THE CARLO ALBERTO NOTEBOOKS

Collegio Carlo Alberto

Information Sales and Strategic Trading

Diego García
Francesco Sangiorgi

Working Paper No. 45
June 2007
www.carloalberto.org

Information sales and strategic trading

Diego García[1] Francesco Sangiorgi[2]

June 2007[3]

[1]Diego García, UNC at Chapel Hill, Chapel Hill, NC, 27599-3490, USA, tel: 1-919-962-8404, fax: 1-919-962-2068, email: diego_garciad@unc.edu, webpage: http://www.unc.edu/~garciadi.

[2]Francesco Sangiorgi, Collegio Carlo Alberto, Via Real Collegio 30, 10131 Moncalieri (To), Italy, tel: +39-011-670-5234, fax: +39-011-670-5082, e-mail: francesco.sangiorgi@carloalberto.org, webpage: http://www.carloalberto.org/people/sangiorgi/.

Abstract

We study information sales in financial markets with strategic risk-averse traders. Our main result establishes that the optimal selling mechanism is one of the following two: (i) sell to as many agents as possible very imprecise information; (ii) sell to a single agent a signal as precise as possible. As noise trading per unit of risk-tolerance becomes large, the *newsletters* or *rumors* associated with (i) dominate the *exclusivity* contract in (ii). The optimal information sales contracts share similar properties in market-orders and limit-orders markets, while models in which competitive behavior is assumed yield qualitatively different equilibria. The endogeneity of the information allocation implies a ranking reversal of the informational efficiency of prices across markets and models. Equilibrium prices become more informative in market-orders than in limit-orders markets, and the model with imperfect competition yields more informative prices than its competitive counterpart. These results are driven by the seller of information offering more precise signals when the externality in the valuation of information is relatively less intense.

JEL classification: D82, G14.

Keywords: markets for information, imperfect competition, share auctions.

1 Introduction

This paper explores the allocation of information that arises when information is sold, via a financial intermediary, to a set of strategic risk-averse traders. We study the problem in the standard market microstructure setting, where agents have CARA preferences and payoffs and signals are normally distributed, across different types of markets (limit-order versus market-order). Our framework allows for strategic trading with multiple informed *risk-averse* agents, and accommodates a large class of information allocations, both important features of the problem. Furthermore, this class of models is one of the few tractable share auctions models with common values,[1] which allows us to link our results to both the finance literature as well as the literature on uniform-price auctions with endogenous informational asymmetries.

Our main finding is that the optimal sales of information take on a particularly simple form in a limit-order market: (i) sell to as many agents as possible very imprecise information; (ii) sell to a single agent a signal as precise as possible. Whether one form or the other dominate depends on the level of noise trading per unit of risk-tolerance of the bidders: as this becomes small, the *exclusivity* contract (ii) dominates the large scale *newsletters* or *rumors* associated with (i). We show that the optimality of one type of contract versus the other is driven by the tradeoff between maximizing aggregate expected profits and ex-ante risk-sharing. The rumors equilibrium maximizes ex-ante risk-sharing by splitting the information in such a way that agents hold very small risky portfolios, at the cost of introducing competition and noise in the information, both of which reduce traders' interim utility. The exclusivity contract maximizes expected trading profits, at the cost of leaving ex-ante risk-sharing gains untapped.

We show that the optimal information sales in a model with market-orders exhibits a similar duality as in the case of limit-orders. For high values of noise trading per unit of risk-tolerance, the seller finds it optimal to sell to as many agents as possible very noisy signals. In this equilibrium the informational properties of asset prices are identical to those that arise under limit-orders. On the other hand, for low values of noise trading per unit of risk-tolerance, the information seller may decide to sell to a small number of traders very accurate signals. Intuitively, traders are always exposed to some execution-price risk when submitting market-orders, no matter how precise their information is. This residual risk opens the possibility of risk sharing among perfectly informed agents, and for positive values of risk aversion the seller may find it optimal to sell precise signals to a small number of them.

Comparing equilibrium properties at the optimal information allocations, we find a ranking reversal of the informational efficiency of prices across markets and models. The ranking in the

[1] Share auctions, first studied in Wilson (1979), allow bidders to receive fractional amounts of the good for sale. Examples in financial markets abound, from auctions of Treasury securities, the actual opening mechanism in markets such as the NYSE, to auctions of equity stakes at IPOs.

exogenous information case is driven by the different usage of private information: limit-orders yield more efficient prices that market-orders because execution-price risk dampens trading aggressiveness (Brown and Zhang, 1997); models in which competitive behavior is assumed are more informative than their imperfect competition counterpart because traders do not internalize their price impact (Kyle, 1989). As the seller of information wants to maximize its value, she finds it optimal to sell *less* information when the negative externality associated with information leakage is relatively more intense. In equilibrium, the monopolist sells sufficiently less information so as to reverse the original ranking, highlighting the importance of endogenizing information allocations when studying how prices aggregate information.

Our analysis yields two other ancillary results. First, the paper provides an example of a large auction market where imperfect competition yields different equilibria than a purely competitive equilibrium concept. In particular, we complement the examples in Kyle (1989) and Kremer (2002),[2] by providing a simple economic problem where the type of limiting economy studied in these papers arises (with precision vanishing as the number of informed agents increases). Second, we find that assuming perfect competition at the trading stage the monopolist seller will always choose to sell noisy newsletters to traders, i.e. the exclusivity contracts that arise when traders are strategic are never optimal if one assumes competitive behavior. Intuitively, it is the fact that the monopolist trader fully internalizes his trades' impact on price (partially with multiple traders) which drives the dominance of the exclusive contracts versus the newsletters. Thereby, models with and without the price-taking assumption yield *qualitatively* different implications.[3]

Our paper contributes to the information sales literature by showing how particularly simple sale strategies are optimal. Furthermore, the types of optimal contracts described above seem to compare well with some of the types of sales we see in markets for information: many financial services firms do sell newsletters that seem to have little informational content (see, for example, Graham and Harvey, 1996; Jaffe and Mahoney, 1999; Metrick, 1999) and many financial consulting services are associated with exclusivity contracts. The duality that the solution exhibits provides a rationale for regime shifts in asset prices. The equilibria with exclusive contracts exhibits low price revelation and low trading volume, whereas the equilibria with noisy newsletters yields more informative prices and high trading volume. A repeated version of the model can thereby generate some of the stylized facts on the time variation and persistence of trading volume and price volatility reported in the empirical asset pricing literature.

[2]See pages 339-340 of Kyle (1989), and in particular Theorem 9.1. Kremer (2002)'s counterexample concludes his paper.

[3]This is in sharp contrast to the results in Kovalenkov and Vives (2004), who argue that the competitive and strategic models we study (in particular the competitive and strategic versions of Kyle, 1989) have similar equilibrium properties. In our application the models are truly different, both quantitatively and qualitatively, even when there are large numbers of agents.

Our paper is closely related to previous work on information sales, as well as the literature on mutual funds and analysts.[4] We model information sales as *direct*, in the sense defined in Admati and Pfleiderer (1986).[5] The paper by Admati and Pfleiderer (1988), who study information sales in a Kyle (1985) framework with risk-averse traders is perhaps the closest to our model. Admati and Pfleiderer (1988) show that in the context of *photocopied* noise (see Admati and Pfleiderer, 1986) a monopolistic seller of information would like to sell to finite number of traders, depending on the risk-aversion of the traders. We extend their analysis by allowing the information seller to add *personalized* noise, which as argued in Admati and Pfleiderer (1986) and Dridi and Germain (1999) is potentially more beneficial in terms of dampening the effects of competition between traders. We go further than these papers by studying information sales across different market settings, allowing the monopolist to choose among a larger class of allocations of information, obtaining remarkably simple solutions.

The second strand of the literature to which our paper contributes is that of share auctions and information aggregation.[6] The main contribution of our paper is to endogenize the allocation of information in two special types of share auctions with risk-averse buyers.[7] Prices in the share auctions we study indeed aggregate the diverse pieces of information that agents receive from the monopolist seller. On the other hand, the type of information received by agents is a non-trivial function of the number of informed agents in the equilibrium that yields optimal rumors, in sharp contrast to most of the limiting equilibria studied in the literature, where signals' precision are typically held constant as new traders are added to the auction. Our results highlight the importance of endogenizing the allocation of information when studying issues of information aggregation.

2 The model

In this section we present the main ingredients of our economy with endogenous asymmetric information. We first discuss the elements of the model as well as the assumptions on information sales, and then characterize the resulting equilibria and the monopolist's problem.

[4] Some early work includes Admati and Pfleiderer (1986), Admati and Pfleiderer (1987), Admati and Pfleiderer (1988), Admati and Pfleiderer (1990), Benabou and Laroque (1992), Fishman and Hagerty (1995). See Ross (2000), Biais and Germain (2002), Veldkamp (2003), Germain (2005), Morgan and Stocken (2003), Cespa (2007), and García and Vanden (2006) for some recent contributions.

[5] In essence, we take the information sales problem of Admati and Pfleiderer (1986), and extend it to the non-competitive markets of Kyle (1985) and Kyle (1989).

[6] Some classic papers on information aggregation include Wilson (1977), Milgrom (1979), and Milgrom (2000). See Pesendorfer and Swinkels (1997), Pesendorfer and Swinkels (2000), Hong and Shum (2004), Reny and Perry (2003) and Cripps and Swinkels (2004) for some recent work on the area.

[7] The work on information acquisition in auctions in general is a rapidly growing field. See Matthews (1984), Hausch and Li (1991), Guzman and Kolstad (1997), Persico (2000), Moresi (2000), Gaier and Katzman (2002), Jackson (2003).

2.1 The share auction setting

All agents have CARA preferences with a risk aversion parameter r. Thus, given a final payoff π_i, each agent i derives the expected utility $\mathbb{E}[u(\pi_i)] = \mathbb{E}[-\exp(-r\pi_i)]$. There is a large number of uninformed traders who participate in the stock market alongside the traders who can become informed. This makes the specification of the economy, post information sales, identical to the one discussed in Kyle (1989) under the assumption of free entry of uninformed speculators. This assumption is equivalent to the existence of a competitive market-making sector that clears the market, as in Kyle (1985). This equivalence allows us to compare the limit- and market-orders based models on equal footing (Bernhardt and Taub, 2006).

There are two assets in the economy: a risk-less asset in perfectly elastic supply, and a risky asset with a random final payoff $X \in \mathbb{R}$ and variance normalized to 1. All random variables in the economy are defined on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$, and unless stated otherwise, are normally distributed, uncorrelated, and have zero mean. There is random noise trader demand Z for the risky asset. This variable is the usual driver in preventing private information to be revealed perfectly to other market participants.[8] We let σ_z^2 denote the variance of Z. We use θ_i to denote the trading strategy of agent i, i.e. the number of shares of the risky asset that agent i acquires. With this notation, the final wealth for agent i is given by $\pi_i = \theta_i(X - P_x)$, where P_x denotes the price of the risky asset.[9]

2.2 The monopolist information seller

There is a single agent who has perfect knowledge about the payoff from the risky asset X, whom we shall refer to as the information seller. We will focus on *direct* sales of personalized information (see Admati and Pfleiderer, 1986), i.e. the case where the seller of information gives agent i a signal of the form $Y_i = X + \epsilon_i$, with ϵ_i i.i.d. and $s_\epsilon \equiv \text{var}(\epsilon_i)^{-1}$ denoting the precision of the signals offered by the monopolist. The information seller can write contracts for the delivery of signals Y_i to N agents, where N is large. We allow the seller of information to ration the market, i.e. to sell to $m < N$ agents, and to freely choose the signal quality s_ϵ. Essentially, we allow the monopolist to sell different pieces of her information to different agents.

Figure 1 sketches the major stages of the model. The monopolist seller of information

[8] We note that the size of the noise trading demand is fixed, in sharp contrast with much of the literature, where some notion of large noise is introduced in order to justify competitive trading behavior (Hellwig, 1980; Verrecchia, 1982; Admati, 1985; García and Urosević, 2007). Furthermore, one could equivalently interpret this noise trading demand as stemming from aggregate supply shocks.

[9] We normalize here, as is customary in the literature, the agents' initial wealth and the risk-free rate to zero. These assumptions are innocuous since the model contains only one period of trading and agents are assumed to have CARA preferences. In addition, there are no borrowing or lending constraints imposed on the agents.

contacts m agents and offers them signals as specified above for a price c. If an agent accepts he pays the fee c, and next period he receives the signal Y_i, which he will use to make his portfolio decision. If an agent declines he trades as an uninformed investor when financial markets open. Traders are not allowed to resell the information they receive to other traders and the precision of the signals is assumed to be contractible. The type of information sales we are considering can be thought as subscriptions to some future advice, for which trades pay some ex-ante price c, and later get to observe information about the risky asset.

We should emphasize that all the assumptions on the information sellers of Admati and Pfleiderer (1986) are in place. In particular, there is no reliability problem between the information seller and the buyers, in the sense that she can commit to truthfully revealing the signal Y_i that she promised. Furthermore, the information seller is not allowed to trade on her information.[10]

2.3 The equilibrium at the trading stage

We describe next the three models that we will study: (1) the limit-orders model of Kyle (1989), (2) the market-orders model of Kyle (1985), (3) the competitive version of Kyle (1989). A linear rational expectations equilibrium is defined by a linear function $P_x : \Omega \to \mathbb{R}$ such that (i) agents' trading strategies are optimal given their information set, (ii) markets clear in all states. We should emphasize that agents do *not* act as price takers in (1) and (2) - they anticipate the dependence of prices on their trading strategies as in the Kyle (1985) and Kyle (1989) models. Studying the competitive version of Kyle (1989) allows us to determine whether such a standard assumption conditions the results we obtain in the particular problem of information sales.

We follow the literature and search for equilibria where prices are linear functions of the primitives in the economy, namely the vector $(X, \{\epsilon_i\}_{i=1}^m, Z)$. We let θ_i denote the number of units of the stock that agent i trades. Without loss of generality, due to the CARA/normal setup, we characterize informed agents' trading strategies in the limit-order model by two positive constants (β, γ), defined by $\theta_i = \beta Y_i - \gamma P_x$, for $i = 1, \ldots, m$; whereas we use $\theta_i = \beta Y_i$ in the market-order model. The parameter β measures the intensity of trading on the basis of private information, whereas γ is the intensity with which they trade as a function of price (including their optimal response to its informational content). From the market-clearing

[10] Following the discussion in Admati and Pfleiderer (1988), we conjecture that allowing the information seller to trade would only condition some of our results on the optimal sales. For high values of the risk-tolerance per unit of noise parameter the information seller would choose to not sell her information and just trade on her own account (analogous to selling to one single agent), whereas for low values she would choose to sell to as many agents as possible.

condition prices will be of the form

$$P_x = \lambda \left(\beta \sum_{i=1}^{m} Y_i - Z \right);$$

for some $\lambda > 0$.

Before proceeding to the characterization of the equilibrium, we follow Kyle (1989) and introduce a set of equilibrium variables that facilitate the presentation of our results. We define the informational content parameter ψ by $\mathrm{var}(X|P_x)^{-1} \equiv \tau_u = 1 + \psi m s_\epsilon$. The variable ψ measures the fraction of the informed agents' precision that is revealed by prices. It is useful to define the product of the number m of signals sold times the precision in each signal, $y \equiv m s_\epsilon$, as the "stock of private information" in the economy, so that $\tau_u = 1 + \psi y$. Furthermore, we define the conditional precision of payoffs and trading profits for an informed agent as $\tau_i \equiv \mathrm{var}(X|\mathcal{F}_i)^{-1}$ and $\tau_\pi \equiv \mathrm{var}(X - P_x|\mathcal{F}_i)^{-1}$ respectively. In the market-order model $\mathcal{F}_i = \sigma(Y_i)$, i.e. an informed agent's information set reduces to the sigma-algebra generated by Y_i; whereas in the limit-order model $\mathcal{F}_i = \sigma(Y_i, P_x)$, i.e. traders can condition their investment decisions on prices, so that $\tau_i = \tau_\pi$. Following Kyle (1989), we also define the informational incidence parameter $\zeta \equiv \beta\lambda \mathrm{var}(X|\mathcal{F}_i)^{-1}/s_\epsilon$. This variable is related to strategic considerations in the following way: ζ measures the change in the price that obtains when an informed agent's valuation of the risky asset goes up by one dollar as a result of a larger realization of his signal Y_i. At its extreme values, $\zeta = 1/2$ corresponds to the case of risk neutral monopoly, $\zeta = 0$ to a perfectly competitive market.

The equilibrium in the limit-order market of Kyle (1989) is characterized, for a given information allocation m and s_ϵ, via the non-linear equation[11]

$$\kappa \sqrt{\frac{\psi}{(1-\psi) m s_\epsilon}} = m \frac{(1-\psi)}{(m-\psi)} \frac{(1-2\zeta)}{(1-\zeta)}; \tag{1}$$

where $\kappa = r\sigma_z$. The corresponding condition in the market-order model is given by

$$\kappa \sqrt{\frac{\psi}{(1-\psi) m s_\epsilon}} = \frac{\tau_\pi (1-\psi)}{\tau_i \tau_u} \frac{(1-2\zeta)}{(1-\zeta)}; \tag{2}$$

and in the competitive version of Kyle (1989) by

$$\kappa \sqrt{\frac{\psi}{(1-\psi) m s_\epsilon}} = m \frac{(1-\psi)}{(m-\psi)}. \tag{3}$$

[11] A technical appendix on the authors' websites contains the details of the solution and the characterization of the equilibrium for the three models considered.

These three models have been studied in the literature with exogenous information allocations, i.e., taking the number of informed traders m and the quality of their information s_ϵ as given. Kyle (1989) shows that prices in the equilibrium with with imperfect competition are less informative than in the equilibrium with perfect competition. This follows immediately by comparing (1) and (3), since the term which includes ζ in (1) is less than 1. The intuition for his result is that agents who internalize their price impact trade less aggressively on their private information. Brown and Zhang (1997) and Bernhardt and Taub (2006) show that limit-order models, of the type characterized by (1), yield more informative prices than market-order models given in (2).[12] The intuition for their results is that agents who can submit demand schedules, instead of market-orders, trade more aggressively on their information because of two reasons. On the one hand they face less execution-price risk due to noise-trading; on the other each of them internalizes the order-reducing effect of his order on the trades of other speculators, increasing competition. This paper builds on these results by endogenizing the allocation of information via a financial intermediary who sells information directly, and characterizing the corresponding equilibrium properties.

2.4 The monopolist's problem

The monopolist seller of information would charge a price c that makes each of the agents just indifferent between accepting the monopolist's offer or trading as an uninformed agent. The profits earned by the seller of information from a particular allocation depend on the number of traders m to which information is sold, and on the value of signals to each trader given by the certainty equivalent of wealth. The next Proposition shows that the monopolist's problem can be equivalently stated in terms of the expected *interim* certainty equivalent.

Proposition 1. *The monopolist's problem can be expressed as*

$$\max_{m\in\{1,\ldots,N\},s_\epsilon\in\mathbb{R}_+} \mathcal{C}(m,s_\epsilon) = \frac{m}{2r}\log\left(1+2r\mathbb{E}[\chi_i]\right); \tag{4}$$

such that one of the equilibrium conditions (1)-(3) holds, where

$$\chi_i = \mathbb{E}[\pi_i|\mathcal{F}_i] - \frac{r}{2}\mathrm{var}(\pi_i|\mathcal{F}_i), \tag{5}$$

and $\pi_i = \theta_i(X-P_x)$ denotes trading profits of agent i.

The Proposition provides a simple expression for the monopolist's profits, which is valid

[12] The models in these two papers are not isomorphic to the ones we study here. Brown and Zhang (1997) look at a version of the Vives (1995) model, with a continuum of informed agents. The analysis in Bernhardt and Taub (2006) differs from ours along two dimensions: they only consider the risk neutral case, and the information structure is of the form $\sum_{i=1}^m Y_i = X$.

across the three models introduced the previous section.[13] The ex-ante consumer surplus $\mathcal{C}$ is a concave function of $\mathbb{E}[\chi_i]$, the expected interim certainty equivalent. The variable χ_i is precisely the objective function that trader i maximizes at the interim stage (this is $t = 2$ in Figure 1). The monopolist seller controls $\mathbb{E}[\chi_i]$ by choosing both s_ϵ and m. More precise signals generate smaller interim discounts due to risk, the second term in (5). On the other hand, the expected profits component may or may not increase in signals' precision: too much information can induce agents into trading very aggressively, and as a consequence eliminate their trading profits. Increasing m the monopolist is also affecting the interim certainty equivalent - more agents will compete more aggressively, thereby reducing expected interim profits. On the other hand, given the concavity of the log function, increasing m increases the ex-ante risk-sharing gains, since noise trader risk is being shared among more risk-averse traders.

The rest of the paper endogenizes the allocation of information in the economy by considering the solution to (4), namely the choice of the number of agents the monopolist sells information to, m, and the quality of the signal she offers, s_ϵ, for the three different market structures introduced in section 2.3. We remark that, given the symmetry assumption and the fact that the information allocation is endogenous, the only two primitives in (4) are the risk-aversion of each trader r and the total amount of aggregate noise σ_z. Although r and σ_z have different effects on the consumer surplus, the optimal information allocation is determined solely by the product of the two, which we denote as $\kappa = r\sigma_z$. We remark that the risk-neutral case implies $\kappa = 0$ for any σ_z. We refer to κ as the noise per unit of risk-tolerance in the economy.

3 Optimal information sales in the limit-order market

3.1 General considerations

The monopolist's problem introduced in the previous section is driven by the interaction of strategic trading, externalities in the valuation of information and risk sharing considerations. Before exploring the full problem, it is worthwhile to look at it for a fixed number of informed traders m. The next Proposition characterizes the optimal stock of information y as a function of m and κ.

Proposition 2. *When $m = 1$, the monopolist seller optimally gives the agent what she knows, i.e. $s_\epsilon = \infty$. Fixing $m \geq 2$, the optimal stock of information y is given by the unique positive solution to the quadratic equation*

$$A(m, \psi)y^2 + B(m, \psi)y + C(m, \psi) = 0; \tag{6}$$

[13] As far as we know this is the first time such an expression has been given in the literature.

where the functions A, B, C are provided in the proof of the Proposition.

The above Proposition shows that if the monopolist sells information to only one trader, then it is optimal not to add any noise to the signal. This is rather intuitive: in absence of competition the informed trader fully internalizes the price impact of his trades, and the seller maximizes the value of the signal giving him full information.

On the other hand, if she were to sell perfect information to multiple traders, speculators would compete very aggressively, making the price reveal all their private information and driving profits to zero (see Kyle, 1989, Theorem 7.5). The seller can avoid this outcome by adding noise to the signals she sells, thereby dampening the competition problem, and allowing informed traders to earn positive profits.[14] However, traders with imperfect signals only partially distinguish price movements related to fundamentals from those related to noise-trading. This risk creates a discount at the interim stage.

The amount of noise that optimizes this trade-off between competition and interim risk-sharing considerations is implicitly defined by Proposition 2. One can verify, solving the system defined by (1) and (6) numerically, that the optimal stock of private information y is an increasing function of κ. Rather intuitively, the higher noise trading per unit of risk tolerance, the smaller the negative externality of price revelation. The monopolist responds by increasing the stock of information she offers to the m traders as a consequence. Similarly, as the number of traders m increases, the negative externality from competition among them increases, and the monopolist optimally cuts down the information offered, i.e. the stock of information y offered is decreasing in the number of agents m that receive information.

Summarizing, there are two opposing forces. On one hand the information seller wishes to maximize interim consumer surplus, by providing agents with high aggregate profits and low trading risk via precise signals. Concentrating the information allocation maximizes the interim certainty equivalent, because traders compete over profits. On the other hand, more traders share the noise-trading risk, so dispersing the information allocation by spreading noisy signals among different agents improves ex-ante risk-sharing. When solving her problem (4), the monopolist seller of information optimally weights these forces.

[14] This resembles the optimality of adding noise from Admati and Pfleiderer (1986). The motivations for adding noise in our paper are related, but not identical to theirs. For instance, the seller would never sell perfect information to a single trader in the competitive model of Admati and Pfleiderer (1986). In section 4.2 we further compare our results to a case closely related to Admati and Pfleiderer (1986), where agents act as price takers.

3.2 Optimal exclusivity contracts and noisy newsletters

In this section we study the problem in (4) for open sets around the risk-neutral and large risk-aversion cases.

Proposition 3. *There exists $\underline{\kappa}$ such that for all $\kappa < \underline{\kappa}$, the monopolist optimally sells to a single agent, $m = 1$, and sets $s_\epsilon = \infty$, i.e. tells the agent what he knows. In this case, informational efficiency and monopolist's profits satisfy:*

$$\mathrm{var}(X|P_x)^{-1} = 2, \tag{7}$$

$$\mathcal{C}_1(r, \sigma_z) \equiv \frac{1}{2r} \log\left(1 + r\sigma_z\right). \tag{8}$$

The Proposition establishes that if risk-aversion or noise trading in the auction are small, the damaging effects of competition are high enough that the monopolist optimally sells her information to one trader. In the risk-neutral case the monopolist's problem reduces to that of maximizing expected profits, and can be solved in closed form. In this case, selling to more traders does not bring any risk-sharing gains, and competition decreases aggregate profits, making the concentrated information allocation with $m = 1$ optimal. When traders are risk averse, most of the analytical tractability disappears. Nevertheless, for low κ risk-sharing gains are still negligible with respect to the costs induced by competition and noisy signals, driving the optimality of the exclusivity contract for an open interval around zero.

Under the optimal information sales, half of the information of the seller gets impounded into prices, i.e. the conditional volatility of the risky asset is exactly 1/2 the unconditional volatility, irrespective of the level of noise trading. This follows from the fact that speculator's effective risk aversion is zero as he receives a signal with no noise and can submit limit orders. As the risk neutral monopolist in Kyle (1985), he optimally adjusts his trading strategy so as to offset any variation in noise trading, making price informativeness independent of noise trading.

Next Proposition describes the allocation of information that arises with a large number of traders, and establishes its optimality when the monopolist faces an economy with highly risk-averse traders and/or an asset with large amounts of noise.

Proposition 4. *There exists some $\bar{\kappa}$ such that for all $\kappa > \bar{\kappa}$ the monopolist's problem (4) is solved for $m = N$. As $N \uparrow \infty$, the optimal stock of private information sold is*

$$y = \frac{1 - 2\psi_\infty^2}{\psi_\infty(1 - \psi_\infty)}; \tag{9}$$

where ψ_∞ is the unique real solution in $[0, 1/2]$ *to*

$$\psi_\infty^4 - \psi_\infty^3 + \frac{(\kappa^2 - 2)}{8}\psi_\infty^2 + \frac{1}{2}\psi_\infty - \frac{1}{8} = 0. \tag{10}$$

As $N \uparrow \infty$, informational efficiency and monopolist's profits satisfy:

$$\operatorname{var}(X|P_x)^{-1} = 1 + \frac{1 - 2\psi_\infty^2}{(1 - \psi_\infty)}; \tag{11}$$

$$\mathcal{C}_\infty(r, \sigma_z) \equiv \frac{(1 - 2\psi_\infty^2)(1 - 2\psi_\infty)}{2r\psi_\infty\,(2(1 - \psi_\infty^2) - \psi_\infty)}. \tag{12}$$

The Proposition shows that the solution for the monopolist's problem when either risk-aversion or the amount of noise trading in the economy are large is to sell to as many agents as possible very imprecise signals. More risk averse speculators trade less aggressively on information, and more noise trading makes prices less informative, so for κ large the negative effects of competition are relatively small. The monopolist could still sell perfect information to a single trader, maximizing the interim certainty equivalent, but the ex ante value of information would be highly discounted due to large risk aversion or large noise-trading risk. As a consequence, risk sharing gains dominate competition effects, driving the optimality of selling to N agents for large noise trading per unit of risk-tolerance. The optimal allocation of information with large number of traders does indeed resemble noisy newsletters, as individual precision in each trader's signal vanishes in the large N limit.

The fraction of information that prices reveal, ψ_∞, is a decreasing function of κ. As a consequence, from (9) the stock of private information sold y is increasing in κ. As noise trading per unit of risk tolerance increases, the dilution of the value of information by price revelation decreases. The seller optimally increases the stock of private information sold to the N agents as a consequence. The net effect on price informativeness is negative as long as κ is sufficiently large.[15] Comparing the equilibrium expressions with those from Proposition 3, we see that informational efficiency is always greater than in the exclusivity contract case.[16] Prices aggregate the information dispersed in the economy and reveal more than under the equilibrium with a monopolist trader.

In the optimal contracts in Proposition 4, the monopolist sells signals in such a way as to have a large number of informed agents monopolistically competing against each other as in the leading example of section 9 of Kyle (1989), and the concluding counter-example in Kremer (2002). These two examples are built abstractly by taking the large N limit in an

[15] One can verify, from (9) that $\tau_u = 1 + \psi y = 1 + (1 - 2\psi_\infty^2)/(1 - \phi_\infty)$ is an increasing function of ψ_∞ for $\psi_\infty \geq 1 - 1/\sqrt{2} \approx 0.29$. Since ψ_∞ is strictly decreasing in κ the claim follows.

[16] From (11) one can check that $\operatorname{var}(X|P_x)^{-1} > 2$ for $\kappa > \bar{\kappa}$ and finite.

auction setting letting the precision of the signal vanish as N increases. Proposition 4 presents an economic problem where such a limiting economy arises endogenously. As highlighted by Kyle (1989), even in the large N limit, when agents are "small" in terms of their informational advantage, they internalize their price impact, i.e. the equilibrium informational incidence parameter satisfies $\zeta(\kappa) > 0$ for all $\kappa > \bar{\kappa}$ and finite, whereas the competitive model has $\zeta = 0$.

3.3 The general case

After establishing that the optimal solution is non-interior for two open sets of $\kappa \in \mathbb{R}_+$, we further analyze the problem in this section to assess how tight the bounds $[0, \underline{\kappa})$ and $(\bar{\kappa}, \infty)$ actually are. Proposition 3 only establishes the existence of an open set $[0, \underline{\kappa}]$, whereas in Proposition 4 does not address whether the bound $\bar{\kappa}$ has a finite limit if we let $N \uparrow \infty$. The general problem in (4), for an arbitrary κ, is particularly challenging analytically due to its non-linear nature.[17]

We solve the model numerically, using results from Proposition 2 to reduce the problem for each m to the solution of a single nonlinear equation for ψ. In particular, the characterization in (1), evaluated at $y(m, \psi)$, defines the equilibrium at the optimal allocation of information for each fixed m. Since the equilibrium is uniquely given by the solution for ψ to such equation (Kyle, 1989), it is a trivial numerical exercise to solve such equation and compute the consumer surplus from (4). Figure 2 plots the profits obtained by the monopolist from selling to $m = 2, \ldots, 40$ (dotted lines), as well as the profit functions corresponding to $m = 1$ and $m = \infty$ (solid lines). As Figure 2 makes clear, the profit functions with $m = 1$ and $m = N$ (for N large), form an upper envelope that dominates any allocation of information to m informed agents: the functions $\mathcal{C}_1(r, \sigma_z)$ and $\mathcal{C}_\infty(r, \sigma_z)$ as defined in (8) and (12) yield an upper bound for the value of the monopolist's problem.

The following Theorem summarizes our main findings.

Theorem 1. *There exists κ^* such that Proposition 3 and Proposition 4 hold with $\bar{\kappa} = \underline{\kappa} = \kappa^* \approx 1.74$.*

The optimal information allocation is one of the two discussed in the previous section: the monopolist should offer either the exclusivity contracts from Proposition 3, or the noisy newsletters from Proposition 4. The finite m solution, which compromises ex-ante risk-sharing vis a vis the noisy newsletters equilibria, is dominated by the exclusivity contracts for all $\kappa < \kappa^*$. At the same time, with a finite m the information seller improves on risk-sharing vis a vis the exclusivity contracts for sufficiently large κ, but for such $\kappa > \kappa^*$ the noisy newsletters

[17] For instance, there exist open sets of κ such that optimal profits as a function of m exhibit both local maximum and minimum which are not the global maximum (or minimum).

equilibria already yields higher profits. Although there is a non-trivial tension between the finite m case and either the two limiting cases for different κ, this tension occurs for parameter values for which the other limiting case dominates.

The discontinuity in the equilibrium prices at κ^* creates a rationale for regime shifts between the $m = 1$ and $m = N$ cases. Namely, consider a repeated sequence of economies, identical and independent from each other. Assume the noise-trading parameter σ_z is a stochastic process taking values on the positive real line. As this process crosses the κ^* barrier the allocation of information in the economy will shift from one type of equilibrium to the other. This will in turn generate regimes with different asset pricing properties, depending on whether σ_z is above or below κ^*. In periods of high noise-trading prices will be more informative, aggregate trading profits lower, and trading volume will be high. When noise-trading drops below the cutoff trading volume will dry up, as only one agent will become informed. Trading profits on the other hand will be high, accompanied of low price revelation.

4 Market structure, trading behavior and information sales

In this section we extend our analysis to a setting where agents can execute market-orders instead of limit-orders. We show that the nature of the solution does not change, although now the seller of information may choose to sell signals to a finite number of agents when noise trading per unit of risk-tolerance is low. These agents are offered perfect information, as in the low κ equilibria of section 3. Next, we study our model under the assumption of price-taking behavior. We find that the competitive model yields significantly different answers: the monopolist seller of information will always sell to as many agents as possible very imprecise information, even when traders are almost risk-neutral. We finish by discussing more general allocations of information, as well as the possibility that the monopolist's information is imperfect.

4.1 Market orders

In this section we study how the particular market structure used throughout section 3 affects optimal sales of information. Instead of allowing traders to submit demand schedules, we study the case where traders submit market-orders to a risk-neutral market-maker, who sets the price according to weak form efficiency

$$P_x = \mathbb{E}[X|\omega];$$

where ω denotes total order flow. Using the previous notation, $\omega = \sum_{i=1}^{m} \theta_i + Z$. More critically, in the Kyle (1985) model agents are not allowed to condition their trades on prices, so their optimization problem is over trading strategies θ_i that are Y_i-measurable.

As in section 3, the consumer surplus, for a given information allocation with m informed traders who have signals of precision s_ϵ, is given by (4), where the interim certainty equivalent is given by (5) with $\mathcal{F}_i = \sigma(Y_i)$. As in the previous analysis, consumer surplus is a concave function of the expected value of the interim certainty equivalent χ_i. In contrast to the limit-order model, agents' interim certainty equivalent depends only on their private information Y_i, since neither order flow ω nor the asset price P_x is in their information sets. Notably, even agents who observe the realization of the fundamental X face some residual price-execution-risk in their investment opportunity, because noise trader demand will randomly move the prices at which the order is executed. As a consequence the conditional variance of profits is bounded away from zero.

The information seller's problem is, analogously to the case of limit-orders, to maximize consumer surplus given in (4), subject to the equilibrium constraint, given by (2). The residual uncertainty that traders face with market orders makes the information sales problem more challenging. In fact, contrary to the limit orders case, residual uncertainty prevents effective risk aversion of informed traders from vanishing as the precision of the signal increases. The reason being that, once the order is submitted, traders are exposed to the risk that liquidity traders may push the price against them. Therefore, risk averse traders do not compete away their profits even if perfectly informed on the one hand, and value the reduction in risk that comes with precision on the other. As a consequence, the seller may find it optimal to sell precise signals to more than one trader.

The next Proposition presents open sets parameter statements for the market-order model, as Propositions 3 and 4 did for the limit-order model.

Proposition 5. *Consider the monopolist problem in (4) in the market-order model of Kyle (1985). Then:*

(a) There exists $\underline{\kappa}$ such that for all $\kappa < \underline{\kappa}$ the monopolist optimally sells to one agent, $m = 1$, and gives him perfect information, $s_\epsilon = \infty$;

(b) There exists $\hat{\kappa}$ such that for all $\kappa > \hat{\kappa}$ the monopolist optimally sells to $m = N$ agents. As $N \uparrow \infty$ the solution to the monopolist problem is as stated in Proposition 4.

The risk-neutral case, $\kappa = 0$, can be solved explicitly, and mirrors the conclusion of the limit-order model. As shown in Dridi and Germain (1999), the monopolist seller of information may want to add some noise to her information, in particular if $m \geq 4$. Furthermore, their equilibrium profit expressions immediately imply that consumer surplus is maximized concentrating the information in one single trader, as in Admati and Pfleiderer (1988). The novel feature of the Proposition is the fact that consumer surplus is higher for the $m = N$ equilibria for sufficiently large κ. The optimality of the rumors equilibria is driven by the ex-ante risk-sharing gains - the main difference vis a vis the limit-order model is the extent of these gains. Furthermore, the proposition shows that the equilibrium price informativeness coincides with that in the limit-order model, i.e. the models behave identically in terms of the informational content of prices.

The general case is again challenging analytically, since equilibrium with risk-averse traders in a Kyle (1985) can only be characterized via a non-linear equation (Subrahmanyam, 1991). As in the limit-order model, we solve the model numerically. For each m, we solve for the optimal s_ϵ and the equilibrium price, obtaining the maximum consumer surplus for each m. We do this for a fine grid of values for κ, and report the resulting consumer surplus for different m in Figure 4. Whether it is optimal to set $s_\epsilon = \infty$ or not depends on both m and κ. For $m \leq 3$ it is never optimal to add any noise, so that $s_\epsilon = \infty$ is always optimal. For $m \geq 4$, on the other hand, the seller of information would like to sell noisy signals, $s_\epsilon < \infty$ if and only if $\kappa \leq \bar{\kappa}_m$, where κ_m is increasing in m. Rather intuitively, for a fixed m, the monopolist gives the agents her information if and only if the noise per unit of risk tolerance is sufficiently high. Comparing Figure 4 to Figure 2, we see that the upper envelope now consists on the fragments of six different profit lines, those that encompass $m \leq 5$ and the m large case. We summarize our findings in the following theorem.

Theorem 2. *For $\kappa < \hat{\kappa} \approx 3.1$, the monopolist sells signals with no noise to a finite number of agents (at most five).*[18] *For $\kappa \geq \hat{\kappa}$ it is optimal to sell signals with vanishing precision to an infinite number of traders.*

As in the analysis in section 3, the problem's solution is of the bang-bang nature: (i) either to concentrate the information in the hands of a few traders, or (ii) to disperse it to a large number of them, giving each of them a very noisy signal. In terms of comparing the solution to the limit order model, we see that for low values of κ, the allocation of information in the models coincide: optimal information sales involve $m = 1$ and $s_\epsilon = \infty$. On the other hand, the model with market-orders has interior optima for m for an open set of values of κ. The rationale for the difference with Theorem 1 lies on the fact that even while giving agents perfect information, there are interim risk-sharing gains by spreading this information among multiple agents due to the execution price risk created by noise traders in a market-order exchange. These interim risk-sharing gains make the critical κ higher than in the limit-order model - it

[18] Namely: for $\kappa < \kappa_1 \approx 0.19$ it is optimal to have $m = 1$; for $\kappa_1 < \kappa < \kappa_2 \approx 0.81$, it is optimal to have $m = 2$; for $\kappa_2 < \kappa < \kappa_3 \approx 1.56$, $m = 3$ maximizes profits; for $\kappa_3 < \kappa < \kappa_4 \approx 2.38$, the seller sets $m = 4$; and for $\kappa_4 < \kappa < \hat{\kappa}$, the seller sets $m = 5$.

is only after $\kappa > 3.1$, versus $\kappa > 1.74$, that the noisy newsletters equilibria dominate the one with a finite number of traders with perfect signals.

Figure 3 plots equilibrium values of $\text{var}(X|P_x)^{-1}$ as a function of κ for both the limit-order and the market-order models. For low values of κ, price informativeness is increasing in the noise per unit of risk-tolerance parameter in the market-order model,[19] whereas it is constant at $\text{var}(X|P_x)^{-1} = 2$ with limit-orders. The intuition lies on traders' effective risk-aversion: while they face a riskless arbitrage opportunity in the limit-order market, market orders are risky. This interim risk makes the monopolist seller sell to more agents, and competition among them drives price informativeness up. On the other hand, for large values of κ both models yield the same prediction: price informativeness is decreasing in κ.

4.2 Competitive behavior

All our previous results were derived assuming agents acted strategically when trading, i.e. they anticipated the effect of their trading on equilibrium asset prices. Of course, one could solve the model under the alternative competitive assumption, that is, assuming that agents act like price takers.[20] The previous discussion highlights the fact that with a finite amount of noise σ_z, one cannot get away assuming competitive behavior: in the two classes of equilibria discussed in the previous section agents were partially internalizing their trading strategies' effect on prices, namely $\zeta > 0$. Although the equilibria will have different characterizations, it is not clear to what extent the competitive assumption will affect the qualitative aspects of the optimal contracts. The next Proposition shows that indeed it does: assuming perfect competition the exclusive contracts are never optimal.

Proposition 6. *If agents act as price takers, the optimal sales satisfy $m = N$, i.e. the information seller sells to as many agents as she can. Furthermore, as $N \to \infty$ she sets*

$$y = \frac{\kappa^2 \psi_\infty}{(1 - \psi_\infty)^3} \tag{13}$$

where ψ_∞ solves

$$(1 + \kappa^2)\psi_\infty^3 + (\kappa^2 - 3)\psi_\infty^2 + 3\psi_\infty - 1 = 0; \tag{14}$$

[19] We focus on the "continuous" version of m in this paragraph, since it highlights the intuition of the problem. When treating m as a discrete variable we see that price informativeness exhibits a highly non-monotonic relationship with κ, with large jumps at the points where the solution changes from each finite m, while being decreasing in between. This is intuitive: for a fixed m, an increase in risk-aversion means agents will trade more conservatively. Once there is a change in m price informativeness jumps due to the discrete increase in order flow.

[20] As shown in Kyle (1989) and García and Urosević (2007) this assumption is innocuous in large economies, where noise, measured by σ_z, grows with the number of agents N.

The monopolist's profits are given by

$$\lim_{N \uparrow \infty} \mathcal{C}(m, s_\epsilon) = \frac{1}{2\tau} \frac{(1-\psi_\infty)^2}{\psi_\infty(2+\psi_\infty)}. \tag{15}$$

For any value of the primitives κ, equilibrium prices at the optimal sales are less informative if agents act as price takers than if they act strategically. Moreover, price informativeness is increasing in κ.

Under perfect competition the information seller always chooses to sell to as many agents as possible, controlling the damaging effects of information leakage by giving agents very imprecise signals. The optimality of selling to a single trader disappears. This implies that it is critical to model the price impact of traders in our application. Rather intuitively, it is precisely the fact that the monopolist trader internalizes his trades' impact on prices that drives the optimality of the exclusivity contract in section 3. The result has a similar flavor to that in Admati and Pfleiderer (1986), where it is shown, in a large market with perfectly competitive traders, that the seller of information would always optimally sell to all agents. Proposition 6 highlights that it is not the particular structure of the large market in Admati and Pfleiderer (1986) that drives their result, but rather the competitive assumption.[21]

It is also interesting to note how equilibrium prices are affected across the equilibria discussed in Proposition 6 and the one from our previous section. As shown in the Proposition, prices are less informative under the competitive assumption than under strategic trading. This result may be surprising, since as Kyle (1989) convincingly shows, strategic trading makes agents more cautious with their trades, thereby having less informative prices (see Theorem 7.1 in Kyle, 1989). The intuition for the result reversal is that the information seller will give agents more informative signals under imperfect competition precisely because these traders, in contrasts to competitive traders, will marginally internalize the effect of their trades on prices, i.e. trade less aggressively on their information. In terms of the information revealed by prices this later effect dominates the usual effect of less aggressive trading by the informed.

Moreover, in the competitive case we have that, contrary to the exogenous information model, informational efficiency is *increasing* in risk aversion. The intuition can be grasped by noting that for low risk aversion the seller is forced to sell very noisy signals to control for the dilution in the value of information via information leakage. Risk tolerant agents therefore end up with noisier signals, which makes the equilibrium prices more informative as risk-aversion is increased. This is not only in contrast with the exogenous information model, but also with the model with strategic traders, where for $\kappa > \kappa^*$ we have that price informativeness

[21] Of course, the large market that Admati and Pfleiderer (1986) study, following Hellwig (1980), actually does exhibit competitive behavior even if a non-competitive solution concept is used (Kyle, 1989; García and Urosević, 2007).

decreases in κ. We conclude that the price-taking assumption eliminates the $m = 1$ equilibria and generates contrary comparative static results with respect to the solution where agents anticipate their impact on prices.

4.3 More general information structures

Throughout the paper, we have assumed that the monopolist seller of information markets signals of the form $Y = X + \epsilon_i$ to the agents, where the set $(\epsilon_i)_{i=1}^m$ are i.i.d. with $\epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$. This class of signals are what Admati and Pfleiderer (1986) refer to as allocations with "personalized noise," in contrast to the case where the signals ϵ_i are perfectly correlated (as in Admati and Pfleiderer, 1988). Two natural questions arise: (i) is this information structure without loss of generality? (ii) how does the equilibrium change if the monopolist did not observe X, but rather a noisy signal of the form $X + \delta$, with $\delta \sim \mathcal{N}(0, \sigma_\delta^2)$? The answer to (i) is critical, since our analysis does not subsume the case studied in Admati and Pfleiderer (1988), where the market structure is as in section 4.1, but signals are perfectly correlated. Furthermore, addressing (ii) is also important in terms of checking how robust the corner solutions we have found actually are.

In the most general case, the monopolist seller would have at her disposal allocations of information among m traders of the form $Y_i = X + \delta + \epsilon_i$, with $\epsilon = (\epsilon_i)_{i=1}^m \sim \mathcal{N}(0, \Sigma_\epsilon)$, where her choice variable is the positive semi-definite variance-covariance matrix Σ_ϵ. In principle, her choices are therefore how many agents to sell to, m, as well as the elements of this matrix, a total of $m + m(m-1)/2$ entries, a rather daunting optimization problem. In order to gain some intuition, and at least a partial answer to (i) above, we let $\sigma_\delta = 0$ and study numerically the problem for a fixed m, when the monopolist seller chooses Σ_ϵ to be a symmetric matrix. Essentially this allows us to address to what extend the optimal solution changes when $m > 1$. The characterization of the equilibrium, fixing m and Σ_ϵ is standard. For a fixed m, and under symmetric allocations, the monopolist profits are a function of the actual variance of ϵ_i, σ_ϵ^2, as well as the correlation between signals sold to different traders, which we shall denote by ρ. One can verify that the profits are a decreasing function of ρ: rather intuitively, the monopolist is better off selling signals with low (or negative) correlation (conditional on X), since heterogeneity in information reduces competition among agents. Furthermore, profits are increasing in m for all $\kappa > \kappa^* \approx 1.74$, i.e. the main qualitative result in Theorem 1 is robust to more general allocations of information. Finally, we note that as $N \uparrow \infty$ the optimal allocations of information converge to those stated in Proposition 4. Rather intuitively, the monopolist seller wishes to sell negatively-correlated signals, but since Σ_ϵ needs to be positive semi-definite, as $m \uparrow \infty$ the optimal correlation tends to zero.

Having shown that personalized allocations are indeed optimal, we now turn to discussing the assumption that the monopolist information is perfect, i.e. in the notation introduced above, how does the optimal sales of information change if $\sigma_\delta^2 > 0$? Following our previous analysis, we solve the model fixing m, endowing agents with signals of the form $Y_i = X + \delta + \epsilon_i$, with $\epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$ i.i.d. We then calculate the profits for the monopolist under the given equilibrium, and solve for the optimal amount of noise σ_ϵ^2 for each m. We find that the nature of the solution, as stated in Theorem 1 does not change: for $\kappa > \kappa_n^*(\sigma_\delta)$ the monopolist optimally chooses to sell to $m = N$ agents, with $\lim_{N\uparrow\infty} s_\epsilon = 0$. On the other hand, for $\kappa < \kappa_n^*(\sigma_\delta)$ the monopolist may now optimally sell to a finite number of agents, much as in the case of market-orders discussed in section 4.1.[22] Intuitively, once $\sigma_\delta^2 > 0$ the trading strategies for the agents are risky, so for finite κ, even if small, the solution with a finite m may dominate the $m = 1$ allocation due to risk-sharing gains. Furthermore, when the monopolist has a noisy signal she may choose to sell to $m > 1$ agents signals with no noise added.[23] Finally, we find that the breakpoint κ_n^* changes, namely we have $\kappa_n^* > \kappa^*$, so that noisy signals make the parameter space for which noisy newsletters are optimal smaller.

[22] For example, if $\kappa = 2$ profits are maximized for $m = 2$, with $\sigma_\epsilon^2 = 0.18$.

[23] Consider the following parameter values: $\sigma_\delta^2 = 10$, $\kappa = 0.1$. The optimal allocation of information satisfies $m = 2$, with $\sigma_\epsilon^2 = 0$.

Appendix

The technical appendix in the authors' websites contains a full characterization of the equilibria the three different models studied in the paper. The interested reader is urged to consult this technical appendix for further details on the proofs of the propositions.

Proof of Proposition 1.

For each trader, the certainty equivalent of wealth is the constant c that solves $\mathbb{E}\left[u(\pi_i)\right] = u(c)$, and the CARA assumption implies

$$c = -\frac{1}{r}\log\left(-\mathbb{E}\left[u(\pi_i)\right]\right). \tag{16}$$

The first-order condition of an informed trader θ_i satisfies (Kyle, 1985, 1989)

$$\theta_i = \frac{\mathbb{E}[X - P_x|\mathcal{F}_i]}{r\mathrm{var}[X - P|\mathcal{F}_i] + P_\theta}; \tag{17}$$

for some $P_\theta > 0$ (in the competitive model $P_\theta = 0$).

In order to compute (16) we start by noticing that, at the interim stage, profits are conditionally normal, so we can write

$$c = -\frac{1}{r}\log\left(\mathbb{E}\left[\mathbb{E}[\exp\left(-r\pi_i\right)|\mathcal{F}_i]\right]\right) = -\frac{1}{r}\log\left(\mathbb{E}\left[\exp\left(-r\chi_i\right)\right]\right), \tag{18}$$

where

$$\chi_i = \mathbb{E}[\pi_i|\mathcal{F}_i] - \frac{r}{2}\mathrm{var}(\pi_i|\mathcal{F}_i). \tag{19}$$

Define $\eta \equiv \mathbb{E}[X - P_x|\mathcal{F}_i]$ and rewrite (19), using the definition of profits $\pi_i = \theta_i(X - P_x)$ and the first-order condition (17), as

$$\chi_i = \eta^2\left(\frac{P_\theta + \frac{r}{2}\mathrm{var}[X - P_x|\mathcal{F}_i]}{\left(P_\theta + r\mathrm{var}[X - P_x|\mathcal{F}_i]\right)^2}\right). \tag{20}$$

Since $P_x = \mathbb{E}[X|\mathcal{F}_u]$, we have $\mathbb{E}[\eta] = \mathbb{E}[X - P_x|\mathcal{F}_u] = 0$. Standard results on the expectation of quadratic forms of Gaussian random variables yield

$$\begin{aligned}\mathbb{E}\left[\exp\left(-r\chi_i\right)\right] &= \left(1 + 2r\sigma_\eta^2\left(\frac{P_\theta + \frac{r}{2}\mathrm{var}[X - P_x|\mathcal{F}_i]}{\left(P_\theta + r\mathrm{var}[X - P_x|\mathcal{F}_i]\right)^2}\right)\right)^{-1/2} \\ &= \left(1 + 2r\mathbb{E}[\chi_i]\right)^{-1/2},\end{aligned}$$

where the second equality follows by taking expectations in (20). Using the above expression in (18) and rearranging completes the proof. □

Proof of Proposition 2.

We first show that the monopolist sets $s_\epsilon = \infty$ when $m = 1$. The expected interim certainty equivalent can be expressed as

$$\mathbb{E}[\chi_i] = \frac{1}{2r}\left(\frac{\tau_i}{\tau_u} - 1\right)\frac{(1-2\zeta)}{(1-\zeta)^2} = \frac{1}{2r}\left(\frac{\zeta-\psi}{\psi}\right)\frac{(1-2\zeta)}{(1-\zeta)^2}.$$

Using the equilibrium definitions for ψ and ζ we have that, for any $\kappa \geq 0$, the monopolist's problem when she sells to one agent reduces to

$$\max_{s_\epsilon} \quad \mathcal{C} = \max_{s_\epsilon} \quad \frac{1}{2r}\log\left(1 + r\sigma_z \frac{\sqrt{r^2\sigma_z^2 + 4s_\epsilon(1+s_\epsilon)} - r\sigma_z}{2(1+s_\epsilon)}\right). \tag{21}$$

It is easy to verify that the above function is strictly increasing in s_ϵ, so that the optimal solution is to set $s_\epsilon = \infty$. Furthermore, taking limits in (21) we have that when the monopolist sells to a single agent her profits are given by

$$\mathcal{C} = \frac{1}{2r}\log\left(1 + r\sigma_z\right). \tag{22}$$

Let us introduce the variable ϕ, defined as $\tau_i = 1 + s_\epsilon + (m-1)\phi s_\epsilon$. In order to derive equation (6), we note that, for a fixed m, the problem in (4) can be equivalently stated as

$$\max_{y,\phi,\mu} \quad \mathcal{L} = \frac{m}{2r}\log(1+\Omega) - \mu G(y,\phi) \tag{23}$$

where $\Omega \equiv 2r\mathbb{E}[\chi_i]$, and

$$G(y,\phi) = \kappa\left[\frac{\phi}{y(1-1/m)(1-\phi)}\right]^{1/2} - 1 + 2\phi + \frac{\phi(1+y)}{(m-1)(1+\phi y)}$$

is the equilibrium condition (1) expressed in terms of ϕ, using the definitions of ζ and ψ.

The first-order conditions for y and ϕ from (23) are

$$\Omega\left(\frac{1}{2y} - \frac{\phi}{1+\phi y}\right) = \mu\left[\frac{\phi(1-\phi)}{(m-1)(1+\phi y)^2} - \frac{\Omega m(1+\phi y)}{2y^2}\right];$$

$$\Omega\left(\frac{1}{2\phi(1-\phi)} - \frac{y}{1+\phi y}\right) = \mu\left[2 + \frac{1+y}{(m-1)(1+\phi y)^2} + \frac{\Omega m(1+\phi y)}{2\phi(1-\phi)y}\right].$$

Dividing these two expressions one has that

$$\frac{\Omega m}{2y}\left(\frac{1}{y\phi(1-\phi)} - 1\right) = \frac{2\phi}{1+\phi y} - \frac{1}{y} + \frac{1}{(1+\phi y)^2(m-1)}\left(\phi - \frac{1}{2y}\right) \tag{24}$$

Define $x = \phi y$. Using the constraint $G(y, \phi) = 0$ and (24) one can verify after some simple algebraic manipulations that, at an interior optima, the following must be satisfied

$$Ax^2 + Bx + C = 0;$$

with $A = (m-2)(1-\phi)$, $B = (m-1)\phi(2\phi-1) + 1 + \phi(1-\phi)$, and $C = (m-1)(2\phi^2-1) + \phi^2$. Equation (6) follows immediately from the definitions of ϕ and ψ. This completes the proof. $\square$

Proof of Proposition 3.

First consider the case $m = 1$. From Proposition 2, in particular expression (22) we have that[24]

$$\lim_{r \downarrow 0} \mathcal{C} = \frac{\sigma_z}{2}. \tag{25}$$

Consider now the case $m \geq 2$. Define $x = \phi y$. Taking limits as $\kappa \downarrow 0$ in (4) and (1) one can verify that, fixing $m \geq 2$, the problem for $\kappa = 0$ reduces to

$$\max_{x,\phi} \quad \mathcal{C} = \frac{\sigma_z}{2} \sqrt{\frac{mx}{(1-\phi)(m-1)}} \frac{1}{(1+x)} \tag{26}$$

such that

$$(m-1)(1+x)(1-2\phi) - (\phi + x) = 0. \tag{27}$$

The first-order conditions to the Lagrangian are (μ denotes the multiplier)

$$\mathcal{C}\left(\frac{1}{2x} - \frac{1}{1+x}\right) = \mu[(m-1)(1-2\phi) - 1];$$

$$\frac{\mathcal{C}}{2(1-\phi)} = \mu[-2(m-1)(1+x) - 1].$$

These two equations with (27) characterize the (interior) optima for x and ϕ, for any given $m \geq 2$. Some simple manipulations of these first-order conditions yields the optimum x as a function of m:

$$x = \frac{1}{2(m-1)}\left(-1 + \sqrt{1 + 2(m-1) + 4(m-1)^2}\right) < 1.$$

Letting $w = -1 + \sqrt{1 + 2(m-1) + 4(m-1)^2}$ one further obtains

$$\phi = \frac{2(m-1)^2 + w(m-1) - w}{2(m-1)(1 + 2(m-1) + w)}.$$

[24]Note that profits are zero if $\sigma_z = 0$, so the case $\kappa = 0$ is only of economic interest for $\tau = 0$ and $\sigma_z > 0$.

In order to proof the Proposition, comparing (26) and (25) we see that it is sufficient to verify that

$$\frac{1}{(1+x)}\Gamma(m) \equiv \sqrt{\frac{mx}{(1-\phi)(m-1)}} < 1$$

for all $m \geq 2$. Some simple manipulations, noting that $w \in (2(m-2), 2(m-1))$, show that $\Gamma(m)$ is a strictly decreasing function of m, which achieves a maximum at $\Gamma(2) = 0.79 < 1$. Since this is a strict inequality, and the profit function for any m is a continuous function of κ, this completes the proof. □

Proof of Proposition 4.

We first show that profits are increasing in m for κ large enough. Define $\hat{\kappa} \equiv 1/\kappa$ and let y be given. For (1) to hold as $\hat{\kappa} \to 0$, we must have $\psi \to 0$. The equilibrium condition $\zeta = \psi\tau_i/\tau_u$ can be written as

$$y = \frac{(m-\psi)(\zeta-\psi)}{\psi(1+\psi(m-2)-\zeta(m-\psi))}, \tag{28}$$

For (28) to hold as $\hat{\kappa} \to 0$ we must have both $\zeta \to \psi$ and $(\zeta-\psi)/\psi \to y/m$. As a consequence,

$$\lim_{\hat{\kappa}\downarrow 0} 2r\mathbb{E}[\chi_i] = \lim_{\hat{\kappa}\downarrow 0} \frac{(\zeta-\psi)}{\psi}\frac{(1-2\zeta)}{(1-\zeta)^2} = \frac{y}{m}.$$

Taking the derivative of (4) with respect to m, and using the above expression, we have that as $\hat{\kappa}$ goes to zero[25]

$$\frac{d\mathcal{C}}{dm} = \frac{1}{2r}\left[\log(1+y/m) - \frac{y/m}{(1+y/m)}\right] > 0.$$

From this it is immediate that the monopolist optimally sells to $m = N$ agents. The open set statement in the Proposition then follows from continuity of the problem with respect to $\hat{\kappa}$.

Next we characterize the solution of the monopolist's problem as $m \to \infty$. For (28) to hold as an equality in the limit as $m \to \infty$, it is clear that we must have $\lim_{m\uparrow\infty}\zeta = \lim_{m\uparrow\infty}\psi \equiv \psi_\infty$. Then, taking limits as $m \to \infty$ in (4), one can verify that

$$\lim_{m\uparrow\infty} \mathcal{C} = \frac{y(1-2\psi_\infty)}{2r(1+y\psi_\infty)}, \tag{29}$$

[25]Notice that in the limit as $r \uparrow \infty$ monopolist's profits vanish, so what we are considering here is the limit as σ_z grows large.

and that the constraint (1) reduces to

$$\kappa\sqrt{\frac{\psi_\infty}{y(1-\psi_\infty)}} = 1-2\psi_\infty. \tag{30}$$

Using (30) to eliminate y in (29) the monopolist's problem becomes

$$\max_{\psi_\infty} \quad \mathcal{C} = \frac{\kappa^2}{2r}\frac{\psi_\infty(1-2\psi_\infty)}{\left((1-\psi_\infty)\left(1-2\psi_\infty\right)^2+\kappa^2\psi_\infty^2\right)}$$

Equating to zero the derivative of the above expression yields the optimality condition for ψ_∞ in (10). Moreover, from (10) we have $\kappa^2\psi_\infty^2 = (1-2\psi_\infty)^2(1-2\psi_\infty^2)$, and using this into (30) yields (9) and (11). Finally, using (9) into (29) yields (12). Moreover, $y\phi_\infty = \frac{1-2\phi_\infty^2}{1-\phi_\infty}$. As in this case $\phi_\infty \in [0, 1/2]$, we have that $\tau_u \geq 2$. Applying the implicit function theorem to (10) one can verify that $\phi_\infty(\kappa)$ defines a monotonically decreasing function of κ. This completes the proof. □

Proof of Proposition 5.

Part (a) follows from Dridi and Germain (1999), who study equilibrium properties in a model as described in section 4.1 assuming traders are risk-neutral, $r = 0$ in our notation. In particular, their results imply that for $m \leq 3$ the monopolist does not want to add any noise, and profits are given by

$$\mathcal{C} = \sigma_z\frac{1}{\sqrt{m}(1+m)}. \tag{31}$$

For $m \geq 4$ the monopolist does want to add some noise to his signals, namely set $\sigma_\epsilon = \sqrt{\frac{m-3}{2}}$. The monopolist profits for $m \geq 4$ at the optimal amount of noise are given by

$$\mathcal{C} = \frac{\sigma_z}{2\sqrt{2}}\sqrt{\frac{m}{m-1}} \tag{32}$$

Noting that both (31) and (32) are decreasing functions of m yields the conclusion that $m = 1$ is optimal under risk-neutrality. The open set statement follows from continuity.

Part (b). The optimality of selling to $m = N$ agents follows from the same argument used in the proof of Proposition 4. For (2) to hold as $\hat{\kappa} \to 0$, we must have $\psi \to 0$, implying $\tau_\pi \to \tau_i$. Solving the equilibrium condition $\zeta = \psi\tau_i/(1+\psi s_\epsilon)$ for s_ϵ yields

$$s_\epsilon = \frac{(\zeta-\psi)}{\psi(1-\zeta)}, \tag{33}$$

so that for (33) to hold as $\hat{\kappa} \to 0$ we must have both $\zeta \to \psi$ and $(\zeta-\psi)/\psi \to y/m$. As a consequence,

$$\lim_{\hat{\kappa}\downarrow 0} 2r\mathbb{E}[\chi_i] = \lim_{\hat{\kappa}\downarrow 0} \frac{(\tau_\pi - \tau_u)}{\tau_u} \frac{(1-2\zeta)}{(1-\zeta)^2} = \frac{y}{m}.$$

In order to prove the second part, we show that as $m \to \infty$ the monopolist's problem in the model with market orders and with limit orders coincide. Consider the problem for fixed y. Multiplying both sides of (33) by m yields

$$y = \frac{m(\zeta - \psi)}{\psi(1-\zeta)}.$$

For the above equation to hold as $m \to \infty$, it is clear that we must have $\lim_{m\uparrow\infty} \zeta = \lim_{m\uparrow\infty} \psi = \psi_\infty$. Taking limits as $m \to \infty$ in the profit function one obtains

$$\lim_{m\uparrow\infty} \quad \mathcal{C} = \frac{y(1-2\psi_\infty)}{2r(1+y\psi_\infty)},$$

and the constraint (2) reduces to

$$\kappa\sqrt{\frac{\psi_\infty}{y(1-\psi_\infty)}} = 1 - 2\psi_\infty.$$

As the last two equations are identical to (29) and (30), the problems with limit and market orders coincide. This concludes the proof. □

Proof of Proposition 6.

Using the constraint (3) to eliminate s_ϵ one can verify that, for a fixed m, the monopolist seller of information is solving

$$\max_{\phi} \quad \mathcal{C} = \frac{m}{2r} \log\left(1+\Omega\right); \tag{34}$$

where

$$\Omega = \frac{\kappa^2\phi}{(1-\phi)\left[m-1+\phi+\frac{(\kappa m\phi)^2}{(m-1)(1-\phi)^3}\right]}.$$

The first-order condition for ϕ yields :

$$\frac{(\kappa m\phi)^2}{(m-1)(1-\phi)^3} = \frac{m-1+\phi^2}{1+\phi}. \tag{35}$$

By the envelope theorem

$$\frac{d\mathcal{C}}{dm} = \frac{1}{2r}\left[\log\left(1+\Omega\right) - \alpha\frac{\Omega}{1+\Omega}\right], \tag{36}$$

where

$$\alpha = \frac{m + \dfrac{(m-2)(\kappa m\phi)^2}{(m-1)^2(1-\phi)^3}}{m - 1 + \phi + \dfrac{(\kappa m\phi)^2}{(m-1)(1-\phi)^3}}. \tag{37}$$

From (36), a sufficient condition for profits to be increasing in m is that $\alpha \leq 1$. Substituting from (35) into (37) and rearranging one can verify that indeed $\alpha \leq 1$, so the sufficient condition is satisfied and, as a consequence, the monopolist finds optimal to sell to $m = N$. The rest of expressions in the Proposition are immediate taking limits in (34) and (3), using (35).

Informational efficiency is measured by the precision conditional on the market price, given by $\tau_u = 1 + y\psi$. Notice that in the limit as $m \to \infty$, we have $\phi \to \psi$. In the competitive case we can express (14) as $(\kappa\phi_\infty)^2 = \frac{(1-\phi_\infty)^3}{1+\phi_\infty}$ and substitute into (13) to get $y\phi_\infty = \frac{1}{1+\phi_\infty}$; as $\phi_\infty \in [0,1]$, we have $\tau_u \in [1.5, 2]$. Since $\tau_u \geq 2$ in the strategic model (see Propositions 3 and 4), prices are less informative in the competitive equilibrium. Finally, applying the implicit function theorem to (14) one can verity that, as in the imperfectly competitive case, $\phi_\infty(\kappa)$ is a monotonically decreasing functions of κ. In turn, this implies that τ_u is monotonically *increasing* in κ in the competitive equilibrium. This concludes the proof. □

References

Admati, A. R., 1985, "A noisy rational expectations equilibrium for multi-asset securities markets," *Econometrica*, 53(3), 629–657.

Admati, A. R., and P. Pfleiderer, 1986, "A monopolistic market for information," *Journal of Economic Theory*, pp. 400 – 438.

——— , 1987, "Viable allocations of information in financial markets," *Journal of Economic Theory*, 43, 76–115.

——— , 1988, "Selling and trading on information in financial markets," *American Economic Review*, 78(2).

——— , 1990, "Direct and indirect sale of information," *Econometrica*, 58(4), 901–928.

Benabou, R., and G. Laroque, 1992, "Using privileged information to manipulate markets: insiders, gurus, and credibility," *The Quarterly Journal of Economics*, pp. 921–958.

Bernhardt, D., and B. Taub, 2006, "Kyle vs. Kyle (85 v. 89)," *Annals of Finance*, 2(1), 23–38.

Biais, B., and L. Germain, 2002, "Incentive compatible contracts for the sale of information," *Review of Financial Studies*, 15, 987–1003.

Brown, D. P., and Z. M. Zhang, 1997, "Market orders and market efficiency," *Journal of Finance*, 52, 277–308.

Cespa, G., 2007, "Information sales and insider trading," *Journal of Finance, forthcoming.*

Cripps, M. W., and J. M. Swinkels, 2004, "Efficiency of large double auctions," working paper, Washington University in St. Louis.

Dridi, R., and L. Germain, 1999, "Noise and competition in strategic oligopoly," working paper, London School of Economics.

Fishman, M. J., and K. M. Hagerty, 1995, "The Incentive to Sell Financial Market Information," *Journal of Financial Intermediation*, 4, 95–115.

Gaier, E. M., and B. Katzman, 2002, "Endogeneous information quality in common value auctions," working paper, University of Miami.

García, D., and B. Urosević, 2007, "Noise and aggregation of information in large markets," working paper, UNC at Chapel Hill.

García, D., and J. M. Vanden, 2006, "Information acquisition and mutual funds," working paper, Dartmouth College.

Germain, L., 2005, "Strategic noise in competitive markets for the sale of information," *Journal of Financial Intermediation*, 14, 179–209.

Graham, J. R., and C. R. Harvey, 1996, "Market timing ability and volatility implied in investment newsletters' asset allocation recommendations," *Journal of Financial Economics*, 42, 397–421.

Guzman, R. M., and C. D. Kolstad, 1997, "Auction equilibrium with costly information acquisition," working paper, University of Illinois at Urbana-Champaign.

Hausch, D. B., and L. Li, 1991, "Private values auctions with endogeneous information: revenue equivalence and non-equivalence," working paper, University of Wisconsin.

Hellwig, M. F., 1980, "On the aggregation of information in competitive markets," *Journal of Economic Theory*, 22, 477–498.

Hong, H., and M. Shum, 2004, "Rates of information aggregation in common value auctions," *Journal of Economic Theory*, 116, 1–40.

Jackson, M. O., 2003, "Efficiency and information aggregation in auctions with costly information," *Review of Economic Design*, 8(2), 121–141.

Jaffe, J. F., and J. M. Mahoney, 1999, "The performance of investment newsletters," *Journal of Financial Economics*, 53, 289–307.

Kovalenkov, A., and X. Vives, 2004, "Competitive rational expectations equilibria without apology," working paper, University of North Carolina.

Kremer, I., 2002, "Information aggregation in common value auctions," *Econometrica*, 70(4), 1675–1682.

Kyle, A. S., 1985, "Continuous auctions and insider trading," *Econometrica*, 53(6), 1315–1336.

——— , 1989, "Informed speculation with imperfect competition," *Review of Economic Studies*, 56(3), 317–355.

Matthews, S., 1984, "Information acquisition in discriminatory auctions," in *Bayesian models in economic theory*, ed. by M. Boyer, and R. Kihlstrom. chap. 11, pp. 181–207.

Metrick, A., 1999, "Performance evaluation with transactions data: the stock selection of investment newsletters," *Journal of Finance*, 54(5), 1743–1775.

Milgrom, P. R., 1979, "A convergence theorem for competitive bidding with differential information," *Econometrica*, 47(3), 679–688.

——— , 2000, "Rational expectations, information acquisition and competitive bidding," *The Quarterly Journal of Economics*, 140(2), 341–388.

Moresi, S., 2000, "Information acquisition and research differentiation prior to an open-bid auction," *International Journal of Industrial Organization*, 18, 723–746.

Morgan, J., and P. Stocken, 2003, "An analysis of stock recommendations," *RAND Journal of Economics*, 34(1), 183–203.

Persico, N., 2000, "Information acquisition in auctions," *Econometrica*, 68(1), 135–148.

Pesendorfer, W., and J. M. Swinkels, 1997, "The loser's curse and information aggregation in common value auctions," *Econometrica*, 65(6), 1247–1281.

——— , 2000, "Efficiency and information aggregation in auctions," *American Economic Review*, 90(3), 499–525.

Reny, P. J., and M. Perry, 2003, "Toward a strategic foundation for rational expectations equilibrium," working paper, University of Chicago.

Ross, S. A., 2000, "Markets for agents: fund management," working paper, MIT.

Subrahmanyam, A., 1991, "Risk aversion, market liquidity, and price efficiency," *Review of Financial Studies*, 4(3), 417–441.

Veldkamp, L. L., 2003, "Media frenzies in markets for financial information," working paper, INSEAD.

Verrecchia, R. E., 1982, "Information acquisition in a noisy rational expectations economy," *Econometrica*, 50(6), 1415–1430.

Vives, X., 1995, "Short-term investment and the informational efficiency of the market," *Review of Financial Studies*, 8(1), 125–160.

Wilson, R., 1977, "A bidding model of perfect competition," *Review of Economic Studies*, 44(3), 511–518.

——— , 1979, "Auctions of shares," *The Quarterly Journal of Economics*, 93(4), 675–689.

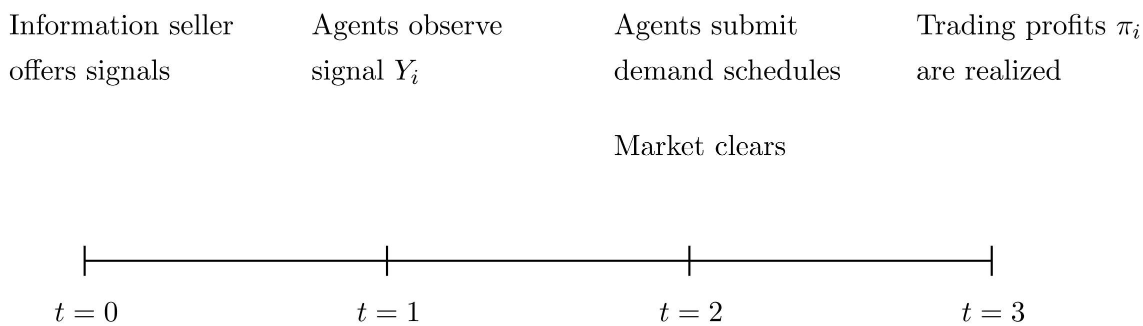

Figure 1: Timeline for information sales model.

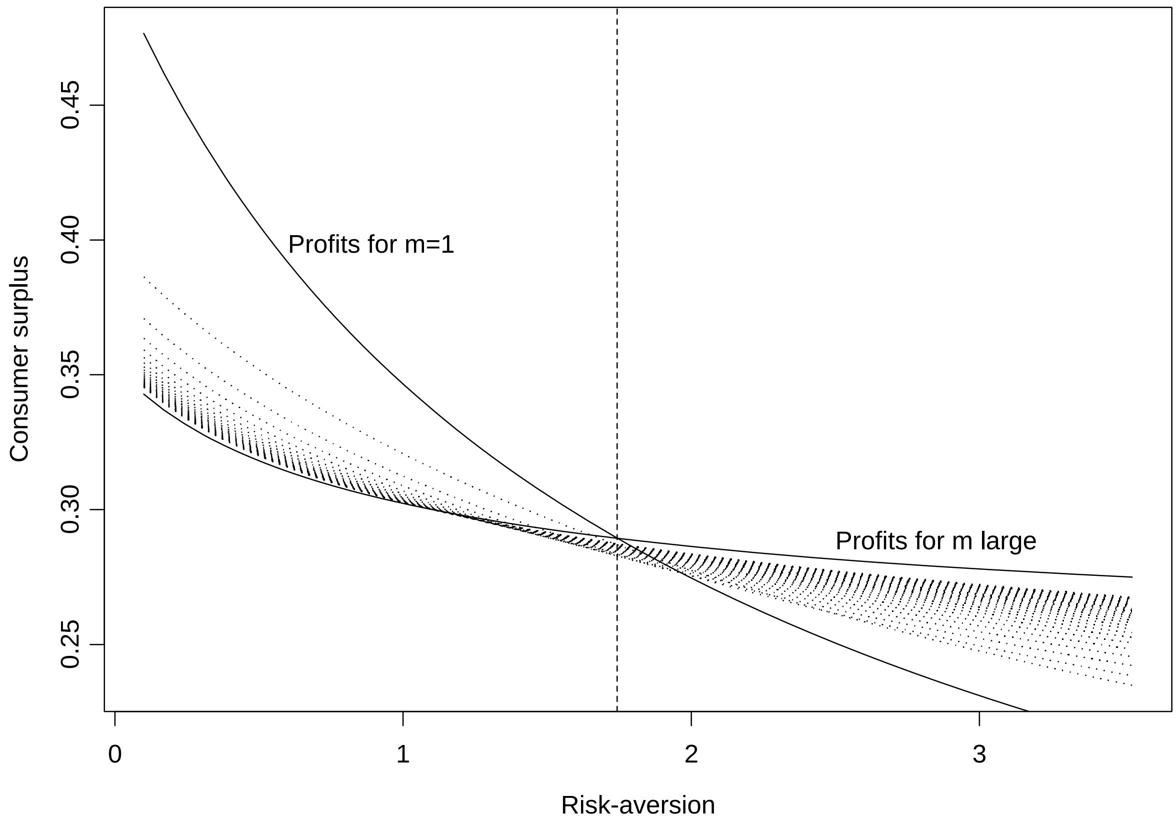

Figure 2: Equilibrium consumer surplus for different values of informed agents m and risk-aversion r, at the optimal noise s_ϵ, in the Kyle (1989) model. The solid lines correspond to consumer surplus $\mathcal{C}$ when $m = 1$ and $m = N$ (for large N), whereas the dashed lines correspond to values of $m = 2, \ldots, 40$. The vertical line gives the breakpoint between regions where different type of information sales, $m = 1$ versus $m = N$, are optimal. Noise trading intensity is set at $\sigma_z = 1$.

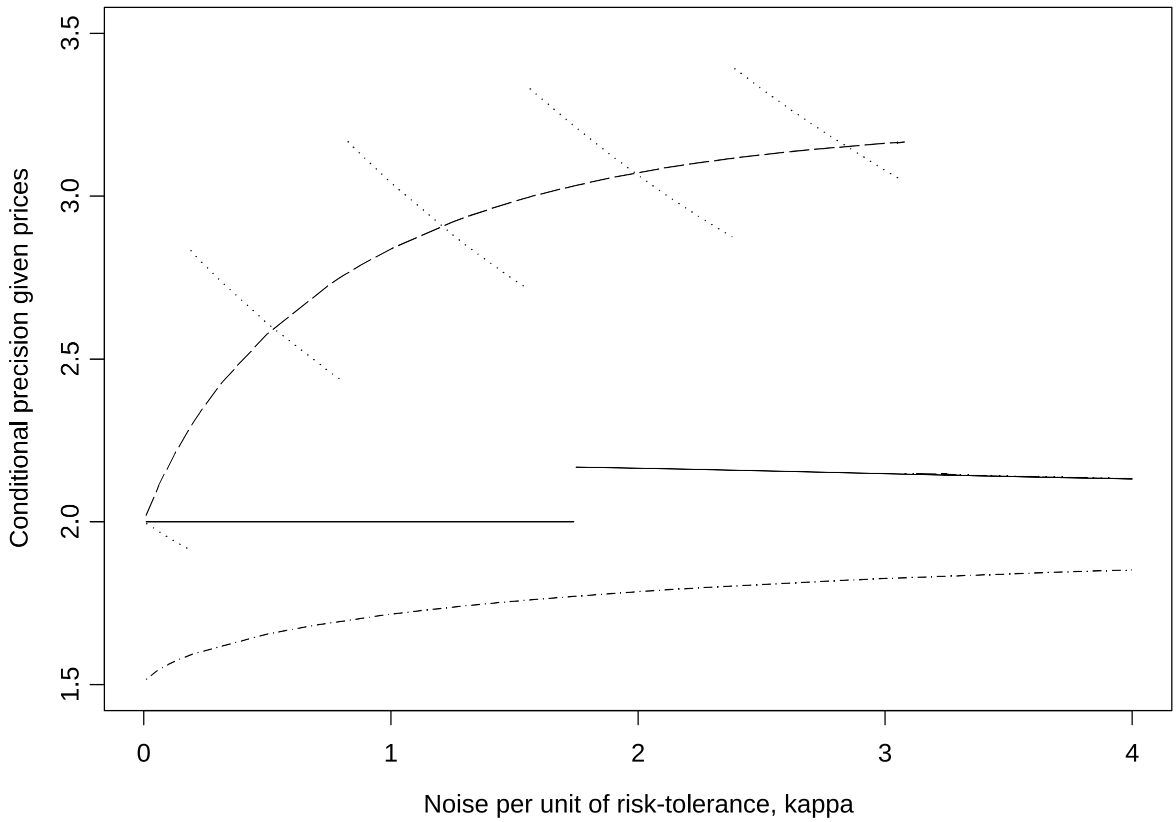

Figure 3: Equilibrium values for $\mathrm{var}(X|P_x)^{-1}$ for different values κ. The solid lines corresponds to the model with strategic traders and limit orders. The dotted and long-dash lines correspond to the model with strategic traders and market orders (dotted for the case where m is treated as an integer, dashed when m is treated as a continuous variable). The line with dashes and dots corresponds to the model where traders are price takers.

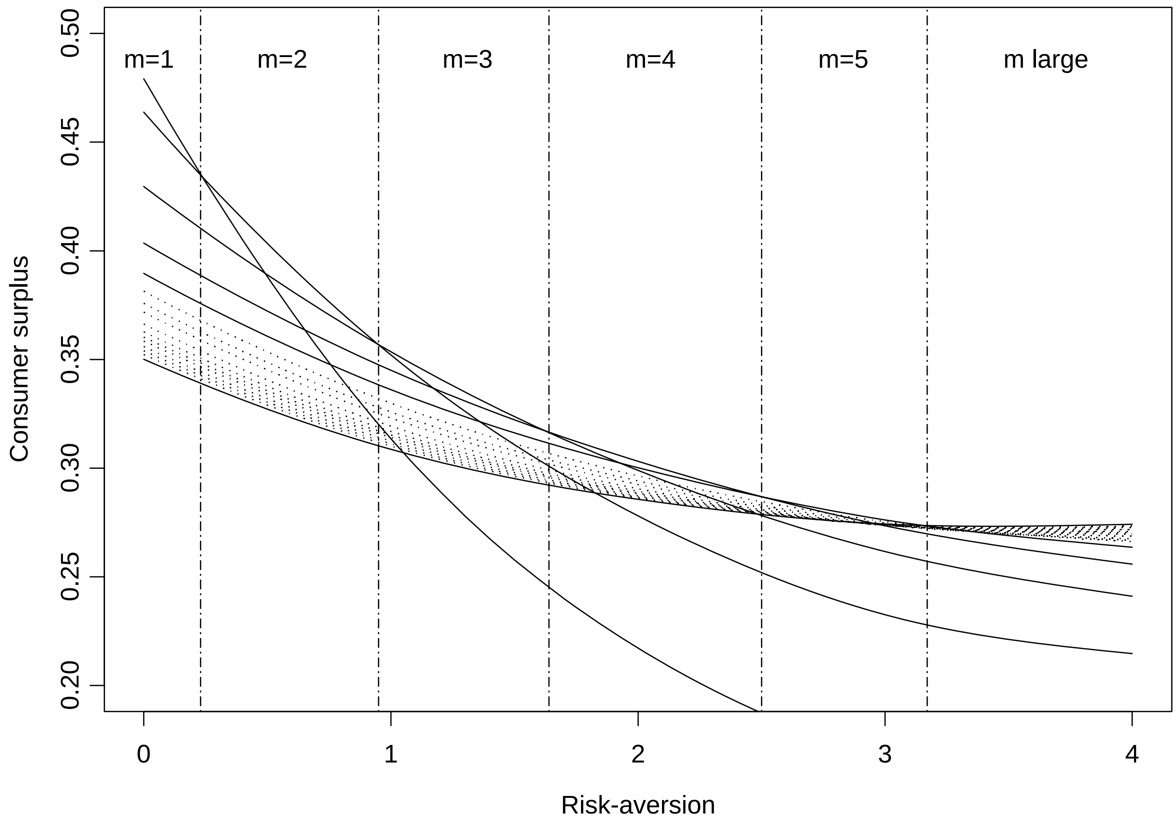

Figure 4: Equilibrium consumer surplus for different values of informed agents m and risk-aversion r, at the optimal noise s_ϵ, in the Kyle (1985) model. The solid lines correspond to consumer surplus $\mathcal{C}$ when $m = 1, 2, 3, 4, 5$ and $m = N$ (for large N), whereas the dotted lines correspond to values of $m = 6, \ldots, 40$. The vertical lines give the breakpoints between regions where different m are optimal. Noise trading intensity is set at $\sigma_z = 1$.