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## Sectoral Differentiation, Allocation of Talent, and Financial Development

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# Sectoral Differentiation, Allocation of Talent, and Financial Development\*

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## Abstract

I present a theory of development in which heterogeneously talented entrepreneurs require credit to start new projects and open new sectors. As the variety of sectors expands during development, the allocation of entrepreneurial talent improves. A key result of the paper is to show that, in addition to increasing the average productivity of the matches between agents and sectors, this process also mitigates informational frictions affecting the functioning of financial markets. Furthermore, the positive impact of sectoral variety on the efficiency of financial markets gives rise to a novel feedback between financial development and R&D effort, which may lead to different types of dynamics. A successful economy typically exhibits a progressive increase in the variety of sectors, which in turn helps to alleviate frictions in the financial markets. However, a poverty trap may also arise. This situation is characterised by a rudimentary productive structure with poor matching of skills to activities, and where the operation of financial markets is severely affected by talent mismatching.

Key Words: Adverse Selection, Informational Frictions, Talent Allocation, Sectoral Diversification, Financial Development.

JEL Codes: D82, O12, O14, O16, O30.

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# 1 Introduction

Over the course of development, the variety of productive activities in the economy tends to increase in conjunction with the aggregate stock of capital and output. This observation implies that economic development manifests itself partly as a process of sectoral diversification and increasing specialisation within the economy, an idea that dates back to Adam Smith (1776) in his discussion of the division of labour and its relation with the size of the market (*The Wealth of Nations*, chapter 3). Such a dynamic pattern is also described by Allyn Young (1928, p. 537), who writes "industrial differentiation has been and remains the type of change characteristically associated with the growth of production." Similarly, Landes (1969, p. 5) argues that the most evident effects brought about by the Industrial Revolution were the gains in productivity and the increase in the variety of products and occupations.

I propose a theory in which this process of sectoral diversification helps to mitigate informational frictions affecting the operation of financial markets. Furthermore, the degree of sectoral variety is itself endogenous to the theory, and it is positively influenced by the efficiency of financial markets. As a result, sectoral differentiation and the efficiency of financial markets appear interrelated in the model, and this positive interaction becomes a fundamental ingredient that shapes the patterns of development followed by different economies.

The paper studies the evolution of an economy populated by heterogeneously talented individuals. In particular, individuals are characterised by distinct comparative advantages concerning entrepreneurial activities. A key assumption is that entrepreneurial skills are private information. In such a context, when agents need credit to start up their projects, asymmetric information gives rise to an adverse selection problem linked to the allocation of skills and prevents the efficient operation of financial markets.

The modelled economy is constituted by different productive sectors. Each of these sectors represents a particular industry or activity, and requires the application of some specific types of entrepreneurial skills. The appearance of new sectors is assumed to be the result of R&D effort and innovations. This assumption reflects the idea that carrying out new productive activities requires first an increase in the stock of knowledge in the society.

The central point in this paper rests on the hypothesis that sectoral variety facilitates the self-selection of talents to sectors. This fact reduces the severity of the adverse selection problem in the credit market, enabling the provision of more satisfactory credit contracts, which fosters entrepreneurial investment. The impact of sectoral variety on credit market efficiency, in turn,

gives rise to a novel positive feedback between financial development and innovation activities. Entrepreneurs are the agents who put innovations into practice in the economy. This means that the level of entrepreneurial investment is what ultimately determines the size of the market for innovations and the returns to R&D effort. As a result, better operation of financial markets spurs the incentives to undertake R&D (by fostering entrepreneurial investment) and, at the same time, higher investment in R&D contributes to financial development (by expanding the variety of sectors in the economy).

Based on this setup, I present two main findings. First, there is a static efficiency result related to the degree of sectoral diversification: a larger variety of sectors helps reduce the informational frictions in the credit market. In particular, given the heterogeneity of skills, sectoral variety allows better matching of agents to activities in the economy, which in turn raises the quality of the pool of credit applicants. In that regard, adverse selection here stems from an underlying problem of relative scarcity of sectors, because this hinders the efficient sorting of (unobservable) talents. When the variety of sectors is limited, a large number of agents have no other choice but to specialise in activities for which they might not be exceptionally talented. Asymmetric information concerning skills, in turn, spreads the negative consequences of talent mismatching to other sectors in the economy, since it prevents the efficient (*ex-ante*) screening of heterogeneous agents in the credit market. In other words, those agents who are not able to exploit their comparative advantages inflict a negative externality (through the adverse selection problem) on those who, in principle, could exercise fully their intrinsic skills.

Second, from a dynamic perspective, the paper shows that some economies might follow successful development paths, while others might get trapped in an underdevelopment equilibrium. In the former case, development is characterised by a continuous process of sectoral differentiation. In addition, alongside development, the allocation of talent improves and financial institutions become increasingly efficient, as adverse selection problems tend to vanish concomitantly with sectoral diversification. On the other hand, in the poverty trap, economies exhibit a rudimentary productive structure, with few active industries, poor allocation of talents and highly inefficient financial institutions. In that sense, the poverty trap is the result of a general organisational failure in the economy, leading to the collapse of several markets.

The paper mostly contributes to two main strands of literature. First, it adds a novel mechanism to the literature on financial market imperfections and poverty, which started with Banerjee and Newman (1993) and Galor and Zeira (1993).<sup>1</sup> These articles stress the influence of

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<sup>1</sup>Other important papers in this literature include: Piketty (1997), Aghion and Bolton (1997), Lloyds-Ellis

wealth distribution on the dynamic behaviour of the economy when agency costs lead to credit rationing. This paper provides a fully micro-founded explanation of why agency costs may arise in a developing economy. Furthermore, the model is able to generate dynamics whereby these agency costs decline as an economy develops. As a result, *rationing* is not just solved because people become rich enough (so that they can afford better credit or insurance contracts), but mainly because financial markets' operation itself becomes more efficient as development progresses.

Related to this literature, the idea that credit markets' efficiency might be influenced by agents' payoffs in other markets of the economy has been suggested by De Meza and Webb (2000) with a model in which payoffs are exogenously set. Ghatak, Morelli and Sjöström (2007) build on this idea, and they explicitly endogenise agents' payoffs, exploiting an interesting general equilibrium interaction between the credit market and the labour market. When the economy is able to provide high wages, *low-quality* entrepreneurs find themselves better off selling their labour in the market. As a result, high wages help to "clean" the pool of credit applicants, reducing informational frictions and enabling better operation of the credit market.

I study the sorting of talents within a multi-sectoral endogenous growth model. Innovation and the creation of new productive activities thus become key features of the model, since they lead to improved sorting of skills to sectors. Two main novel findings result from my model compared to Ghatak *et al.* First, it shows that innovation improves the assignment of skills, which in turn feeds back on innovation by increasing the returns to R&D. Second, it highlights a new role for the innovation process, very different from the one traditionally stressed in the growth literature. Innovations are not only desirable because they *directly* augment the productivity of inputs. They are also desirable because they help to mitigate frictions hindering the operation of financial markets. From that perspective, this paper is also contributing to the literature on sectoral variety and growth by proposing an additional channel whereby increased variety promotes development.<sup>2</sup>

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and Bernhardt (2000), Mookherjee and Ray (2002), and Ghatak and Jiang (2002).

<sup>2</sup>Sectoral differentiation has traditionally been considered to raise aggregate productivity by two distinct channels: 1) permitting the exploitation of *economies of scale* through increasing specialisation (e.g., Smith (1776), Young (1928), Romer (1990), Yang and Borland (1991), Jones(2008)); 2) enabling heterogeneously skilled agents to obtain a better *match* (e.g., Rosen (1978), Miller (1984), Kim (1989)). The contribution of this paper to that literature is then to show that sectoral differentiation brings about an additional positive effect on growth via improved matching, because an increasing variety of activities helps to lessen adverse selection problems linked to the allocation of skills.

The second main strand of literature to which this paper contributes is that on growth and financial deepening. Within that literature, the two most related papers to mine are Acemoglu and Zilibotti (1997, 1999).<sup>3</sup> In the former paper, they construct a model with technological indivisibilities in which the degree of market incompleteness tends to disappear with capital accumulation, and this fosters financial development (in particular, it improves risk sharing). Financial markets are enhanced by sectoral differentiation, because it allows better pooling of sector-specific shocks. The key novelty of my model with respect to theirs is that financial development is the consequence of the alleviation of agency costs due to improvements in the sorting of skills in a context of asymmetric information. In the latter paper, Acemoglu and Zilibotti study the evolution of informational asymmetries and agents' performances over the development path. However, they focus on how a society manages to provide correct incentives to agents, and how incentives become more effective as an economy grows. My paper studies a different problem, that is how the allocation of heterogeneous skills evolves during development and, more importantly, it incorporates innovation decisions into the model, allowing for endogenous variety expansion.

Section 2 describes the basic setup of the model. Section 3 studies the static equilibrium of the economy; in particular it analyses the entrepreneurs' optimal choice in the presence of adverse selection. Section 4 introduces the innovation activities into the model, which endogenises the variety of sectors in the economy. Section 5 proceeds to the dynamic analysis of this economy. Section 6 discusses an important extension to the basic model. Section 7 presents and discusses some stylised facts observed in cross-country data which are consistent with the main predictions of the model. Section 8 concludes. Omitted proofs are provided in the Appendix.

## 2 Environment

The paper considers a small economy enjoying full access to international financial markets. Life evolves over a discrete-time infinite horizon  $t = \{0, 1, \dots, \infty\}$ . In each period  $t$  a single-period lived continuum of agents with mass normalised to 2 is alive.

The economy contains a continuum of *sectors* indexed by the letter  $i \in [0, 1]$ . Each sector  $i$  represents a particular industry where a final good is produced. The set of sectors  $[0, 1]$  is constant over time; however, not all sectors are necessarily *active* at any moment in time. In

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<sup>3</sup>Other papers in that literature with some connection to mine are Greenwood and Jovanovic (1990) and Saint-Paul (1992).

particular, at time  $t$  only a fraction  $n_t$  of all sectors are able to enjoy the activity of productive industries. Hereafter,  $\mathcal{A}_t \subset [0, 1]$  will denote the set of sectors with active industries at time  $t$ . The set  $\mathcal{A}_t$  has Lebesgue measure  $n_t$ .

The availability of productive industries is the result of innovations (either generated during the past or in the present). This assumption reflects the idea that in order to produce a new type of good, we first need to create the knowledge required to produce this new good. Once the industrial activity that corresponds to sector  $i$  is *created* by an innovation, it never disappears (i.e., if sector  $i \in \mathcal{A}_t$ , then sector  $i \in \mathcal{A}_{t+\delta} \forall \delta \geq 0$ ). To ease notation, henceforth I skip the use of time-subscripts when creating no confusion. Sectors belonging to  $\mathcal{A}$  will be referred to as *active sectors* (and the remaining sectors will accordingly be called *inactive sectors*).

A sector  $i \in \mathcal{A}$  provides the agents in the economy the chance to invest in an entrepreneurial project called Project- $i$ . The return of Project- $i$  is random, subject to an idiosyncratic shock. Project- $i$ 's return also depends on the application of some specific entrepreneurial skills, and on the amount of capital invested in the project. A full description of Project- $i$  is provided in the following subsection (equations (1) and (2) ahead in the text).

Each generation comprises two different groups of individuals, each one with unit mass:

1. *Entrepreneurs*: These agents are endowed with entrepreneurial skills which, are needed to organise and undertake the production of final goods.
2. *Inventors*: They carry out R&D in order to generate new ideas that can be used by the entrepreneurs in the production of new final goods.<sup>4</sup>

## 2.1 Entrepreneurs

At any time  $t$ , there exists a continuum of (prospective) entrepreneurs who are indexed by the letter  $i \in [0, 1]$ . Henceforth, the entrepreneur  $i$  will be referred to as the type  $i$ .

The cohort- $t$  of entrepreneurs is alive during period  $t$ . A new cohort is born just at the end of the previous cohort's lifespan. Each (dying) entrepreneur procreates one (new) entrepreneur. For the moment, I assume agents are non-altruistic and are born with zero initial wealth (in Section

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<sup>4</sup>To illustrate this distinction, take the Pharmaceutical Industry as an example. The innovator would be represented by a biochemist whose task consists in designing a formula to produce a new drug. On the other hand, the pharmaceutical company would represent the entrepreneur. This agent organises the production process of the drug and takes it to the market, turning the (abstract) formula into a final good ready for consumption.

6 this assumption is relaxed). All entrepreneurs are risk-neutral, sharing identical preferences over consumption. Accordingly, they all seek to maximise their expected consumption.

Entrepreneurs are heterogeneous with respect to their entrepreneurial skills. More precisely, if type  $j \in [0, 1]$  invests  $k$  units of capital in Project- $i \in \mathcal{A}$ , then his Project- $i$ 's gross return ( $y_{i,j}$ ) is given by:

$$y_{i,j} = \theta_{i,j} f(k_{i,j}).$$

The function  $f(k)$  is strictly increasing, strictly concave, twice continuously differentiable, and satisfies Inada conditions. The variable  $k_{i,j}$  represents the amount of capital invested in Project- $i$  by type  $j$ . Capital fully depreciates during the process of production. Finally,  $\theta_{i,j}$  denotes the realisation of a random variable with support  $\{0, 1\}$ . The value taken by  $\theta_{i,j}$  is governed by the following distribution function:

$$\theta_{i,j} = \begin{cases} 1 & \text{with probability } p_{i,j} \\ 0 & \text{with probability } 1 - p_{i,j}, \end{cases}$$

where,

$$\begin{aligned} p_{i,j} &= 1 && \text{for all } i, j \in [0, 1] \text{ if } j = i, \\ p_{i,j} &= p \in (0, 1) && \text{for all } i, j \in [0, 1] \text{ if } j \neq i. \end{aligned}$$

In short, type  $i$  is an agent with intrinsic comparative advantage in Project- $i$ .<sup>5</sup> Gross returns of Project- $i$  are thus given by:

$$y_{i,i} = f(k_{i,i}) \tag{1}$$

$$y_{i,j}(\theta_{i,j}) = \begin{cases} f(k_{i,j}) & \text{with probability } p \\ 0 & \text{with probability } 1 - p \end{cases}, \text{ where } j \neq i \tag{2}$$

Diversification among entrepreneurial projects is not feasible. In other words, agents must *specialise* in, at most, one particular Project- $i \in \mathcal{A}$ .

Concerning the informational structure in the economy, entrepreneurial types are assumed private information. Project outcomes, on the contrary, are publicly observable. In addition to that, I assume types are intergenerationally uncorrelated, implying that parents' historical outcomes provide no information whatsoever about the type of a child.

Lastly, I assume that everybody has access to a "backyard" activity which requires no initial investment and yields net return equal to  $v$  with certainty. Without loss of generality, I set  $v = 0$  (implying that the corresponding participation constraint will never bind).<sup>6</sup>

<sup>5</sup>The concept of comparative advantage is defined in terms of average productivity (the average productivity of type  $i$  in Project- $i$  is higher than the average productivity of type  $j \neq i$  in Project- $i$ ).

<sup>6</sup>If  $v > 0$ , agents would have access to an outside option with positive payoff, hence their participation constraint



## 2.2 Inventors

In addition to the entrepreneurs, in any period  $t$ , there is also a continuum of agents with unit mass (the *inventors*) who are born with the particular skill to be able to produce *new* ideas. New ideas, in turn, materialise in *innovations* and expand the set of active sectors in the economy in period  $t$ ,  $\mathcal{A}_t$ . This means that the set  $\mathcal{A}_t$  is the result of the stock of innovations generated during the history of the economy up to  $t$ . The presentation of the inventors' optimisation problem will be postponed until Section 4.

## 2.3 Credit Markets

Since agents in the economy are born with zero wealth, they will need to rely on credit markets in order to undertake their investment projects. The rest of the world will provide local agents with the needed funds. All credit market transactions with the rest of the world are mediated by some firms called *financial intermediaries*. The local credit market is characterised by free-entry and absence of set-up or sunk costs. Since the economy is small and there is perfect international capital mobility, financial intermediaries are able to draw liquid funds from international credit markets facing a perfectly elastic supply at the international (net) interest rate  $R^f$ . In the sake of algebraic simplicity, let  $R^f = 0$ .

Financiers will offer loan contracts stipulating the payment to be made to them, conditional on the outcome of the entrepreneurial project. Individuals in the economy are protected by limited liability. As a result, since in the event of failure projects yield zero output, entrepreneurs would be able to pay back a positive amount to the financiers *only* in the case of success. Equilibrium loan contracts will thus display the following structure:  $(l_j, r_j) \in \mathbb{R} \times \mathbb{R}$ , where  $l_j$  represents the loan extended to type  $j$  and  $r_j$  stands for the (net) interest rate charged on  $l_j$  in the event of success. In other words, the entrepreneur  $j$  must pay back  $l_j(1 + r_j)$  in the state of success, while if the project fails he goes bankrupt and the financier recovers 0 income.<sup>7</sup>

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may bind in equilibrium. This might have some minor implications on the type of credit contracts observed in equilibrium, however, none of the main results and insights of the paper would be altered by letting  $v > 0$ .

<sup>7</sup>Nothing in the model would change if entrepreneurs raised capital by issuing equity, as each share will pay zero in the event of failure and a strictly positive dividend in the event of success that is identical for all entrepreneurial projects.

### 3 Static Equilibrium Analysis

Throughout this section the set of active sectors  $\mathcal{A}_t$  is taken as exogenously given. Thus, the paper focuses on the optimal behaviour of the entrepreneurs, and on the set of credit contracts offered by the financial intermediaries, given  $\mathcal{A}_t$ . This course of action will yield the equilibrium solution of the model at some specific period of time  $t$ . In the next sections I proceed to study the dynamic evolution of the economy. This will require explicitly incorporating the inventors' optimisation problem, which endogenises the set  $\mathcal{A}_t$ .

Let  $\mathcal{C}_t$  denote the set of credit contracts offered by financial intermediaries in period  $t$ . An entrepreneur  $j \in [0, 1]$  alive during  $t$  will choose an allocation  $[(r_j, l_j)^*, k_{i,j}^* : i \in \mathcal{A}_t]$ , solving the following two-stage optimisation problem:

- **First-Stage (specialisation decision):**  $j \in [0, 1]$  selects sector  $i \in \mathcal{A}_t$  in which to invest.
- **Second-Stage (optimal investment in sector  $i$ ):**<sup>8</sup>

$$\begin{aligned} \max_{k_{i,j}, (r_j, l_j)} : E_i(U_j) = & p_{i,j} \max \{0, f(k_{i,j}) - (1 + r_j)l_j + (l_j - k_{i,j})\} \\ & + (1 - p_{i,j}) \max \{0, -(1 + r_j)l_j + (l_j - k_{i,j})\} \end{aligned} \quad (\text{I})$$

$$\begin{aligned} \text{subject to: } & k_{i,j} \leq l_j \quad (\text{budget constraint}), \\ & k_{i,j} \geq 0 \quad (\text{feasibility constraint}), \\ & (r_j, l_j) \in \mathcal{C}_t \quad (\text{set of offered credit contracts}). \end{aligned}$$

**Definition 1 (Equilibrium at time  $t$ )** *Given the set  $\mathcal{A}_t$ , an equilibrium at time  $t$  is a set of entrepreneurial allocations  $[(r_j, l_j)^*, k_{i,j}^* : i \in \mathcal{A}_t]_{j \in [0,1]}$  and a set of offered credit contracts  $\mathcal{C}_t$ , such that the following two conditions are satisfied:*

- 1) **Entrepreneurs' optimal allocation:** *Given the set  $\mathcal{C}_t$ ,  $\forall j \in [0, 1]$  alive in period  $t$ , the allocation  $[(r_j, l_j)^*, k_{i,j}^* : i \in \mathcal{A}_t]$  solves the two-stage optimisation problem (I).*
- 2) **Credit markets (competitive) equilibrium:** *(i) No credit contract belonging to  $\mathcal{C}_t$  makes negative expected profits; and (ii) there exists no other feasible credit contract  $\mathfrak{z}$ , such that  $\mathfrak{z} \notin \mathcal{C}_t$ , and which, if offered in addition to  $\mathcal{C}_t$ , would make positive expected profits.*

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<sup>8</sup>  $E_i(U_j)$  denotes the expected utility of type  $j$  when he invests in Project- $i$  (recall that the success probability  $p_{i,j}$  depends on the match between the type and the sector).

### 3.1 Credit Market Equilibrium Contracts

Following the literature on adverse selection in financial markets (e.g., Rothschild and Stiglitz (1976), Wilson (1977), and Milde and Riley (1988)), one would reasonably expect two different kinds of equilibria to possibly arise in this model's credit market: 1) a *pooling equilibrium*, in which all types receive an identical credit contract; 2) a *separating equilibrium*, in which types are *screened*, receiving distinctive contracts which induce truthful self-revelation of their (unobservable) skills.

**Lemma 1** *Assume the set of inactive sectors at time  $t$  is non-empty (i.e.,  $\mathcal{A}_t \neq [0, 1]$ ). Take any sector  $i \in \mathcal{A}_t$  and any sector  $j \notin \mathcal{A}_t$ . Then, there can never exist an equilibrium at  $t$  in which type  $i$  and type  $j$  are offered different credit contracts.*

Lemma 1 means that there cannot exist a separating equilibrium in this model. As a consequence, if an equilibrium is to exist at all, it should entail pooling credit contracts. This result stems from the conjunction of four different assumptions: *i*) risk-neutrality, *ii*) the limited-liability constraint, *iii*) agents being born with zero initial wealth (so they can place no collateral), and *iv*) the fact that the outside option yields  $v = 0$ . Intuitively, given a set of credit contracts, any contract that maximises net returns for (1) must also necessarily maximise expected net returns for (2) (since, in the presence of limited liability and no collateral, expected net returns when (2) holds are proportional to net returns when (1) prevails).<sup>9</sup>

Given the set of active sectors at time  $t$ ,  $\mathcal{A}_t \subset [0, 1]$ , we may split the population of entrepreneurs alive during  $t$  in two disjoint subsets: the first subset composed by all those types- $i \in [0, 1]$ , such that sector  $i \in \mathcal{A}_t$ ; the second one by all those types- $j \in [0, 1]$ , such that sector  $j \notin \mathcal{A}_t$ . The first group of agents would be able to exploit fully their comparative skills, whereas the second one have to specialise in a sector for which they are not (exceptionally) talented. Abusing a bit of the language utilised in the adverse selection literature, I will call the first group the *good types*, while the second group will be denoted as the *bad types*.<sup>10</sup>

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<sup>9</sup>See Ghatak, Morelli and Sjöström (2007), and also Grüner (2003), for models that obtain pooling contracts in a similar fashion. Pooling contracts are especially attractive in this context because they lead to a very neat and smooth characterisation of the main results of this paper. Yet, pooling contracts, and in particular the assumptions required for pooling to arise, are by no means crucial. What is essential here is the fact that as more sectors become active and the matching of skills improves, the informational frictions in the credit market are eased, which in turn permits the provision of credit contracts closer to the first-best contracts.

<sup>10</sup>More rigorously:  $good\ types_t = \{h \in [0, 1] \mid sector\ h \in \mathcal{A}_t\}$  and  $bad\ types_t = \{h \in [0, 1] \mid sector\ h \notin \mathcal{A}_t\}$ . Notice that in this paper whether a particular Type- $h \in [0, 1]$  is a good type or a bad type is not fixed, but it is

In a pooling equilibrium, all entrepreneurs receive an identical credit contract  $(l, r)$ . Notice then that  $\mathcal{C}_t$  must comprise one single element; namely:  $\mathcal{C}_t = (l, r)$ . Additionally, in any (competitive) pooling equilibrium, credit contracts must necessarily verify the following two properties. First, the contract must make non-negative expected profits; otherwise this contract would simply be withdrawn. Second, the contract must maximise the expected utility of the good types; otherwise financial intermediaries could offer a different contract such that it makes non-negative profits and, at the same time, it makes these agents better off.

Assume for the moment that type  $i$  chooses to specialise in sector  $i \in \mathcal{A}$  (as it will become clear later on, this will necessarily be true in equilibrium). Then, given  $\mathcal{C}_t = (l, r)$ , his optimisation problem boils down to:

$$\begin{aligned} \max_{k_{i,i} \geq 0} & : \max \{0, f(k_{i,i}) - (1+r)l + (l - k_{i,i})\} & (I') \\ \text{s.t.} & : k_{i,i} \leq l \quad (\text{budget constraint}). \end{aligned}$$

Note now that because  $r \geq 0$  (otherwise financiers would make losses on entrepreneurial loans), entrepreneurs will borrow *only* with the intention to invest in a project. As a consequence,  $k_{i,i} = l$  will hold in the optimum and Problem (I') will yield the following (standard) first-order condition:

$$f'(k^*) = (1+r) \quad (3)$$

From (3), we can then obtain the optimal amount of capital invested in the project, given the interest rate  $r$ . That is,  $k^*(r)$ ; where  $k'(r) < 0$  since  $f''(\cdot) < 0$ . An equilibrium pooling contract will, therefore, display the following structure:  $(l, r) = (k^*(r), r)$ . (So that it maximises the expected utility of the good types.)

### 3.2 The Equilibrium Interest Rate

The pair  $(k^*(r), r)$  characterises the equilibrium credit contract, given the interest rate  $r$ . Therefore, in order to determine the exact credit contract that holds in  $t$ , it still remains to find the equilibrium value of  $r$  in  $t$ . Let us denote this variable by  $r_t^*$ .

Consider sector  $i \in \mathcal{A}_t$  and suppose the type  $i$  alive in  $t$  decides to invest in Project- $i$ . Then, given  $r$ , his consumption  $(c_{i,i})$  would be determined by:

$$c_{i,i} = f(k^*(r)) - (1+r)k^*(r). \quad (4)$$

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contingent of the set  $\mathcal{A}_t$ . In that sense, from a dynamic point of view, everyone could eventually become a good type, if the set of active sectors constantly expands over time.

Now, suppose this type  $i$  chooses to invest in Project- $x \in \mathcal{A}_t$ , where  $x \neq i$ . In that case, his consumption ( $c_{x,i}$ ) would be given by  $c_{x,i} = p[f(k^*(r)) - (1+r)k^*(r)]$ . It is then straightforward that  $c_{i,i} > c_{x,i}$ , no matter the value of  $r$ . Hence, as long as sector  $i \in \mathcal{A}_t$ , this type  $i$  will specialise in Project- $i$ .

Consider now sector  $j \notin \mathcal{A}_t$  and the type  $j$  alive in period  $t$ . This agent could invest in any Project- $x$ , such that sector  $x \in \mathcal{A}_t$ , obtaining as expected consumption:

$$c_{x,j} = p[f(k^*(r)) - (1+r)k^*(r)]. \quad (5)$$

Since  $p > 0$  the equation (5) yields  $c_{x,j} > 0$ , irrespective of the value taken by  $r$ . This implies that it will always be desirable for type  $j$  to invest  $k^*(r)$  in Project- $x$ .

From the previous discussion, it follows that a fraction  $n_t$  of the population of entrepreneurs (the good types) will always pay back the financial intermediaries the agreed amount  $(1+r)k^*(r)$ . On the other hand, the remaining fraction  $1-n_t$  (the bad types) will go bankrupt with probability  $1-p$ . Being protected by limited-liability, the bad types are *expected* to pay back financiers only the amount  $p(1+r)k^*(r)$ .

Perfect competition in the credit market naturally implies that financiers must make zero profits in equilibrium. Then, the *zero-profit condition* on entrepreneurial loans is given by:  $n_t(1+r_t^*)k^*(r_t^*) + (1-n_t)p(1+r_t^*)k^*(r_t^*) = (1+R^f)k^*(r_t^*)$  (where, recall that for algebraic simplicity,  $R^f = 0$  will be assumed).

**Proposition 1** *The equilibrium interest rate charged on credit contracts offered to entrepreneurs is a decreasing function of the fraction of active sectors. More precisely,*

$$r_t^* = r^*(n_t) = \frac{(1-n_t)(1-p)}{n_t + (1-n_t)p}. \quad (6)$$

From (6), it can also be noted that:  $r^*(0) = (1-p)/p$ ,  $r^*(1) = 0$ , and  $r''(n_t) > 0$ .

Proposition 1 represents one the key insights of this paper. A larger number of active sectors leads to a more efficient operation of credit markets, because a higher value of  $n_t$  improves the sorting of entrepreneurial skills, alleviating the adverse selection problem in the credit market. Intuitively, as the set  $\mathcal{A}_t$  expands, a higher fraction of agents find it feasible to specialise in the sector they are most talented at. This, in turn, reduces the *average default rate* in the economy, enabling financiers to charge a lower interest rate on the loans they extend to entrepreneurs, without incurring in expected losses.<sup>11</sup>

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<sup>11</sup>Notice that  $r^*$  represents also the *risk premium* in the economy. In that regard, it is the risk premium on entrepreneurial loans what diminishes as  $n$  goes up due to the better sorting of talent.

### 3.3 Entrepreneurial Consumption Level / Net Returns

Take again some type  $i \in [0, 1]$ , such that sector  $i \in \mathcal{A}$  (a good-type representative). His consumption level will be dictated by (4). Denote by  $U_g(r)$  the utility achieved by an entrepreneur who belongs to the subset of good types. Differentiating (4) with respect to  $r$ , and taking (3) into account, we get:

$$U'_g(r) = -k^*(r). \quad (7)$$

Select now some type  $j \in [0, 1]$ , such that sector  $j \notin \mathcal{A}$  (a bad types representative). His expected consumption will be given by (5). Hence, letting  $U_b(r)$  denote the expected utility reached by a bad type, we obtain:

$$U'_b(r) = -pk^*(r), \quad (8)$$

where derivation of (8) also makes use of (3).

**Lemma 2** *Let  $\Delta(r) \equiv U_g(r) - U_b(r)$ . Then,  $\Delta(r) > 0$  and  $\Delta'(r) < 0$ , for all possible values  $r$  may take in equilibrium.*

The proof of Lemma 2 is straightforward from inspection of (7) and (8). The derivative  $\Delta'(r) < 0$  means that good types benefit from a fall in the interest rate  $r$  more than bad types do. The reason for this result rests on the fact that good types never go bankrupt, thus they will appropriate the full cost-reduction induced by a lower  $r$ . On the other hand, since bad types go bankrupt with probability  $(1 - p)$ , they will profit from a smaller  $r$  only with probability  $p < 1$ . Lemma 2 will play a key role in the inventors' optimisation problem (in the following section).

## 4 Inventors, Market for Ideas, and Innovations

I model the appearance of *new* active sectors as the result of innovations. Following the Endogenous Growth Theory paradigm, innovations result from deliberate profit-maximising R&D policies undertaken by private agents which I refer to as *inventors*.<sup>12</sup> I will focus only on horizontal innovations, as those are the kind of innovations that will lead to improvements in the allocation of agents' talents, which is the key mechanism at work in this theory.

In each period  $t$  there is a continuum of single-period lived inventors with unit mass. Inventors are non-altruistic and risk-neutral. Each (dying) inventor gives birth to a (new) inventor. Inventors are able to generate *new* ideas (this is their specific skill). Think of an idea as a

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<sup>12</sup>E.g. Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992).

blueprint or design, which contains the information needed to produce new types of goods. As previously done with sectors and entrepreneurs, let inventors be indexed by  $i \in [0, 1]$ . Except for their particular index  $i$ , all inventors within the same cohort are *ex-ante* identical. I suppose the inventor  $i$  can only possibly innovate for sector  $i$ . Since vertical innovations are assumed away, the subset of inventors who (would) innovate for sectors which were already *active* in period  $t - 1$  will thus not play any relevant role during  $t$ .

In order to come up with a new idea, an inventor needs first to carry out R&D, which is costly. A new idea, however, does not *per se* modify the technological frontier of the economy; for that to happen, the idea must be *applied* by an entrepreneur.<sup>13</sup> When the *idea* designed by inventor  $i$  is put into practice by some entrepreneur  $j \in [0, 1]$ , this idea becomes *technology*, and materialises as Project- $i$  (turning sector  $i$  into an active sector).

Technology is a pure public-good; that is, its use is non-rival and non-excludable. More precisely, once some particular entrepreneur  $j \in [0, 1]$  applies a new idea, the underlying knowledge becomes readily (and instantly) available to all the other entrepreneurs from  $t$  onwards. On the contrary, an idea is excludable, since the inventor who has generated it can keep his idea *undisclosed* for as long as he wants, simply by not spelling it out to any other agent.

An inventor who comes up with a new idea, will then try to sell it to an entrepreneur. I assume entrepreneurs pay the inventors after production takes place and that the transaction between an inventor and an entrepreneur is not observable to the financiers. Given the public nature of technology, only type  $i$  would be willing to pay a positive price to obtain the idea generated by inventor  $i$ .<sup>14</sup> To see this, recall from Lemma 2 that  $\Delta(r) > 0$  for any possible value that  $r$  may take in equilibrium. This  $\Delta(r)$  equals the increment in (expected) utility that the type  $i$  would get by applying the idea generated by inventor  $i$  (were this idea given to him for free!). Notice  $\Delta(r)$  is a surplus resulting from a *bilateral monopoly* relationship between type  $i$  and inventor  $i$ . In principle, the surplus  $\Delta(r)$  could be distributed between the two parties according to various rules. For simplicity, let the whole surplus  $\Delta(r)$  be appropriated by the

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<sup>13</sup>This complementarity between inventors and entrepreneurs is in line with the view of economic development by Joseph A. Schumpeter (1934, pp. 88-89); he writes, "*Entrepreneurship must be distinguished from 'invention'. As long as they are not carried out into practice, inventions are economically irrelevant. And to carry any improvement into effect is a task entirely different from the inventing of it, and requiring different kinds of aptitudes. Although entrepreneurs of course may be inventors, it would not be by nature of their function but by coincidence.*"

<sup>14</sup>Notice that this, in turn, implies that inventors will not face any adverse selection problem in the market for ideas, as they can be certain that *only* the correct types would be willing to buy their ideas.

inventor, leaving the entrepreneur just indifferent between buying or not the new idea (in other words, assume the inventor makes a take-it-or-leave-it-offer to the entrepreneur for the transfer of the idea).<sup>15</sup>

#### 4.1 Inventors' Optimisation Problem

Inventors must expend effort in order to generate new ideas. Effort generates disutility. Let  $\iota_{i,t}$  denote the effort cost (measured in units of consumption) spent in R&D activities by the inventor  $i$  alive during period  $t$ . Additionally, denote by  $\Pr(I_i = 1)$  the probability that the inventor  $i$  will generate a new idea. Consider sector  $i \notin \mathcal{A}_{t-1}$ ; the probability that inventor  $i$  generates an idea for sector  $i$  in period  $t$  is given by (henceforth, I skip the use of time subscripts on  $\iota_{i,t}$  to ease notation):

$$\Pr(I_i = 1) = \beta(\iota_i), \quad (9)$$

where:  $\beta'(\iota) > 0$ ,  $\beta''(\iota) < 0$ ,  $\beta(0) = 0$ ,  $\lim_{\iota \rightarrow \infty} \beta(\iota) \leq 1$ , and  $\lim_{\iota \rightarrow 0} \beta'(\iota)$  is finite.

Given that sector  $i \notin \mathcal{A}_{t-1}$ , inventor  $i$  will choose the value of  $\iota_i$  so as to maximise the expected profits derived from the *generation* and *sale* of new ideas.<sup>16</sup> Denote by  $\bar{\iota}_t$  the level of R&D effort chosen by all the inventors belonging to the subset  $-\mathcal{A}_{t-1}^{-i}$ , where  $-\mathcal{A}_{t-1}^{-i} = \{j \in [0, 1] \mid j \neq i \text{ and sector } j \notin \mathcal{A}_{t-1}\}$ .<sup>17</sup>

Having managed to produce a new idea, inventor  $i$  will optimally charge a price  $\Delta(r^*(n_t))$  when selling this idea to the type  $i$ . Notice that, assuming that all new ideas are sold to entrepreneurs (which will be true in equilibrium),  $n_t = n_{t-1} + \beta(\bar{\iota}_t)(1 - n_{t-1})$ .<sup>18</sup> Hence, we can rewrite  $\Delta(r^*(n_t)) = \Psi(n_{t-1}, \bar{\iota}_t)$ . Lemma 3 characterises the optimisation problem faced by inventor  $i$ .

**Lemma 3** *Consider sector  $i \notin \mathcal{A}_{t-1}$ , and take the inventor  $i$  alive during  $t$ . He solves:*

$$\max_{\iota_i \geq 0} : \Pi_{i,t}(\iota_i, n_{t-1}, \bar{\iota}_t) = \beta(\iota_i) \cdot \Psi(n_{t-1}, \bar{\iota}_t) - \iota_i \quad (\text{II})$$

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<sup>15</sup>Nonetheless, as long as it is assumed that the inventor's income is increasing in the total surplus  $\Delta(r)$ , none of the main findings of this paper would be affected if the entrepreneur could actually appropriate part of  $\Delta(r)$  (for instance, if the surplus were split following a Nash-bargaining rule).

<sup>16</sup>If sector  $i \in \mathcal{A}_{t-1}$ , then the inventor  $i$  alive in  $t$  trivially chooses  $\iota_i = 0$ .

<sup>17</sup>This  $\bar{\iota}_t$  should actually be a mapping  $\bar{\iota}_t : -\mathcal{A}_{t-1}^{-i} \rightarrow [0, \infty)$ , summarising the choice of  $\iota$  for each inventor belonging to  $-\mathcal{A}_{t-1}^{-i}$ . However, in the optimum, all these inventors will select the same value of  $\iota$ . Hence, a *singleton*  $\bar{\iota}_t$  turns out to be sufficient to represent their aggregate behaviour.

<sup>18</sup>This is because: 1) the sectors that were already active in  $t - 1$  remain active in  $t$ , and 2) a fraction  $\beta(\bar{\iota}_t)$  among the inactive sectors in  $t - 1$  become active in  $t$ .



Where the function  $\Psi(n_{t-1}, \bar{\iota}_t) : [0, 1] \times \mathbb{R}^+ \rightarrow \mathbb{R}^+$  is increasing in both of its arguments. More precisely: (i)  $\Psi'_n(\cdot) > 0, \forall n_{t-1} \in [0, 1]$  and  $\bar{\iota}_t \geq 0$ ; and (ii.a)  $\Psi'_{\bar{\iota}}(\cdot) > 0, \forall n_{t-1} \in [0, 1]$  and  $\bar{\iota}_t \geq 0$ , (ii.b)  $\Psi'_{\bar{\iota}}(\cdot) = 0$  if  $n_{t-1} = 1$ .

From Lemma 3 it follows that  $\Pi_{i,t}(\iota_i, n_{t-1}, \bar{\iota}_t)$  must be increasing in both  $n_{t-1}$  and  $\bar{\iota}_t$ . To grasp some intuition, notice that, since active sectors never revert to inactive, the higher  $n_{t-1}$  is, the higher  $n_t$  is expected to be. As a result, relatively high values of  $n_{t-1}$  will tend to be associated with relatively low levels of  $r_t^*$  (Proposition 1). This, in turn, implies that the surplus generated by new innovations,  $\Delta(r_t^*)$ , is expected to be large (Lemma 2), allowing inventors to charge a relatively high price for their ideas. Similarly, larger values of  $\bar{\iota}_t$  are also associated with less severe adverse selection leading to lower  $r_t^*$  and higher  $\Delta(r_t^*)$ . In this case, the reason is that a larger  $\bar{\iota}_t$  means more innovations will actually be produced, raising thus the value of  $n_t$  (from the given  $n_{t-1}$ ). In addition to that, note  $\Psi'_{\bar{\iota}}(\cdot) > 0$  implies that there exists a positive externality across inventors. This externality arises because when an inventor  $j \in [0, 1]$  comes up with a new idea, this may turn sector  $j$  into an active sector, increasing the value of  $n_t$  (something which all inventors will benefit from).

Problem (II) leads to the following first-order condition:

$$\beta'(\iota_i^*) \cdot \Psi(n_{t-1}, \bar{\iota}_t) \leq 1 \quad \text{and} \quad \iota_i^* [\beta'(\iota_i^*) \cdot \Psi(n_{t-1}, \bar{\iota}_t) - 1] = 0 \quad (10)$$

**Proposition 2** Let  $\iota_i^* \equiv \arg \max_{\iota_i} \{\Pi_{i,t}(\iota_i, n_{t-1}, \bar{\iota}_t)\}$ . Then,  $\iota_i^* = \iota_i^*(n_{t-1}, \bar{\iota}_t) : [0, 1] \times \mathbb{R}^+ \rightarrow \mathbb{R}^+$ , and it exhibits the following two properties: 1)  $\iota_i^*(n_{t-1}, \bar{\iota}_t)$  is (weakly) increasing in  $n_{t-1}$ ; 2)  $\iota_i^*(n_{t-1}, \bar{\iota}_t)$  is (weakly) increasing in  $\bar{\iota}_t$ .

Results in Proposition 2 are straightforward implications of Lemma 3 and equation (10). Intuitively, as  $\partial \Pi_{i,t}(\cdot) / \partial \iota_i$  is increasing in both  $n_{t-1}$  and  $\bar{\iota}_t$ , larger values of these variables will induce inventors to increase the optimal amount of effort spent in R&D.

The positive impact of  $n_{t-1}$  on  $\iota_i^*$  represents the key result of this section. This feature is the underlying force generating the novel positive feedback between financial development and innovation activities proposed here. Essentially, a larger  $n_{t-1}$  is associated with weaker distortions in the credit market, thereby leading to higher entrepreneurial investment which raises profit to inventors. This induces higher R&D effort which, in turn, leads to a faster rate of innovations, feeding back on  $n_t$ . This positive feedback gives rise to the possibility of non-ergodic dynamics in the model, as it will be discussed in detail in Section 5.

For the remainder of the paper, it proves convenient to restrict the parameters configuration such that the following two conditions hold:

**Assumption 1.**  $\exists \bar{n} \in (0, 1)$ , such that:  $\beta'(0) \Psi(\bar{n}, 0) = 1$ .

**Assumption 2.**  $\exists \underline{n} \in (0, 1)$ , such that:  $\beta'(0) [\lim_{\bar{t} \rightarrow \infty} \Psi(\underline{n}, \bar{t})] = 1$ .

**Corollary 1** *If Assumption 1 holds, then: (i)  $\forall n_{t-1} \leq \bar{n} : \bar{t}_t = 0 \Rightarrow \iota_i^* = 0$ ; (ii)  $\forall n_{t-1} > \bar{n} : \iota_i^* > 0$ , regardless of the value taken by  $\bar{t}_t$ .*

**Corollary 2** *If Assumption 2 holds, then:  $\forall n_{t-1} \leq \underline{n} : \iota_i^* = 0$ , regardless of the value taken by  $\bar{t}_t$ . (Notice Lemma 3 implies  $\underline{n} < \bar{n}$ .)*

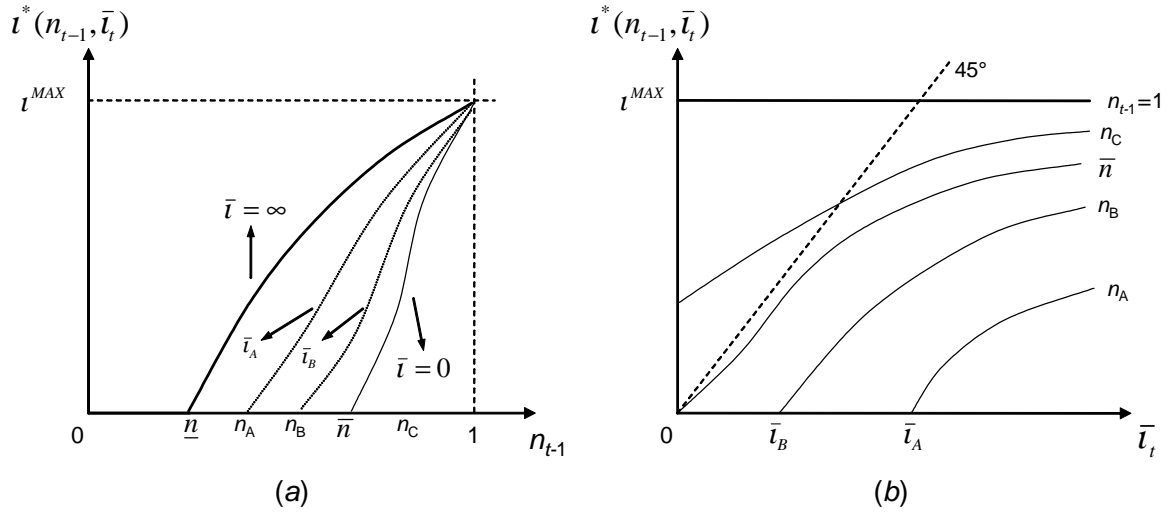


FIGURE 1: *Optimal R&D effort as a function of  $n_{t-1}$  and  $\bar{t}_t$ .*

FIGURE 1 illustrates the results stated in Proposition 2. The left panel plots  $\iota_i^*$  against  $n_{t-1}$ , given four different values of  $\bar{t}_t$  (these values are:  $0 < \bar{t}_B < \bar{t}_A < \infty$ ). Analogously, the right panel plots  $\iota_i^*$  against  $\bar{t}_t$ , given five different values of  $n_{t-1}$  ( $n_A < n_B < \bar{n} < n_C < 1$ ). Notice that the notation in both panels is consistent with each other (i.e., the value  $\bar{t}_A$  in panel (a) corresponds to the value  $\bar{t}_A$  in panel (b), and so on and so forth). Additionally, in FIGURE 1.b (although not plotted) for  $n_{t-1} = \underline{n}$  we should have  $\iota_i^*(n_{t-1}, \bar{t}_t) = 0$  for all values of  $\bar{t}_t$ . (The 45° line is just plotted for future reference.)

## 4.2 Inventors Nash Equilibrium Solution

FIGURE 1 characterises the result of the optimisation problem faced by inventor  $i$  alive in period  $t$  when sector  $i \notin \mathcal{A}_{t-1}$ , given  $n_{t-1}$  and the (expected) behaviour of the other inventors.

Nevertheless, I haven't yet discussed whether inventors' expectations, summarised by  $\bar{v}_t$ , are indeed *correct*. In fact, expectations play an important role in the model because R&D effort by a particular inventor exerts a positive externality on the others. More specifically, as stated in Proposition 2, the optimal policy of an inventor positively depends on the value of  $\bar{v}_t$ . As a result, we must restrict the attention only to those solutions of Problem (II) which also represent a *Nash Equilibrium* (NE) when we consider the whole set of inventors.

Given the structure of the model, any NE will be *symmetric* (SNE). The SNE are determined by the intersections between the 45° line and the curves plotted in FIGURE 1.b. For some ranges of  $n_{t-1} \in (\underline{n}, 1)$ , the model might lead to *multiple* SNE.<sup>19</sup> Equilibrium multiplicity may arise because inventors are subject to *strategic complementarities* (Cooper and John (1988)). FIGURE 2 shows two possible SNE schedules as a function of  $n_{t-1}$  (only the SNE schedule for an inventor  $i$  alive in  $t$  such that sector  $i \notin \mathcal{A}_{t-1}$  is plotted). In FIGURE 1.(b) and 2.(b) the parameters configuration leads always (i.e., for all values of  $n_{t-1}$ ) to unique SNE.<sup>20</sup> On the other hand, in FIGURE 2.(a) multiple equilibria emerge for values of  $n_{t-1} \in (\hat{n}, \bar{n})$ . Two equilibria are possible in this case: one where  $v_t^* = 0$ , and another one in which  $v_t^* > 0$ . Bear in mind that, as it can be deduced from Corollary 2, for any  $n_{t-1} \leq \underline{n}$ , the SNE must necessarily be unique and encompass  $v_t^* = 0$ . Furthermore, for values of  $n_{t-1}$  sufficiently close to 1, the SNE must also necessarily be unique (since  $\lim_{n \rightarrow 1} \Psi'_v = 0$ ); but comprising  $v_t^* > 0$  (because  $0 < \bar{n} < 1$ ).

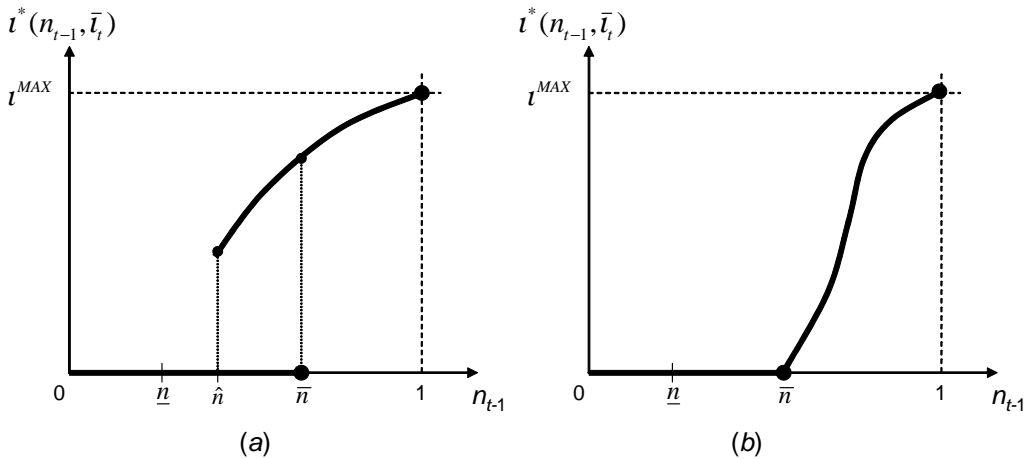


FIGURE 3: Inventors' Symmetric Nash Equilibrium.

<sup>19</sup>In what follows I restrict the analysis only to *stable* SNE though.

<sup>20</sup>A sufficient condition for *uniqueness* of SNE is that:  $\frac{\partial v_t^*}{\partial \bar{v}_t} = -\frac{\beta'(v_t^*) \Psi'_v}{\beta''(v_t^*) \Psi(n, \bar{v}_t)} < 1$ ,  $\forall n \in [0, 1]$  and  $\bar{v}_t \geq 0$ . Generally speaking, *uniqueness* requires innovators' external effects not to be too strong, so that the curves plotted on FIGURE 1.b do not ever cross the 45° line from below – see Cooper and John (1988).

*Remark.* Since the optimal R&D effort is a function of the bilateral surplus,  $\Delta(r_t^*)$ , which has been assumed to be fully appropriated by the inventor, all the previous results of this section in terms of  $\iota_i^* = \iota_i^*(n_{t-1}, \bar{\iota}_t)$  will remain unchanged if inventor  $i$  and entrepreneur  $i$  were in fact the *same* agent. All that is needed in that case is to reinterpret  $\iota_{i,t}$  as the R&D effort cost by entrepreneur  $i$  alive in period  $t$ .

## 5 Aggregate Dynamic Analysis

The analysis in Section 3 has been conducted within a static framework (the set  $\mathcal{A}_t$  was taken as given). Section 4 provides the bridge between the static and the dynamic analysis of the economy, since the inventors' behaviour determines the evolution of the set  $\mathcal{A}_t$  which, in turn, dictates the exact equilibrium that holds at any time  $t$  according to Definition 1. In this section, I present the dynamics of  $\mathcal{A}_t$ . Since agents are born with zero initial wealth and all sectors are (ex-ante) symmetric,  $n_t$  turns out to be the only variable whose behaviour we need to study in order to keep track of the dynamics of the economy.

**Definition 2 (Dynamic Equilibrium)** *A dynamic equilibrium is a sequence of static equilibria, linked together across time by the "law of motion" of  $n_t$  specified in (11).*

$$\mathbf{Law\ of\ Motion:} \quad n_t = n_{t-1} + \beta(\iota_t^*)(1 - n_{t-1}); \quad (11)$$

where  $\iota_t^*$  denotes the R&D effort by inventor  $h \in [0, 1]$  alive in  $t$  when sector  $h \notin \mathcal{A}_{t-1}$ , resulting from the SNE in Section 4.2.

### 5.1 Stagnation vs. Development (Multiple Dynamic Equilibria)

This subsection investigates the characteristics of the dynamic paths followed by economies that differ in terms of their initial conditions. In particular, it studies whether economies that differ in terms of  $n_0$  may follow divergent dynamic paths, reaching different long-run equilibria. For this reason, I impose here the following condition on the parameters configuration (so that the inventors' SNE will always be unique, leading to a situation as the one in FIGURE 2.b).

**Assumption 3 (sufficient condition for uniqueness of SNE).**

$$\frac{\partial \iota^*}{\partial \bar{\iota}} = -\frac{\beta'(\iota^*) \Psi'_i}{\beta''(\iota^*) \Psi(n, \bar{\iota})} < 1, \text{ for all } n \in [0, 1] \text{ and } \bar{\iota} \geq 0.$$

**Proposition 3 (Stagnation vs. Development)** *Suppose Assumptions 1 and 3 hold. Then:*

(i) *Any economy that starts off with  $n_0 \leq \bar{n}$  remains forever at  $n_0$  and displays no innovation activities. That is, if  $n_0 \leq \bar{n}$ , then:  $n_t = n_0$  for all  $t \geq 0$ , while  $\iota_t^* = 0$  for all  $t > 0$ .*

(ii) *In any economy in which  $n_0 > \bar{n}$ ,  $n_t$  will continuously grow over time, converging monotonically to  $n_\infty = 1$ .*

**Secular Stagnation:** Take an economy for which  $n_0 \leq \bar{n}$ . The equilibrium in  $t = 1$  encompasses  $\iota_1^* = 0$ . In addition to zero R&D effort and absence of innovations, this economy will exhibit highly inefficient credit provision and low levels of entrepreneurial investment. The credit market inefficiency is the consequence of severe adverse selection problems, which derive from the high degree of sector incompleteness. On the other hand, repressed entrepreneurship is the result of both lack of opportunities (few active sectors) and inadequate credit provision.

From (11), since  $\iota_1^* = 0$ , then  $n_1 = n_0$ . This implies that  $\iota_2^* = 0$  will hold again at  $t = 2$ , in turn leading to  $n_2 = n_1 = n_0$ . Furthermore, in the absence of any substantial exogenous shock, this stagnant equilibrium will perpetuate itself for all  $t \in \{0, 1, \dots, \infty\}$ .

**Prosperity and Development:** Consider now an economy in which  $n_0$  is large enough; more specifically,  $n_0 > \bar{n}$ . In this case, the equilibrium at  $t = 1$  displays  $\iota_1^* > 0$ . Intuitively, since  $n$  is relatively large, the adverse selection problem does not become too serious, and the operation of the economy does not turn out to be severely distorted (in particular, innovation activities do not completely disappear).

From (11),  $\iota_1^* > 0$  implies that some additional sectors become active during  $t = 1$ . As a result,  $n_1 > n_0 > \bar{n}$ , and  $\iota_2^* > \iota_1^* > 0$ . Moreover, this prosperous dynamics will perpetuate *ad infinitum*, and this economy will eventually reach a long-run equilibrium characterised by all sectors being active ( $n_\infty = 1$ ). During the transition period, the economy experiences development and growth; this manifests itself as a process of progressive sectoral differentiation and better sorting of entrepreneurial skills. At the same time, financial market operation concomitantly improves, as adverse selection problems tend to vanish as  $n_t$  rises.

## 5.2 History vs. Expectations (Multiple Static Equilibria)

Section 4.2 has shown that, within the range of  $n_{t-1} \in (\underline{n}, 1)$ , for some set of parameters configurations the model might display multiple SNE in the inventors game. As a particular example, in FIGURE 2.a, for  $n_{t-1} \in [\hat{n}, \bar{n}]$ , where  $\hat{n} \in (\underline{n}, \bar{n})$ , we find two possible (stable)

SNE. Multiplicity of the inventors' SNE will lead to multiplicity of static equilibria in this model. It is beyond the scope of this paper to study this sort of equilibrium multiplicity, as the main intention here is to analyse how dynamic paths may depend on the initial conditions. Nevertheless, I provide below a brief discussion of the equilibrium characteristics of an economy whose parameters configuration leads to a situation as the one depicted in FIGURE 2. *a*.

When parameters in the model lead to a situation as the one plotted in FIGURE 2. *a*, then if the value of  $n_0 \in [\hat{n}, \bar{n}]$ , this economy will be subject to multiple static equilibria. Equilibrium multiplicity will be driven by inventors' expectations. In particular, if expectations coordinate in  $\bar{\iota}_1 = 0$ , then  $\iota_1^* = 0$  will prevail. Besides this "bad" equilibrium, we can observe that there also exists some specific value  $\bar{\iota}_1^* > 0$ , which would lead to a "better" equilibrium comprising  $\iota_1^* = \bar{\iota}_1^* > 0$ . More importantly, from a dynamic perspective, whether expectations in  $t = 1$  lead to  $\iota_1^* = 0$  or  $\iota_1^* > 0$  may carry dramatic future consequences. Dynamically,  $\iota_1^* = 0$  entails that  $n_t$  stays stagnant during period  $t = 1$ ; as a result, initial conditions in  $t = 2$  would identically replicate those faced in  $t = 1$ , with the economy still at risk of suffering from coordination failures. On the other hand,  $\iota_1^* > 0$  means that  $n_1 > n_0$  and, consequently, this could possibly shoot up  $n_1$  above  $\bar{n}$ , and ignite a process of continuous prosperity and development thereafter. For an economy with  $n_{t-1} \in [\hat{n}, \bar{n}]$ , the larger  $n_{t-1}$  is, the higher the chances that  $n_t > \bar{n}$  will hold if  $\iota_t^* > 0$ . Hence, within  $[\hat{n}, \bar{n}]$ , both *history* and *expectations* matter in the sense of Krugman (1991), and the economy might display periods of growth and technical change, followed by periods of stagnation.

## 6 Incorporating Wealth into the Model

So far it has been supposed that all individuals are born with zero initial wealth. In many aspects this assumption might seem far too extreme. Nevertheless, the zero initial wealth assumption has allowed the model to completely isolate the impact of the fraction of active sectors on the operation of the economy.

In this section, I let agents be born with positive initial wealth; furthermore, I allow initial wealth to differ across individuals of the same cohort. Individuals are *warm-glow* altruistic and, accordingly, bequeath a fraction of their *net life-time income* to their offspring (this bequest will constitute the next generation's initial wealth) – see Andreoni (1989). In short, this section shows that none of the main results and insights presented earlier will be altered when we permit agents' initial wealth to be positive, stemming from parental bequests.

Let  $w_{i,t}$  denote the initial wealth of the type  $i$  alive in period  $t$ . Initial wealth is assumed publicly observable, and is distributed in the population of entrepreneurs according to the cumulative distribution function  $\Omega_t(w)$ .<sup>21</sup> Since types are assumed to be intergenerationally uncorrelated, then, in a steady state, initial wealth and types will turn out to be uncorrelated as well (accordingly, the specific value of  $w_{i,t}$  will provide no information about the  $i$ 's type).

## 6.1 The Participation Constraint

When initial wealth is positive we need to take care of the *participation constraint* ( $PC$ ) in the credit market. In particular, when  $w > 0$  a bad type might prefer not to engage in any credit market transaction, and behave as if he were in complete *autarky*, since he may now invest a positive amount of capital ( $k \leq w$ ) in a project, without the need to borrow.

Suppose a bad type with initial wealth  $w$  must choose his portfolio allocation in autarky. In such case, he will solve:

$$\max_{0 \leq k \leq w} : pf(k) + (w - k).$$

This optimisation problem yields the following investment policies: *i*)  $k^* = w$  if  $w \leq k_B^*$ , *ii*)  $k^* = k_B^*$  if  $w > k_B^*$ . Where  $f'(k_B^*) = p^{-1}$  (i.e.,  $k_B^*$  is the bad types' *first-best* investment level).

Imagine now that this bad type decides to participate in the credit market. In this case, he will invest  $k_P^*(r)$  units of capital in the project, paying an interest rate  $r$  on the borrowed amount ( $k_P^*(r) - w$ ); where  $r$  corresponds to the interest rate that would hold in a pooling equilibrium. The function  $k_P^*(r)$  stems from the first-order condition  $f'(k_P^*) = 1 + r$ ; analogous to (3) in the main model. Notice that  $1 + r \leq p^{-1}$ , hence  $k_P^*(r) \geq k_B^*$ .

A bad type will participate in the credit market only if his  $PC$  is not violated; this requires that:  $p[f(k_P^*(r)) - (1 + r)(k_P^*(r) - w)] \geq pf(k_B^*) + (w - k_B^*)$ , for  $w > k_B^*$ .<sup>22</sup> From this condition, it follows that a he will participate in the credit market if and only if his initial wealth does not surpass the threshold  $\hat{w}(r) \in (k_B^*, k_P^*(r))$ ; that is, if and only if  $w < \hat{w}(r)$ , where:

$$\hat{w}(r) \equiv \frac{p[f(k_P^*(r)) - f(k_B^*) - (1 + r)k_P^*(r)] + k_B^*}{1 - p(1 + r)}.$$

<sup>21</sup>The presence of positive initial wealth will only affect the equilibrium in the economy through its effect on the entrepreneurs. Accordingly, without any loss of generality, we can restrict the attention here only to the initial wealth distribution among the population of entrepreneurs.

<sup>22</sup>The participation constraint also requires that:  $p[f(k_P^*(r)) - (1 + r)(k_P^*(r) - w)] \geq pf(w)$ , for all  $w \leq k_B^*$ . Nevertheless, this last condition never binds.

## 6.2 The Incentive Compatibility Constraint

Take now an entrepreneur whose  $w \geq \widehat{w}(r)$ . If he is a good type, he must get a *separating* credit contract (paying an interest rate equal to  $R^f = 0$ ), as no bad type with  $w \geq \widehat{w}(r)$  desires to participate in the credit market at the (pooling) interest rate  $r$ . Despite that, a good type with  $w \geq \widehat{w}(r)$  will not necessarily obtain a *first-best* credit contract. For this to happen, an equally rich bad type should find no incentives to imitate the good-type first-best behaviour. Denote with  $k_G^*$  the result deriving from the first-order condition  $f'(k_G^*) = 1$ ; i.e.,  $k_G^*$  designates the first-best investment level of the good types. Notice that  $k_G^* \geq k_P^*(r)$ , because  $1+r \geq 1$ . A good type will thus receive a first-best credit contract if and only if:  $p[f(k_G^*) - (k_G^* - w)] < pf(k_B^*) + (w - k_B^*)$ . This last condition requires that his initial wealth is larger than the threshold  $\widetilde{w} \in (\widehat{w}(r), k_G^*)$ ; that is, it calls for  $w > \widetilde{w}$ , where:

$$\widetilde{w} \equiv \frac{p[f(k_G^*) - f(k_B^*) - k_G^*] + k_B^*}{1-p}.$$

What happens to a good type whose  $w \in [\widehat{w}(r), \widetilde{w}]$ ? This agent will certainly receive a separating contract. However, he won't be able to get a first-best contract, as this would violate the *incentive-compatibility constraint (IC)* of the bad types with identical  $w$ . In fact, the *IC* will bind for those entrepreneurs whose  $w \in [\widehat{w}(r), \widetilde{w}]$ . As a result, the credit contract received by a good type with  $w \in [\widehat{w}(r), \widetilde{w}]$  stems from the following condition:

$$p[f(k_S^*) - (k_S^* - w)] = pf(k_B^*) + (w - k_B^*). \quad (12)$$

Equation (12) (implicitly) yields a function  $k_S^*(w)$ ; which displays the following properties: (i)  $dk_S^*/dw = \frac{1-p}{p} (f'(k_S^*) - 1)^{-1} > 0$ , (ii)  $d^2k_S^*/(dw)^2 > 0$ , and (iii)  $\lim_{w \rightarrow \widetilde{w}} k_S^*(w) = k_G^*$ .

TABLE 1.A summarises the main features of the credit contracts offered to entrepreneurs.<sup>23</sup>

TABLE 1.A: Equilibrium Contracts (main features)

	$w < \widehat{w}(r)$	$w \in [\widehat{w}(r), \widetilde{w}]$	$w > \widetilde{w}$
type of credit contract	pooling	sub-optimal separating	first-best separating
investment by <i>good types</i>	$k_P^*(r)$	$k_S^*(w)$	$k_G^*$
interest rate (on credit)	$0 < r < \frac{1-p}{p}$	0	0

<sup>23</sup>The underlying reason why richer agents receive more favourable credit contracts is the same as in the literature on financial markets imperfections and poverty cited in the Introduction. Namely, since richer agents have more of their own wealth at stake in the projects, their incentives are more closely aligned to those of lenders.



### 6.3 Entrepreneurial Consumption and Sketch of Dynamics

As in Section 3.3, denote by  $\widehat{U}_g$  ( $U_b$ ) the expected utility level achieved by a good type (bad type). When initial wealth is incorporated into the model, it will naturally be the case that (expected) utility will depend on  $w$  as well – i.e.,  $U_g = U_g(r, w)$  and  $U_b = U_b(r, w)$ . TABLE 1.B summarises how entrepreneurial expected utility depends on  $w$  (and  $r$ ).

TABLE 1.B: Entrepreneurial Consumption –  $U_g(r, w)$  and  $U_b(r, w)$ .

	$w < \widehat{w}(r)$	$w \in [\widehat{w}(r), \widetilde{w}]$	$w > \widetilde{w}$
<i>good types</i>	$f(k_P^*(r)) - (1+r)(k_P^*(r) - w)$	$f(k_S^*(w)) - (k_S^*(w) - w)$	$f(k_G^*) - (k_G^* - w)$
<i>bad types</i>	$p[f(k_P^*(r)) - (1+r)(k_P^*(r) - w)]$	$pf(k_B^*) + (w - k_B^*)$	$pf(k_B^*) + (w - k_B^*)$

From the results presented in TABLE 1.B, this lemma follows.

**Lemma 4** *Let  $\Delta(r, w) \equiv U_g(r, w) - U_b(r, w)$ . Then: (i)  $\Delta(\cdot) > 0$ ,  $\forall w, r \geq 0$ ; (ii)  $\Delta'_r(\cdot) < 0$ ,  $\forall r \geq 0$  and  $w \in [0, \widehat{w}(r)]$ ; (iii) a)  $\Delta'_w(\cdot) > 0$ ,  $\forall w \in [0, \widetilde{w})$  and  $r \geq 0$ ; b)  $\Delta'_w(\cdot) = 0$ ,  $\forall w \geq \widetilde{w}$ .*

Lemma 4 represents the counterpart of Lemma 2, when entrepreneurs start their lives with positive wealth. On the one hand, Lemma 4 shows that Lemma 2's key result  $\Delta'_r(\cdot) < 0$  holds as well when  $w > 0$ . On the other hand, it shows that the surplus  $\Delta(\cdot)$  is (weakly) increasing in  $w$ , which implies that richer entrepreneurs benefit from a larger  $n_t$  more than poorer entrepreneurs do. Furthermore, recall that the larger  $\Delta(\cdot)$  is, the higher the incentives for inventors to undertake R&D (Lemma 3 and Proposition 2). Therefore,  $\Delta'_w(\cdot) > 0$  entails that, for a given value of  $n_t$  –which, following Proposition 1, will determine  $r^*(n_t)$ –, the aggregate distortions generated by the adverse selection problem in the credit market will become less severe the wealthier the economy is. FIGURE 3 plots the surplus  $\Delta(r, w)$  against  $w$  at four different values of  $r$  (namely:  $1/p > r_H > r_L > 0$ ), to illustrate Lemma 4.<sup>24</sup>

From a dynamic perspective, notice finally that economies exhibiting a larger  $n_t$  tend to be richer as well. This is the case because the larger the fraction of active sectors, the higher the average productivity in the economy. As a result, introducing wealth dynamics into the model (by means of bequests, or any other reason that would still generate positive serial correlation in  $w_t$ ) will not invalidate any of the main findings of this paper. In fact, as  $n_t$  and wealth affect the

<sup>24</sup>Recall  $r = p^{-1}$  when  $n = 0$ , and  $r = 0$  when  $n = 1$ . Additionally, notice  $\widehat{w}(r) < 0$ , where  $\lim_{r \rightarrow p^{-1}} \widehat{w}(r) = k_B^*$  and  $\lim_{r \rightarrow 0} \widehat{w}(r) = \widetilde{w}$ .

economy's performance in the same direction, the presence of bequests will actually reinforce the dynamics previously discussed in Section 5.

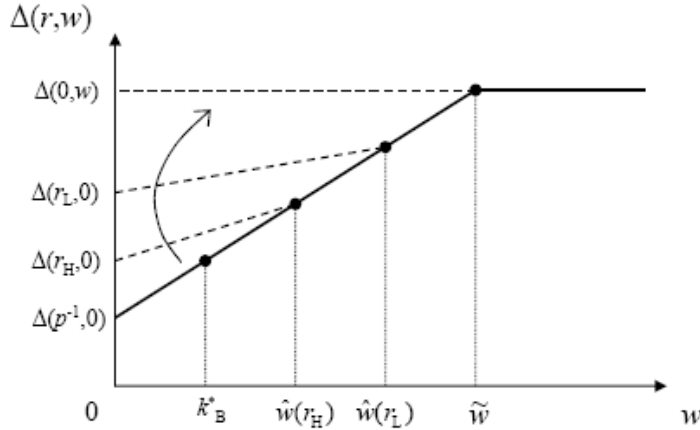


FIGURE 3:  $\Delta(r, w)$  against  $w$  at four different levels of  $r$ .

#### 6.4 Dynamics with Positive Bequests

Suppose preferences are given by  $U_{i,t} = c_{i,t}^{1-\delta} b_{i,t}^\delta$ , where  $c_{i,t}$  denotes the consumption of agent  $i$  alive in  $t$ ,  $b_{i,t}$  represents the bequest left to his offspring, and  $\delta \in (0, 1)$ . Given those preferences, individuals will optimally bequeath a fraction  $\delta$  of their lifetime income to their offspring. The amount  $b_{i,t}$  will in turn fully determine the initial wealth of  $i$ 's son; i.e.,  $w_{i,t+1} = b_{i,t}$ . Henceforth, we split the population of entrepreneurs in *lineages* indexed by the letter  $i \in [0, 1]$ . Since types are intergenerationally uncorrelated, the initial wealth transition equations for any lineage  $i$  of entrepreneurs are given by:

$$\begin{aligned}
 w_{i,t+1} &= \begin{cases} \delta [f(k_P^*(r_t)) - (1+r_t)(k_P^*(r_t) - w_{i,t})] & \text{with Pr} = n_t + p(1-n_t) \\ 0 & \text{with Pr} = (1-p)(1-n_t) \end{cases} & \text{if } w_{i,t} < \widehat{w}(r_t) \\
 w_{i,t+1} &= \begin{cases} \delta [f(k_S^*(w_{i,t})) - k_S^*(w_{i,t}) + w_{i,t}] & \text{with Pr} = n_t \\ \delta [f(k_B^*) - k_B^* + w_{i,t}] & \text{with Pr} = p(1-n_t) \\ \delta [w_{i,t} - k_B^*] & \text{with Pr} = (1-p)(1-n_t) \end{cases} & \text{if } w_{i,t} \in [\widehat{w}(r_t), \widetilde{w}] \\
 w_{i,t+1} &= \begin{cases} \delta [f(k_G^*) - k_G^* + w_{i,t}] & \text{with Pr} = n_t \\ \delta [f(k_B^*) - k_B^* + w_{i,t}] & \text{with Pr} = p(1-n_t) \\ \delta [w_{i,t} - k_B^*] & \text{with Pr} = (1-p)(1-n_t) \end{cases} & \text{if } w_{i,t} > \widetilde{w}
 \end{aligned}$$

When  $w$  is linked across generations by bequests, the dynamics of the economy can no longer be solely determined by the value of  $n_t$  but also depend on the initial wealth distribution  $\Omega_t(w)$ .

In particular, the economy's dynamic path is now dictated by the following system:

$$n_t = n_{t-1} + \beta(\iota_t^*)(1 - n_{t-1}) \quad (13)$$

$$\Omega_{t+1}(w) = \Gamma_t[\Omega_t(w)]. \quad (14)$$

Where:

$$\iota_t^* = \arg \max_{\iota} \left\{ \beta(\iota) \int_{\Omega_t(w)} \Delta(r_t, w) d\Omega_t(w) - \iota \right\}. \quad (15)$$

*Remark.* For this section we continue assuming that the NE of the inventors' game is always unique (or, alternatively, that coordination failures, even if possible, do not arise). Accordingly, from (15), we can write  $\iota_t^* = \iota_t^*(n_{t-1}, \Omega_t(w))$ , as the function that pins down the optimal  $\iota_t$ , given  $n_{t-1}$  and the initial wealth distribution  $\Omega_t(w)$ . (Recall, once again, that  $n_{t-1}$  determines  $n_t$  which in turn determines  $r_t$ ; hence we can write  $r_t$  as a function of  $n_{t-1}$ ).

The operator  $\Gamma_t[\cdot]$  maps the initial wealth distribution prevailing in period  $t$  into the initial wealth distribution holding in  $t+1$ , based on the transition equations specified above. Notice that this operator changes over time, since the transition equations and their associated occurrence probabilities both depend on the value of  $n_t$ . Additionally, the dynamic behaviour of  $n_t$  is affected by  $\Omega_t(w)$  through (15). These two features of the dynamic system described by (13) and (14) make it non-stationary and highly complicated to study. However, the most important general results can be proven without much difficulty.

**Lemma 5** (i) Consider two different initial wealth distributions  $\Omega_t(w)$  and  $\Omega'_t(w)$ , and suppose  $\Omega_t(w)$  first-order stochastically dominates  $\Omega'_t(w)$  – henceforth denoted as  $\Omega_t(w) \succeq \Omega'_t(w)$ . Then:  $\iota_t^*(n_{t-1}, \Omega_t(w)) \geq \iota_t^*(n_{t-1}, \Omega'_t(w))$ .

(ii) Consider two economies (A and B) with identical initial wealth distribution, i.e.  $\Omega_t^A(w) = \Omega_t^B(w) = \Omega_t(w)$ . Suppose also that  $n_t^A > n_t^B$ . Then:  $\Omega_{t+1}^A(w) \succeq \Omega_{t+1}^B(w)$ .

Lemma 5 (i) states that, all other things equal, wealthier economies tend to spend more in R&D, and its underlying intuition is straightforward from Lemma 4.<sup>25</sup> On the other hand, Lemma 5 (ii) says that economies with a larger fraction of active sectors tend to be richer too. The reason for this result lies in two combined effects: first, a higher  $n_t$  means that more agents are able to find a sector in which they have a comparative advantage, increasing the

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<sup>25</sup>Notice that given the shape of  $\Delta(r, w)$  as plotted in FIGURE 3, we cannot say much about the effect of higher moments of  $\Omega_t(w)$  on  $\iota_t^*$ . In particular, since  $\Delta(r, w)$  has initially a convex segment (with respect to  $w$ ), followed by a concave segment, the effect on  $\iota_t^*$  of subjecting  $\Omega_t(w)$  to a *mean-preserving spread* is ambiguous.

average productivity in the economy; second, a higher  $n_t$  leads to the provision of better credit contracts, spurring entrepreneurial investment. Lemma 5 thus formally proves that introducing wealth dynamics into the model (through bequests motives) reinforces the dynamics that have been described before in Section 5.

**Proposition 4** *Suppose Assumption 1 holds, where we should now interpret  $\Psi(n_{t-1}, l_t^*) = \Delta(r_t^*(n_{t-1}, l_t^*), 0)$ , and let  $\Omega_{\tilde{w}}$  ( $\Omega_0$ ) denote the degenerate distribution function in which  $w_i = \tilde{w}$  ( $w_i = 0$ ) for all  $i \in [0, 1]$ . Then:*

- (i) *If  $n_{t-1} > \bar{n}$ ,  $n_t$  will converge monotonically to  $n_\infty = 1$ , regardless of  $\Omega_t(w)$ .*
- (ii) *Suppose  $\Omega_t(w) = \Omega_{\tilde{w}}$ . Then, there exists  $\bar{n}_{\tilde{w}} < \bar{n}$  such that if  $n_{t-1} > \bar{n}_{\tilde{w}}$ ,  $n_t$  will converge monotonically to  $n_\infty = 1$*
- (iii) *Suppose  $\Omega_0 \preceq \Omega_t(w) \preceq \Omega_{\tilde{w}}$ . Then,  $\exists \bar{n}_{\Omega(w)} \in [\bar{n}_{\tilde{w}}, \bar{n}]$  such that if  $n_{t-1} > \bar{n}_{\Omega(w)}$ ,  $n_t$  will converge monotonically to  $n_\infty = 1$ . Furthermore, consider  $\Omega_t(w) \succeq \Omega'_t(w)$ , then  $\bar{n}_{\Omega(w)} \leq \bar{n}_{\Omega'(w)}$ .*

Proposition 4 firstly shows that the main result in Proposition 3 still holds true when we incorporate standard wealth dynamics into the model – when  $n_t$  is sufficiently large, the economy embarks in a process of long-run development, regardless of the wealth distribution in  $t$ . Secondly, it shows that initial wealth acts as a *partial substitute* for  $n_t$ . This last result stems from the fact that both  $n_t$  and  $w_t$  contribute to alleviate adverse selection problems in the credit market. Notice that Proposition 4 (ii) and (iii) imply that the minimum degree of sectoral variety needed to guarantee long-run growth turns out to be smaller the richer the economy is. This result can be interpreted as saying that the importance of sectoral diversification as a factor improving the operation of financial markets is relatively higher at initial stages of development, and tends to decrease as the economy develops and becomes wealthier.

## 7 Further Discussion: Some Stylised Facts in the Data

This section discusses some empirical patterns and observations that are consistent with the main predictions of the model. In particular, it focuses on three main predictions of the model and it poses the following questions:

- (i) Does the variety of sectors grow along the path of development? In addition, is sectoral diversification more pronounced at early stages of development (as Section 6 would suggest)?
- (ii) Are sectoral diversification and financial deepening positively correlated? In addition, does the magnitude of this correlation change as development progresses (as Section 6 would suggest)?

(iii) As the economy grows and the variety of sectors expands, does income volatility decline as the improved allocation of talents would predict?

## 7.1 Sectoral Diversification and Development

The key anecdotal observation that motivates this paper and, at the same time, one of the main predictions of the model is that the variety of sectors increases as economies grow and develop.

For a panel of 67 countries, Imbs and Wacziarg (2003) show that *sectoral concentration* (the opposite of *sectoral diversification*) drastically falls at early stages of development, following a *U-shaped* relationship with respect to income per head.<sup>26</sup> They conclude that, along the development path, economies initially experience a long process of sectoral diversification, which eventually reaches a maximum beyond where the process begins to revert.

FIGURE 4, which relies on Imbs and Wacziarg’s dataset, presents an overview of the association between sectoral diversification and income per head. (See also figures 1, 2 and 3 in their paper, p. 69.) Sectoral concentration is measured by their Gini coefficients for employment shares based on the UNIDO 3-digit dataset; a smaller Gini coefficient thus reflects a more diversified economy in terms of manufacturing industries. Income per head is measured by GDP per capita in thousands of PPP 1985 US dollars (from Summers and Heston (1991)).

To allow for the possibility of a non-monotonic relationship, I run a fifth-order polynomial regression. I also show the results of a quadratic regression. Both regressions additionally control for country fixed effects. We can observe the pattern described in Imbs and Wacziarg: sectoral concentration initially decreases with income, eventually reaching a turning point beyond which the relationship partially reverts.

Given the implications of the model, two key observations need to be stressed here: (i) the *turning point* in the diversification process tends to occur at relatively high levels of income per capita (the authors argue that this point is located roughly at the income per head of Ireland in 1992); (ii) the eventual re-concentration process *only* partly offsets the effect of the initial diversification phase.

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<sup>26</sup>Imbs and Wacziarg (2003) use the non-parametric *lowess* technique to capture the association between sectoral concentration and income per capita. They build five different concentration indices based on employment shares (Gini coefficient, Herfindahl index, log-variance of sector shares, coefficient of variation, and the max-min spread). These indices are constructed for three different datasets: 1-digit level (9 sectors) from the International Labor Office (ILO), 3-digit level (28 sectors) from the United Nations Industrial Development Organization (UNIDO), and 2-digit level (20 sectors) from the OECD. For the UNIDO and OECD datasets, value added per sector is also available and utilized. All their results are robust to the use of different indices and datasets.

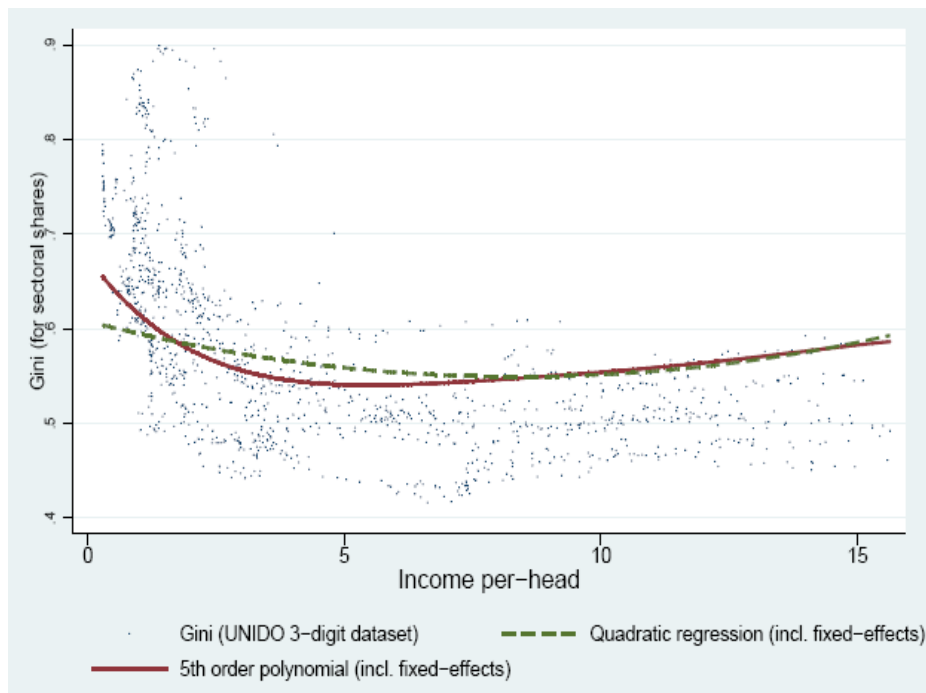


FIGURE 4: Sectoral Concentration and Income Per Head.

The Gini coefficient in the picture measures the degree of sectoral concentration in the economy in terms of employment shares across 28 manufacturing sectors (3-digit level disaggregation, UNIDO dataset).

### 7.1.1 The Non-monotonic Diversification Path

One of the most interesting findings in Imbs and Wacziarg (2003) is the non-monotonic relationship between income and sectoral diversification. Although, strictly speaking, this paper does not predict such non-monotonic relationship, the results presented in Section 6 can yet help shedding some light on the possible reasons leading to this non-monotonicity. In particular, that section has shown that sectoral variety is *most* relevant to alleviate informational frictions at early stages of development. As the economy grows and accumulates wealth, the use of collateral can substitute for the improved self-selection of skills allowed by sectoral variety. As a consequence, if there exist also gains from regional specialisation (such as increasing returns to scale), at some point in the development path, economies might find it worthwhile to sacrifice some degree of sectoral variety in order to better exploit increasing returns to scale, leading to a non-monotonic path as that in FIGURE 4.

## 7.2 Financial Development and Sectoral Diversification in the Data

Another important prediction of the model is the positive feedback between the degree of sectoral diversification and the level of development of financial markets. This feedback implies that those two variables should display positive correlation in cross-country data.

In this subsection, I present some evidence of this correlation for an unbalanced panel of countries during years 1975-92. I consider three different indicators traditionally used in the literature of financial deepening and growth: 1) the logarithm of the ratio of private credit by financial institutions to GDP,  $\text{Log}(\text{Credit}/\text{GDP})$ ; 2) the logarithm of the ratio of stock market capitalisation to GDP,  $\text{Log}(\text{SMK}/\text{GDP})$ ; 3) the logarithm of the ratio of stock market value traded to GDP,  $\text{Log}(\text{SMVT}/\text{GDP})$ .<sup>27</sup> To measure the degree of sectoral concentration, I follow again Imbs and Wacziarg (2003) and use the Herfindahl indices for the employment shares across the 28 manufacturing sectors in the UNIDO 3-digit dataset.<sup>28</sup> The summary statistics in TABLE 2 indicate that there is substantial variation in the variables.

In columns (1), (3) and (5) in TABLE 3, each of the financial development indicators is regressed against the sectoral concentration index. Each of the regression equations additionally controls for country fixed-effects and GDP per capita. Country fixed-effects are included so as to track individual economies over their own path of development. GDP per head controls for the fact that financial indicators and sectoral diversification might be moving together just as consequence of income shocks affecting both variables simultaneously.

From columns (1) and (3) we can observe that the estimated coefficient for the Herfindahl exhibits the expected negative sign, being also significant. According to those two regressions, sectoral diversification is positively and significantly correlated with financial development *within* each country, even after controlling for the possibility of common income shocks.

When the stock market value traded to GDP ratio is used as a proxy for financial development in column (5), the estimate turns out to be insignificant and displays the opposite sign. In that regard, it can be argued that the ratio of credit to GDP and the ratio of stock market

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<sup>27</sup>All data on financial indicators is taken from Beck *et al* (1999). Refer to this paper for a detailed description of those indicators.

<sup>28</sup>The reason why we are using here the Herfindahl instead of the Gini to measure sectoral concentration is that the former displays more variability than the latter, so it permits a more precise estimation of the coefficients in TABLE 3. In particular, the coefficient associated to the *interaction term* in TABLE 3 cannot be precisely estimated if using the Gini, while this is not the case if using the Herfindahl. To have an idea of the problem, the correlation between the interaction term ( $Y \times \text{Gini}$ ) and  $Y$  is 0.98, while the correlation between ( $Y \times \text{Herfindahl}$ ) and  $Y$  is 0.74.

capitalisation to GDP are better proxies for the level of financial development than the ratio of stock market value traded to GDP, which seems more to account for the liquidity of the stock market rather than for the size of it.

Section 6 has shown that the *wealth effect* operates in the same direction as an expanding variety of sectors. Therefore, for a given degree of diversification, richer economies would tend to suffer from less severe adverse selection, displaying accordingly higher financial development. This implies that sectoral diversification should play a more important role in poorer economies compared to richer ones. In order to capture evidence consistent with the presence of this effect, regressions (2), (4) and (6) in TABLE 3 additionally include an *interaction term* between the log of GDP per head and the degree of sectoral concentration,  $Y \times Herfindahl$ . All the estimates for the interaction term display the expected positive sign, being also highly significant. Furthermore, in column (6), including the interaction term turns the coefficient associated to the Herfindahl index negative, as predicted by the model (although it still remains statistically insignificant).<sup>29</sup>

Lastly, the results in TABLE 3 only aim at eliciting a positive correlation between sectoral diversification and financial development, even after controlling for some additional covariates. A stronger result is present in Ramcharan (2006), who shows a positive and significant causal effect of sectoral diversification on financial deepening, using countries' topographical characteristics to instrument for diversification.

**TABLE 2: Summary Statistics**

Dependent Variable: Log(Cred/GDP) - 1212 observations						Dependent Variable: Log(SMK /GDP) - 484 observations					
Variable	Mean	Median	Std. Dev.	Min	Max	Variable	Mean	Median	Std. Dev.	Min	Max
Log(Credit/GDP)	-1.346	-1.275	1.194	-15.12	0.554	Log(SMKT/GDP)	-2.123	-2.028	1.377	-7.13	0.339
Log Income per head	1.33	1.40	0.98	-1.24	3.00	Log Income per head	1.72	1.83	0.81	-0.15	2.90
Herfindahl Index	0.118	0.086	0.098	0.060	0.877	Herfindahl Index	0.092	0.079	0.055	0.060	0.450

Dependent Variable: Log(SMVT/GDP) - 534 observations					
	Mean	Median	Std. Dev.	Min	Max
Log(SMVT/GDP)	-4.065	-3.719	2.103	-10.78	1.867
Log Income per head	1.74	1.85	0.78	-0.21	2.90
Herfindahl Index	0.087	0.077	0.042	0.060	0.423

Note: Log income per head equal to -1.24 corresponds to income per head 290 in 1985 PPP US dollars (this is the income per head in PPP of Ethiopia in 1967).  
Log income per head equal to 2.90 corresponds to income per head 18,095 in 1985 PPP US dollars (this is the income per head in PPP of US in 1989).

<sup>29</sup>Using five-year intervals for the fixed-effects regressions does not significantly alter the pointwise estimates reported in TABLE 3, although their precision naturally falls due to the lower number of observations. The results are also quite robust to the inclusion of a linear time trend on the level of financial development (the coefficient on the time trend is always positive and significant). These results are available from the author upon request.



**TABLE 3: Sectoral Diversification and Financial Development**

Independent Variable	Dependent Variable					
	Log(Credit/GDP)		Log(SMK/GDP)		Log(SMVT/GDP)	
	(1)	(2)	(3)	(4)	(5)	(6)
Sectoral Concentration (Herfindahl)	-0.783*** (0.285)	-1.122*** (0.294)	-3.45* (1.93)	-4.19** (1.93)	5.78 (3.84)	-1.164 (3.11)
Log Income per head ( $Y$ )	0.841*** (0.047)	0.627*** (0.062)	2.89*** (0.19)	2.10*** (0.34)	4.66*** (0.363)	2.85*** (0.673)
$Y \times$ Herfindahl (interaction term)		1.924*** (0.421)		8.00*** (2.62)		19.39*** (5.74)
R squared (within)	0.21	0.23	0.42	0.44	0.29	0.30
Obs. / Countries	1212 / 57	1212 / 57	484 / 36	484 / 36	534 / 40	534 / 40

Note: robust standard errors parentheses. All regressions include an intercept and country fixed-effects.

Regressions are run on an unbalanced panel during 1975-92. Log(SMVT/GDP) is the log of Stock Market Value Traded to GDP.

Log(Cred/GDP) is the logarithm of Total Private Credit to GDP. Log(SMK/GDP) is the log of Stock Market Capitalization to GDP.

Log Income per head is the log of GDP per head in PPP in 1,000 of 1985 US dollars from Summers and Heston (1991).

The Herfindahl coefficients are based on the UNIDO 3-digit employment dataset from Imbs and Wacziarg (2003)

\* significant at 10% level, \*\* significant at 5% level, \*\*\* significant at 1% level.

### 7.3 Allocation of Talent, Growth and Income Volatility

One other main prediction of the model is that the allocation of skills improves during the process of development. This feature is consistent with the evidence that income volatility falls as output per capita rises documented, for example, by Ramey and Ramey (1995) and Acemoglu and Zilibotti (1997, section II).

Even more compelling for the prediction of improved allocation of talents, is the cross-country evidence using sectoral data in Koren and Tenreyro (2007). They point out that a significant part of the volatility differential between rich and poor countries is due to lower volatility *within* sectors in the former compared to the later.<sup>30</sup> This is exactly what the model predicts. More precisely, as the variety of activities expands, allowing improved matching of skills to activities, the sectoral failure risk should decline accordingly. In other words, better assignment of skills translates into a higher rate of entrepreneurial success, which in turn reduces output volatility within sectors (and, as a consequence, in the aggregate economy too).

The model then predicts that income volatility within activities (or firms) decreases during

<sup>30</sup>Koren and Tenreyro (2007), page 271, estimate that between 40% and 41% of the difference in total income volatility between countries in the top 5 percentile versus those in the bottom 5 percentile is attributable to what they call *idiosyncratic sectoral risk*. This component captures the volatility of a *specific* sector in a *specific* country (for example, if apparel is particularly volatile in country  $x$  but not in country  $z$ , this will be reflected in this component, after weighting that volatility by the employment share in the apparel industry in each country).

development. This differs from Acemoglu and Zilibotti (1997), where volatility within activities actually rises over the process of development because agents choose to take on larger risks when they have access to more efficient financial markets. More significantly, my paper predicts that an exogenous increase in the variety of sectors (for example after a countries open up to trade) would lead to lower income variability within sectors, which would in turn lower the cost of financial intermediation and expand financial transactions. Therefore, lower volatility might arise purely as the result of technological diversification, and without any (exogenous) improvements in the financial markets. This is in line with the evidence in Koren and Tenreyro (2008), who found that the negative correlation between volatility and development takes place at all levels of financial deepening.

## 8 Concluding Remarks

This paper has proposed a theory in which the efficiency of financial markets is a key condition for growth and development. I have suggested that an expanding variety may be an important factor leading to financial development. In particular, this theory has stressed a side effect associated with the innovation process that had not been explored before, but which could exert a significant impact on development. Innovation activities can lead to a reduction of frictions in the financial markets and foster financial development, because by expanding the variety of productive activities, they concomitantly facilitate the allocation of skills, alleviating adverse selection problems.

The core model that illustrates this theory has made use of several simplifying assumptions. One assumption that may seem particularly worrying is the fact that individuals are born with no initial wealth. In that regard, Section 6 has shown that none of the model's main findings would be affected if we let agents be born with positive wealth. Despite not altering its main results, introducing wealth may carry some interesting additional implications within a more general model. Imagine that we gave room for increasing returns to scale and international trade. If sectoral diversification really matters as a mechanism to solve adverse selection only at early stages of development (as suggested by Section 6), then in the presence of increasing returns and trade, at some point in the development path, economies might find it worthwhile to reverse the diversification tendency and start re-specialising in some specific sectors. This feature would in fact be consistent with the evidence found by Imbs and Wacziarg (2003), providing a sound explanation for the *non-monotonic* relation between sectoral diversification and income

per head shown initially in FIGURE 4.

Another feature that deserves further discussion is the behaviour of financial intermediaries. In the model, financiers respond "passively" to the environment. However, it can be argued that the operation of the financial system improves during development not only because frictions are alleviated, but also because the screening capacity of the financiers gets better. The paper has abstracted from the latter mechanism. One remark concerning this omission is worth noting, though. The amount of screening effort is an endogenous choice, and it will certainly be influenced by the cost of screening. This paper states that screening effort is eased by sectoral variety, as this allows heterogeneous agents to self-select better. However, this does not necessarily imply that richer economies should conduct less credit screening than poorer ones. In fact, as sectoral variety decreases the cost of screening, in some cases, more screening effort could be the optimal response by lenders to the new environment, rather than simply denying credit so as to avoid the screening cost fully.

From a policy perspective, an important implication concerns poverty-alleviation programmes. Section 5 has shown that some economies might get stuck in a peculiar type of poverty trap. This is the result of a "deep-rooted" organisational failure, affecting several markets at the same time. Underdevelopment is characterised by few sectors in which individuals can specialise, inefficient financial markets, and scant innovation activities. The market failure contaminating the operation of the economy stems from the incapacity of some individuals to find an activity for which they are comparatively talented. Most theories on poverty traps imply that economies can be easily rescued from poverty by receiving a sufficiently large wealth transfer. In contrast, my theory suggests that foreign aid should presumably also include important transfers of technology and know-how, as standard wealth transfers alone might not suffice to suppress the adverse selection problem (at least in a reasonably short time frame).

## Appendix: Proofs

**Proof of Lemma 1.** Take two different credit contracts  $(l^*, r^*) \in \mathbb{R}^+ \times \mathbb{R}^+$  and  $(\tilde{l}, \tilde{r}) \in \mathbb{R}^+ \times \mathbb{R}^+$ , such that  $f'(k = l^*) \geq 1$  and  $f'(k = \tilde{l}) \geq 1$ .<sup>31</sup> Hence, in equilibrium, all the amount that is borrowed will be invested in the entrepreneurial projects. Accordingly, let's denote:  $k^* = l^*$  and

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<sup>31</sup>It must be straightforward to notice that entrepreneurs only borrow in order to finance entrepreneurial investment. Therefore, in equilibrium, they would never borrow beyond the point  $f'(k) = 1$ .

$\tilde{k} = \tilde{l}$ . Assume that:

$$f(k^*) - (1 + r^*)k^* > f(\tilde{k}) - (1 + \tilde{r})\tilde{k} \quad (\text{L.1.1})$$

Then, from (L.1.1), if type  $i$  decides to specialise in sector  $i \in \mathcal{A}$ , he will prefer contract  $(k^*, r^*)$  to contract  $(\tilde{k}, \tilde{r})$ .

Take now type  $j$ . Since sector  $j \notin \mathcal{A}$ , he will specialise (indifferently) in any sector  $h \in [0, 1]$ , such that sector  $h \in \mathcal{A}$ . Given *limited liability*, type  $j$  will (weakly) prefer contract  $(\tilde{k}, \tilde{r})$  to contract  $(k^*, r^*)$ , if and only if:

$$p[f(\tilde{k}) - (1 + \tilde{r})\tilde{k}] \geq p[f(k^*) - (1 + r^*)k^*] \quad (\text{L.1.2})$$

But, since  $p > 0$ , (L.1.2) contradicts (L.1.1). Hence, it cannot be true that, while type  $i$  prefers contract  $(k^*, r^*)$  to contract  $(\tilde{k}, \tilde{r})$ , type  $j$  prefers  $(\tilde{k}, \tilde{r})$  to  $(k^*, r^*)$  instead. Finally, since  $(\tilde{k}, \tilde{r})$  and  $(k^*, r^*)$  can be any credit contracts; whenever type  $i$  prefers  $(k^*, r^*)$  to  $(\tilde{k}, \tilde{r})$ , then type  $j$  also prefers  $(k^*, r^*)$  to  $(\tilde{k}, \tilde{r})$ , and no equilibrium can possibly encompass separating credit contracts among those two types. ■

**Proof of Proposition 1.** The expression in (6) follows from the previous discussion in Section 3.2. Then, differentiating (6) with respect to  $n_t$ :  $dr_t^*/dn_t = -(1 - p)[n_t + (1 - n_t)p]^{-2} < 0$ . ■

**Proof of Lemma 3.** Assume that the inventor  $i \in [0, 1]$  alive in  $t$  expends  $\iota_i$  units of effort. If he manages to generate a new idea, then from Lemma 2 it should be straightforward that he will optimally charge a price equal to  $\Delta(r_t^*)$  to transfer the idea (to the type  $i$ ) - this is the maximum price the inventor  $i$  could charge, while the type  $i$  is still willing to buy the new idea. Making use of Proposition 1, we can write  $\Delta(r_t^*) = \Delta(r^*(n_t)) \equiv \tilde{\Delta}(n_t)$ , where  $\tilde{\Delta}'(n_t) = \Delta'(\cdot) \frac{dr_t^*}{dn_t} > 0$  (from Proposition 1 and Lemma 2). How is the value  $n_t$  determined? Suppose all inventors belonging to  $-\mathcal{A}_{t-1}^i$  choose  $\bar{\iota}_t$ . Since active sectors in  $t - 1$  never revert to inactive in  $t$ , and recalling (9), then:

$$n_t = n_{t-1} + (1 - n_{t-1})\beta(\bar{\iota}_t) \equiv \Phi(n_{t-1}, \bar{\iota}_t) \quad (\text{L.3.1})$$

Notice that, because  $\beta(\bar{\iota}_t)$  is bounded away from 1, (L.3.1) implies  $\Phi(\cdot)$  is increasing in both  $n_{t-1}$  and  $\bar{\iota}_t$ . Now, plugging  $\Phi(\cdot)$  from (L.3.1) into  $\tilde{\Delta}(n_t)$ , we can rewrite  $\tilde{\Delta}(\Phi(n_{t-1}, \bar{\iota}_t)) \equiv \Psi(n_{t-1}, \bar{\iota}_t)$ . From where it follows that: (i)  $\Psi'_n = \tilde{\Delta}'(n_t)(1 - \beta(\bar{\iota}_t))n_{t-1} > 0$ ; (ii)  $\Psi'_{\bar{\iota}} = \tilde{\Delta}'(n_t)(1 - n_{t-1})\beta'(\bar{\iota}_t)$ , which leads to  $\Psi'_{\bar{\iota}} > 0$  if  $n_{t-1} \in [0, 1)$  and  $\Psi'_{\bar{\iota}} = 0$  if  $n_{t-1} = 1$ .

Finally, noting that having exerted effort  $\iota_i$ , inventor  $i$  will succeed in generating a new idea with probability  $\beta(\iota_i)$ , we may write:  $\Pi_{i,t}(\iota_i, n_{t-1}, \bar{\iota}_t) = \beta(\iota_i) \cdot \Psi(n_{t-1}, \bar{\iota}_t) - \iota_i$ ; which is the expression stipulated in Lemma 3. ■

**Proof of Proposition 2.** Part 1). Consider two values of  $n_{t-1}$ ;  $n_0, n_1 \in [0, 1]$ , such that  $n_0 < n_1$ . Denote:  $\iota_0^* \equiv \iota_i^*(n_0, \bar{\iota}_t)$  and  $\iota_1^* \equiv \iota_i^*(n_1, \bar{\iota}_t)$ ; where  $\bar{\iota}_t \geq 0$ . Finally, suppose  $\iota_0^* > \iota_1^*$ . Thus, from (10), it follows that:

$$\beta'(\iota_1^*)\Psi(n_1, \bar{\iota}_t) \leq \beta'(\iota_0^*)\Psi(n_0, \bar{\iota}_t). \quad (\text{P.2.1})$$

Since,  $\beta''(\iota) < 0$ , when  $\iota_0^* > \iota_1^*$ ,  $\beta'(\iota_0^*) < \beta'(\iota_1^*)$  must then hold. As a result, (P.2.1) necessarily requires that:  $\Psi(n_0, \bar{\iota}_t) > \Psi(n_1, \bar{\iota}_t)$ , which contradicts  $\Psi'_{n_{t-1}} > 0$  for all  $\bar{\iota}_t \geq 0$  proved in Lemma 3. Consequently,  $n_0 < n_1 \Rightarrow \iota_0^* \leq \iota_1^*$ .

Part 2). Take two values of  $\bar{\iota}$ ;  $\bar{\iota}_a, \bar{\iota}_b \in \mathbb{R}^+$ , such that  $\bar{\iota}_a > \bar{\iota}_b$ . Denote:  $\iota_a^* \equiv \iota_i^*(n_{t-1}, \bar{\iota}_a)$  and  $\iota_b^* \equiv \iota_i^*(n_{t-1}, \bar{\iota}_b)$ ; where  $n_{t-1} \in [0, 1]$ . Finally, suppose  $\iota_a^* < \iota_b^*$ . Then, from (10), it follows that:

$$\beta'(\iota_a^*)\Psi(n_{t-1}, \bar{\iota}_a) \leq \beta'(\iota_b^*)\Psi(n_{t-1}, \bar{\iota}_b). \quad (\text{P.2.2})$$

In addition to that,  $\beta''(\iota) < 0$  implies that, if  $\iota_a^* < \iota_b^*$ , then  $\beta'(\iota_a^*) > \beta'(\iota_b^*)$ . As a result, (P.2.2) necessarily requires:  $\Psi(n_{t-1}, \bar{\iota}_a) < \Psi(n_{t-1}, \bar{\iota}_b)$ , which contradicts  $\Psi'_{\bar{\iota}} > 0$  for all  $n_t \in [0, 1]$  (and  $\Psi'_{\bar{\iota}} = 0$  when  $n_t = 1$ ), proved in Lemma 3. Therefore,  $\bar{\iota}_a > \bar{\iota}_b \Rightarrow \iota_a^* \geq \iota_b^*$ . ■

**Proof of Corollary 1.** (i) Since, from Lemma 3,  $\Psi'_n(\cdot) > 0$ , setting  $\bar{\iota}_t = 0$  we obtain:

$$\beta'(0)\Psi(n_{t-1}, 0) \leq \beta'(0)\Psi(\bar{n}, 0) = 1, \quad \forall n_{t-1} \leq \bar{n}. \quad (\text{C.1.1})$$

Thus, given  $\beta''(\iota) < 0$  and the conditions stated in (10), (C.1.1) entails that  $\iota_i^* = 0$  must necessarily prevail for any value of  $n_{t-1} \leq \bar{n}$  when  $\bar{\iota}_t = 0$ .

(ii) Since  $\Psi'_n(\cdot) > 0$ , it follows that:

$$\beta'(0)\Psi(n_{t-1}, 0) > \beta'(0)\Psi(\bar{n}, 0) = 1, \quad \forall n_{t-1} > \bar{n}. \quad (\text{C.1.2})$$

Therefore, given  $\beta''(\iota) < 0$ , (C.1.2) implies that  $\iota_i^* > 0$  must necessarily hold for any  $n_{t-1} > \bar{n}$  when  $\bar{\iota}_t = 0$ , so that to comply with (10). Finally, since  $\Psi'_{\bar{\iota}}(\cdot) \geq 0$ ,

$$\beta'(0)\Psi(n_{t-1}, \bar{\iota}_t) \geq \beta'(0)\Psi(n_{t-1}, 0) > \beta'(0)\Psi(\bar{n}, 0) = 1, \quad \forall n_{t-1} > \bar{n} \text{ and } \bar{\iota}_t > 0.$$

Hence, in order to comply with (10),  $\iota_i^* > 0$  must hold for all  $n_{t-1} > \bar{n}$  and  $\bar{\iota}_t \geq 0$ . ■

**Proof of Corollary 2.** Since  $\Psi'_{\bar{\iota}}(\cdot) \geq 0$ , then:  $\beta'(0)\Psi(n_{t-1}, \infty) \geq \beta'(0)\Psi(n_{t-1}, \bar{\iota}_t)$ , for all values of  $\bar{\iota}_t \geq 0$  and  $n_{t-1} \in [0, 1]$ . As a result, if  $\beta'(0)\Psi(\bar{n}, \infty) = 1$ , it must be the case that:

$$\beta'(\iota_i)\Psi(n_{t-1}, \bar{\iota}_t) \leq \beta'(0)\Psi(n_{t-1}, \infty) \leq 1, \quad \forall n_{t-1} \leq \bar{n}, \text{ and } \iota_i, \bar{\iota}_t > 0. \quad (\text{C.2.1})$$

Thus, given (10), from (C.2.1) it follows  $\iota_t^* = 0$  must hold for all  $n_{t-1} \leq \bar{n}$  and  $\bar{t}_t \geq 0$ . ■

**Proof of Proposition 3.** (i) Take an economy in which  $n_0 \leq \bar{n}$  and focus on equilibrium  $t = 1$ . Given Assumption 1, Corollary 1 implies there must exist a SNE for the inventors game in which  $\iota_1^* = 0$ . On the other hand, Assumption 3 entails that this SNE is unique. Since  $\beta(0) = 0$ , then (11) implies that  $n_1 = n_0 \leq \bar{n}$ . As a result, in  $t = 2$  conditions for the inventors game remain identical as they were at  $t = 1$ ; thus,  $\iota_2^* = 0$  represents again the unique SNE in  $t = 2$ . Repeating the same argument *ad infinitum*, it follows that:  $n_t = n_0 \forall t \geq 0$  and  $\iota_t^* = 0 \forall t > 0$ .

(ii) Take an economy where  $n_0 > \bar{n}$  and focus on  $t = 1$ . Given Assumption 1, Corollary 1 implies that  $\iota_1^*(n_0, 0) > 0$ . As a result, there must necessarily exist a SNE for the inventors game in  $t = 1$  in which  $\iota_1^* > 0$ . Given Assumption 3, then this  $\iota_1^* > 0$  represents the unique SNE. Since  $\iota_1^* > 0$ , from (11) it follows that  $n_1 = n_0 + \beta(\iota_1^*)(1 - n_0)$ ; hence,  $n_1 > n_0$ . In particular, this leads to  $n_1 > n_0 > \bar{n}$ . Proposition 2 then implies that  $\iota_2^* > \iota_1^* > 0$ . As a result of this,  $n_2 > n_1$ . Repeating this argument *ad infinitum*, we can observe that:  $\bar{n} < n_0 < n_1 < n_2 < \dots < n_\infty$ . Furthermore, since  $\beta(\iota_t^*)(1 - n_{t-1}) \rightarrow 0$  as  $n_t \rightarrow 1$ , and because  $\beta(\iota_t^*)(1 - n_{t-1})$  is bounded away from zero for any  $n_{t-1} \in [0, 1)$  and  $\iota_t^* > 0$ ; then it follows that  $\lim_{t \rightarrow \infty} n_t = 1$ . ■

**Proof of Lemma 4.** Available from the author upon request.

**Proof of Lemma 5.** (i) The expression in (15) yields the first-order condition:

$$\beta'(\iota_t^*) \int_{\Omega_t(w)} \Delta(r_t, w) d\Omega_t(w) = 1. \quad (\text{L.5.1})$$

Since  $r_t$  is a decreasing function of  $n_t$ , and  $n_t$  is an increasing function of  $n_{t-1}$  for all  $n_{t-1} \in [0, 1)$ ; restating  $\Delta(r_t, w) \equiv \Lambda(n_{t-1}, w)$ , (L.5.1) can be rewritten as:

$$\beta'(\iota_t^*) \int_{\Omega_t(w)} \Lambda(n_{t-1}, w) d\Omega_t(w) = 1, \quad (\text{L.5.2})$$

where  $\partial\Lambda/\partial n_{t-1} > 0$  for all  $n_{t-1} \in [0, 1)$ , and  $\partial\Lambda/\partial w = \partial\Delta/\partial w \geq 0$  (Lemma 4). As a result, from (L.5.2) it follows that if  $\Omega_t(w) \succeq \Omega_t'(w)$ , then:  $\int_{\Omega_t(w)} \Lambda(n_{t-1}, w) d\Omega_t(w) \geq \int_{\Omega_t'(w)} \Lambda(n_{t-1}, w) d\Omega_t'(w)$ , which in turn implies  $\iota_t^*(n_{t-1}, \Omega_t(w)) \geq \iota_t^*(n_{t-1}, \Omega_t'(w))$ . ◆

(ii) We need to prove the following: for all  $w \geq 0$ , and for all  $n^A, n^B \in [0, 1]$ , such that  $n^A > n^B$ : then,  $\forall x \geq 0$ ,  $P(w, [0, x] | n^B) \geq P(w, [0, x] | n^A)$ ; where  $P(w, [0, x] | n)$  denotes the probability that when  $w_t = w$ , then  $w_{t+1} \in [0, x]$ , conditional on  $n_t = n$ .

Step 1: Suppose  $w \in [0, \widehat{w}(r)]$ . Let  $y(n_t, w_t) \equiv \delta [f(k_P^*(r_t)) - (1 + r_t)(k_P^*(r_t) - w_{i,t})]$ ; where the fact that  $r_t^* = r(n_t)$  is taken into account when defining  $y(\cdot)$ . Notice that  $\partial y / \partial n_t > 0$  and  $\partial y / \partial w_t = (1 + r_t) > 0$ . Additionally, define the following index-function:

$$\mathbb{I}_{y(n,w) < x} = \begin{cases} 1 & \text{if } y(n, w) < x \\ 0 & \text{otherwise} \end{cases}. \quad (\text{L.5.3})$$

Notice that, because  $\partial y / \partial n > 0$ , then the following two properties hold: 1)  $\mathbb{I}_{y(n^A, w) < x} = 1 \Rightarrow \mathbb{I}_{y(n^B, w) < x} = 1$ ; 2)  $\mathbb{I}_{y(n^B, w) < x} = 0 \Rightarrow \mathbb{I}_{y(n^A, w) < x} = 0$ . Hence, if  $\mathbb{I}_{y(n^B, w) < x} \neq \mathbb{I}_{y(n^A, w) < x}$ , it must be the case that  $\mathbb{I}_{y(n^B, w) < x} = 1$  while  $\mathbb{I}_{y(n^A, w) < x} = 0$ . Using (L.5.3), thus:

$$\begin{aligned} P(w, [0, x] \mid n^B) - P(w, [0, x] \mid n^A) &= [(1-p)n^B + p] \mathbb{I}_{y(n^B, w) < x} \\ &\quad - [(1-p)n^A + p] \mathbb{I}_{y(n^A, w) < x} + (1-p)(n^A - n^B). \end{aligned} \quad (\text{L.5.4})$$

Hence, if  $\mathbb{I}_{y(n^A, w) < x} = 0$ , the right-hand side in (L.5.4) yields a strictly positive number. Alternatively, if  $\mathbb{I}_{y(n^A, w) < x} = 1$ , then the right-hand side of (L.5.4) equals zero. Therefore,  $P(w, [0, x] \mid n^B) \geq P(w, [0, x] \mid n^A)$  for all  $w \in [0, \widehat{w}(r)]$ .

Step 2: Suppose  $w \geq \widehat{w}(r)$ . First, note that either if  $\delta [f(k_S^*(w)) - k_S^*(w) + w] < x$  when  $w \in [\widehat{w}(r), \widetilde{w}]$ , or if  $\delta [f(k_G^*(w)) - k_G^*(w) + w] < x$  when  $w > \widetilde{w}$ ; then in both cases:  $P(w, [0, x] \mid n^B) = P(w, [0, x] \mid n^A) = 1$ . Second, when the opposite results hold, three different cases may arise:

Case 1:  $\delta(w - k_B^*) > x$ . Then,  $P(w, [0, x] \mid n^B) = P(w, [0, x] \mid n^A) = 0$ .

Case 2:  $\delta [f(k_B^*) - k_B^* + w] > x$  and  $\delta(w - k_B^*) < x$ . Now,  $P(w, [0, x] \mid n) = (1-p)(1-n)$ ; thus:  $P(w, [0, x] \mid n^B) - P(w, [0, x] \mid n^A) = (1-p)(n^A - n^B) > 0$ .

Case 3:  $\delta [f(k_B^*) - k_B^* + w] < x$  and  $\delta(w - k_B^*) < x$ . Now,  $P(w, [0, x] \mid n) = (1-n)$ ; hence:  $P(w, [0, x] \mid n^B) - P(w, [0, x] \mid n^A) = (n^A - n^B) > 0$ .

Therefore, as a result of all these four possible cases, we can deduce that:  $P(w, [0, x] \mid n^B) \geq P(w, [0, x] \mid n^A)$  for all  $w \geq \widehat{w}(r)$  as well. ■

**Proof of Proposition 4.** (i) Let  $\Theta$  denote the set of all feasible distribution functions  $\Omega(w)$ . Suppose  $\Omega_t(w) = \Omega_0$ . Since  $n_{t-1} > \bar{n}$ , then  $\iota_t^* > 0$ . Furthermore, since  $\Omega_t(w) \succeq \Omega_0$  for any  $\Omega_t(w) \in \Theta$ , then from Lemma 5 (i) it follows that:  $\iota_t^* > 0$  for any  $\Omega_t(w) \in \Theta$ . Therefore,  $n_t > n_{t-1} > \bar{n}$ , implying, in turn, that  $\iota_{t+1}^* > 0$  for any  $\Omega_{t+1}(w) \in \Theta$ . Repeating the same argument *ad infinitum*, the claimed result obtains. ♦

(ii) When  $n_{t-1} = \bar{n}$ , we have that  $\beta'(0)\Delta(\bar{r}, 0) = 1$ ; where  $\bar{r} = r^*(\bar{n})$ . Thus,  $\iota_t^*(\bar{n}, \Omega_0) = 0$ . Furthermore, from (15) notice that  $\iota_t^*(\bar{n}, \Omega_{\widetilde{w}})$  is the solution to:

$$\beta'(\iota_t^*(\bar{n}, \Omega_{\widetilde{w}})) \Delta(r^*(n_t), \widetilde{w}) = 1, \quad \text{where } n_t = \bar{n} + (1 - \bar{n}) \beta(\iota_t^*(\bar{n}, \Omega_{\widetilde{w}})) \quad (\text{P.4.1})$$

From Lemma 4, and the fact that  $r^*(n_t) \geq \bar{r}$ , it follows that  $\Delta(r^*(n_t), \tilde{w}) > \Delta(\bar{r}, 0)$ . Therefore, to comply with (P.4.1),  $\iota_t^*(\bar{n}, \Omega_{\tilde{w}}) > 0$  must hold. As a result, there must exist  $\bar{n}_{\tilde{w}} < \bar{n}$ , such that  $\iota_t^*(\bar{n}_{\tilde{w}}, \Omega_{\tilde{w}}) > 0$  and  $\bar{n} = \bar{n}_{\tilde{w}} + (1 - \bar{n}_{\tilde{w}}) \beta (\iota_t^*(\bar{n}_{\tilde{w}}, \Omega_{\tilde{w}}))$ ; from which it follows that if  $n_{t-1} > \bar{n}_{\tilde{w}}$  when  $\Omega_t(w) = \Omega_{\tilde{w}}$ , then  $n_t$  will grow over time, converging monotonically to  $n_\infty = 1$ .  $\blacklozenge$

(iii) From Lemma 4 (i), it follows that:  $\iota_t^*(\bar{n}, \Omega_t(w)) \geq 0$  and  $\iota_t^*(\bar{n}_{\tilde{w}}, \Omega_t(w)) \leq \iota_t^*(\bar{n}_{\tilde{w}}, \Omega_{\tilde{w}})$ . As a result, there must exist  $\bar{n}_{\Omega(w)} \in [\bar{n}_{\tilde{w}}, \bar{n}]$ , such that  $\iota_t^*(\bar{n}_{\Omega(w)}, \Omega_t(w)) \geq 0$  and  $\bar{n} = \bar{n}_{\Omega(w)} + (1 - \bar{n}_{\Omega(w)}) \beta [\iota_t^*(\bar{n}_{\Omega(w)}, \Omega_t(w))]$ ; from which it follows that if  $n_{t-1} > \bar{n}_{\Omega(w)}$  when  $\Omega_t(w)$  holds, then  $n_t$  will grow over time, converging monotonically to  $n_\infty = 1$ . Finally, applying Lemma 4 (i) again  $\iota_t^*(\bar{n}_{\Omega(w)}, \Omega_t(w)) \geq \iota_t^*(\bar{n}_{\Omega(w)}, \Omega'_t(w))$  obtains, from where  $\bar{n}_{\Omega(w)} \leq \bar{n}_{\Omega'(w)}$  if  $\Omega_t(w) \succeq \Omega'_t(w)$  immediately follows.  $\blacksquare$

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