversion", European Economic Review, XLVI (2002), 1539-1557

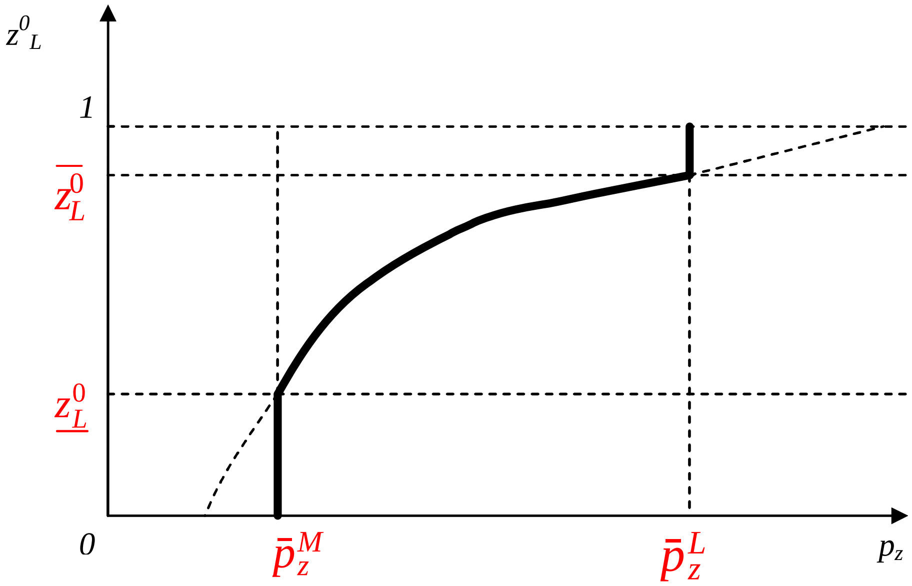

Figure 1:

Average Monthly Equity Premium

Jul 1926 - Dec 2001	0.62
Gen 1946 - Dec 2001	0.57
Gen 1980 - Dec 2002	0.55
Apr 1991 - Mar 2001	0.76
Apr 2001 - Dec 2002	-1.06

Figure 2