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Income Insurance and the Equilibrium Term-Structure of Equity*

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ABSTRACT

This paper documents that GDP, wages and dividends are co-integrated but feature term-structures of risk respectively flat, increasing and decreasing. Income insurance within the firm from shareholders to workers explains those term-structures: distributional risk smooths wages and enhances the short-run risk of dividends. A simple general equilibrium model, where labor rigidity affects dividend dynamics and the price of short-run risk, reconciles standard asset pricing facts with the term-structures of equity premium and volatility and those of macroeconomic variables, at odds in leading models. Income insurance also helps to explain dividend growth predictability, cross-sectional value premia, counter-cyclical Sharpe-ratios, and interest rates term-premia.

Keywords. term structure of equity · income insurance · dividend strips · distributional risk · equilibrium asset pricing

JEL Classification. D53 · E24 · E32 · G12

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I. Introduction

Leading equilibrium asset pricing models can describe some important features of financial markets, such as i) the high equity premium, ii) the excess return volatility, and iii) the low and smooth risk-free rate. These models have different rationale (e.g. habit formation, time-varying expected growth, disasters, prospect theory) but share an important feature: priced risk comes from variation in long-run discounted cash-flows.[1] An key implication of this is that these models are inconsistent with some recent empirical evidence, such as i) the decreasing variance ratios of dividends, ii) the downward sloping term-structures of equity risk and premia (at least at short and medium horizons), and iii) the cyclicality and mean-reversion of the dividend-share of consumption.[2] This is important because the term-structures of both cash-flows and equity returns provide information about how prices are determined in equilibrium. Hence, a term-structure perspective offers additional testable implications to asset pricing frameworks and can help us to understand the macroeconomic determinants of asset prices.

The main contribution of the paper is twofold. First, I propose a simple model of labor relations that reconciles the two sets of stylized facts mentioned above in the context of a parsimonious and closed-form general equilibrium model. Second, I empirically support the idea that income insurance within the firm is at the heart of the *timing* of macroeconomic risk. The rationale behind this is as follows. The paper argues that the short-term risk of dividends and the dynamics of the dividend-share are due to a "distributional risk". The model assumes that workers benefit of income insurance from shareholders and this leads to counter-cyclical labor-share dynamics.[3] As a consequence, the dividend-share is negatively correlated with the labor-share and, hence, dividends are more pro-cyclical and volatile. Such a *cyclicality effect* implies that equity is a bad hedge against aggregate risk and, thus, commands a high compensation. While undiscovered by the previous literature, also a *term-structure effect* takes place as long as income insurance preserves the stationarity of the labor- and dividend-shares, as in the real data. The transitory component of aggregate risk is shared asymmetrically among workers and shareholders, whereas the permanent component is faced by both. Namely, shareholders bear a high short-term risk because dividend risk shifts toward the short horizon, whereas workers bear a strong long-run risk because the wage risk increases with the horizon. Finally, under standard preferences, such a term-structure effect is inherited by financial markets and downward-sloping term-structures of equity risk and premia obtain.

An empirical investigation supports the main model mechanism. Consistently with the assumption

[1] Examples are the seminal works by Campbell and Cochrane (1999), Barberis, Huang, and Santos (2001), Bansal and Yaron (2004) and Gabaix (2012), among others.

[2] Belo, Collin-Dufresne, and Goldstein (2014) document the term-structure of aggregate dividend volatility; van Binsbergen, Brandt, and Koijen (2012) and van Binsbergen, Hueskes, Koijen, and Vrugt (2013) provide empirical evidence about the term-structures of equity returns; Boldrin and Horvath (1995), Longstaff and Piazzesi (2004), Danthine, Donaldson, and Siconolfi (2006), Santos and Veronesi (2006) and Ríos-Rull and Santaeulàlia-Llopis (2010) investigate the labor- and dividend- shares.

[3] A long tradition, since Knight (1921), Baily (1974) and Azariadis (1978), emphasizes distributional risk and suggests that the very role of the firm is that of insurance provider. On the one hand, labor's claim on output is partially fixed in advance and, hence, shareholders bear most of aggregate uncertainty and, on the other hand, in exchange of income insurance, shareholders gain flexibility in labor supply.

of income insurance from shareholders to workers, in the real data the labor-share is counter-cyclical and mean-reverting and is the main determinant of the fluctuations in the dividend-share.[4] Moreover, real GDP growth rates have approximately a flat term-structure of variance, whereas wages and dividends feature variances which, respectively, increase and decrease with the horizon. In accord with the model, these term-structures and the co-integrating relationship among GDP, wages, and dividends support the idea that both workers and shareholders are subject to permanent shocks but they share transitory shocks in such a way that workers are subject more to long-run risk than short-run risk and shareholders more to short-run risk than long-run risk. Such a *term-structure effect of income insurance* is confirmed by the fact the times-series of the wage-to-dividend ratios strongly explain the joint dynamics of wage and dividend variance ratios. In addition, the labor-share predicts dividend growth and excess stock returns, coherently with the positive and negative intertemporal relationships implied by the model.[5]

Standard real business cycle models fail to explain financial markets mainly because Walrasian labor markets lead to constant labor-share. In turn, dividends include a hedge component and, hence, do not deserve a sizeable premium. Instead, labor rigidity due to income insurance leads to a source of *short-run but persistent* risk which drives the term-structures of both macroeconomic variables and equity returns.

The paper provides two additional noteworthy contributions. First, the model offers an extensive analysis of asset pricing in spite of a peculiar perspective, based on the equilibrium term-structures. Consider the following question: whence the equity premium? As suggested by van Binsbergen et al. (2012), the properties of the single discounted cash-flows at each future horizon provide a lot of information about the way stock prices are formed. I derive a closed-form equilibrium density which describes the relative contribution of each time horizon to the whole equity premium.

Insert Figure 1 about here.

The left panel of Figure 1 shows that a long-run risk model (blue line) –as well as other leading models– implies that long-run cash-flows are the main determinant of the equity premium. Instead, the model of this paper (red line) implies a strong shift of priced risk toward the short-run. Namely, in the long-run risk model, the density peaks at about 15 years far in the future, whereas in the model of this paper the immediate future mostly contributes to the market compensation. Why do investors feature such a different perception of priced risk? The right panel of Figure 1 reports the variance ratios of dividends. While the model of this paper (left axis) roughly matches the empirical evidence (black dashed lines), the long-run risk model overestimates long-horizon dividend risk by two orders of magnitude. This example clearly shows that disregarding the information from the term-structures of risk of macroeconomic fundamentals is highly misleading.[6]

[4] In particular, I show that at the aggregate level, the shareholders remuneration, as a residual choice of the firm, is mainly affected by variation in the remuneration of human capital and only marginally by investment and financing decisions.

[5] A similar analysis of the non-financial corporate sector data confirms the results concerning the total economy and provides further robustness.

[6] Beyond the specific calibration in Figure 1, long-run risk models feature upward-sloping variance-ratios of dividends

As a second additional contribution, the paper analyzes the joint effect on the equilibrium term-structures of both recursive preferences and flexible dynamics of fundamentals. Exogenous uncertainty represents the total resources of the representative firm which should be distributed to workers and shareholders. Such a process is usually stationary in real business cycle literature and integrated in asset pricing literature. This paper analyzes the role of both transitory and permanent shocks jointly and shows how they can differently affect equilibrium outcomes. Under standard preferences, transitory and permanent shocks contribute, respectively, negatively and positively to the slope of the term-structures of equity volatility and premia. A model with both types of shocks can reconcile an otherwise standard model with the evidence about dividend and equity term-structures, as long as short-run but persistent risk is large enough. A simple model of labor relations provides a rationale.

The model works as follows. In spirit of Danthine and Donaldson (1992, 2002), Boldrin and Horvath (1995) and Gomme and Greenwood (1995), I model labor relations through a contract rule that produces, as a result of a simple bargaining model, income insurance to workers as long as those are more risk averse than shareholders, in line with the empirical literature (see Moskowitz and Vissing-Jørgensen (2002)). Consequently, the labor-share has counter-cyclical dynamics and enhances the riskiness of owning capital. Indeed, income insurance endogenously leads to a strong desire for the shareholders to smooth consumption intertemporally.[7] Income insurance loads on shareholders most of the fluctuations due to the transitory component of aggregate risk. Then, distributional risk due to labor relations increases the short-run riskiness of dividends even if the latter are not riskier than the firm's operational cash-flows in the long-run. Dividends are risky at short horizons and feature downward sloping term structures of volatility. This result is consistent with the empirical work by Guiso, Pistaferri, and Schivardi (2005) which documents economically significant income insurance from employers to employees, mostly related to transitory productivity shocks. Similar results are documented by Han, Maug, and Schneider (2013).

Equilibrium returns on equity depend on the marginal rate of substitution of shareholders, who act as a representative agent under limited market participation. In presence of both long-run and short-run persistent shocks, the term-structure of equity volatility is non-monotone. Long-run persistent risk contributes positively to the slope of the equilibrium term-structure, whereas the short-run persistent risk contributes negatively if the elasticity of intertemporal substitution is larger than unity and vice-versa. Negative slopes at short and medium horizons obtain as long as income insurance enhances short-run persistent risk. Moreover, the model generates "long-run" excess return volatility over dividends and consumption (Beeler and Campbell (2012)).

The term-structures of equity premia depend on risk preferences. Short-run persistent risk contributes

because the key ingredient, i.e. time-varying expected growth and/or volatility, induces uncertainty that integrates with the horizon. Instead, in actual data long-horizon dividend growth rates are about 4 or 5 times less risky than short-horizon ones.

[7] Income insurance essentially substitutes for exogenous approaches such as habit formation or some form of market incompleteness. To make unambiguous the equilibrium implications of labor relations, production is modeled in reduced form and limited market participation is assumed such that workers do not own capital. This prevents alternative forms of consumption smoothing which can hide the role of income insurance.

negatively to the slope if the intertemporal substitution effect dominates the wealth effect and vice-versa. Instead, long-run persistent risk contributes negatively to the slope if either i) the elasticity of intertemporal substitution is larger than one and shareholders have preference for the late resolution of uncertainty or ii) the elasticity of intertemporal substitution is smaller than one and shareholders have preference for the early resolution of uncertainty. Both the cases have poor equilibrium implications about standard asset pricing facts in the context of a long-run risk model. Instead, the model presented in this paper reconciles the equilibrium term-structures with the standard asset pricing facts by accounting for *both* long-run and short-run persistent shocks, given the usual parametrization of preferences.

The model calibration exploits the information from the term-structures of wage and dividend risk. On the one hand, they represent additional stylized facts captured by the model, and, on the other hand, they provide information about the persistence of both transitory and permanent shocks –a main issue in asset pricing models.

Interestingly, the model leads to four important sets of additional testable implications. First, the model improves on the description of the term-structure of interest rates. The real yield increases with the horizon even if the intertemporal substitution effect dominates the wealth effect. This is not the case in the long-run risk model but is important for two reasons. On the one hand, increasing real yields help to describe a positive nominal term-spread –as in the real data– without referring only to inflation risk. On the other hand, increasing real yields help to produce downward sloping premia on the equity yields, in line with the empirical findings by van Binsbergen, Hueskes, Koijen, and Vrugt (2013).

Second, the model provides a rationale for the value premium and offers an equilibrium explanation to the framework of Lettau and Wachter (2011). They define growth and value firms through the longer and shorter duration of their cash-flows and then obtain a value premium by an exogenously specified pricing kernel. This paper endogenizes the pricing kernel and shows that capturing the downward-sloping term-structure of equity automatically leads to cross-sectional equity prices featuring a value premium. A growth premium would obtain if labor relations do not enhance enough the effect of short-run risk.

Third, van Binsbergen et al. (2013) and Muir (2014) refine the findings of van Binsbergen et al. (2012) suggesting that the negative slope of equity premia is an unconditional property of the data, whereas conditional equity premia are slightly increasing in good times and strongly decreasing in bad times. I show that such a dynamics obtains in the model –without deteriorating other results– once fundamentals feature countercyclical heteroscedasticity.

Fourth, income insurance leads to a countercyclical dynamics of conditional premia, volatility, and Sharpe ratios of stock returns. Moreover, the labor-share predicts dividend growth and excess stock returns, respectively, with positive and negative signs, in line with the results of the empirical analysis.[8]

Related literature. A number of recent papers points out the importance of labor relations for asset

[8]For the sake of exposition and analytic tractability, these results are presented under the special case of power utility. They also hold under recursive preferences as long as numerical solutions avoid the log-linearization of dividends dynamics.

pricing. Namely, the high risk of owning capital is due to a large extent to the smooth dynamics of wages. The latter can obtain because of preference heterogeneity as in Danthine and Donaldson (1992, 2002), infrequent wage resettling as in Favilukis and Lin (2012), search frictions as in Kuehn, Petrosky-Nadeau, and Zhang (2012), labor mobility as in Donangelo (2014). All these models imply an explicit or implicit income insurance from shareholders to workers, which takes place within the firm and is consistent with the empirical findings of Guiso et al. (2005) and Han et al. (2013).

This paper complements such a literature studying the term-structure implications of income insurance in general equilibrium. Differently from those works, I provide *both empirical* and *theoretical* support to the idea that labor relations alter not only the *cyclicality* of dividends but also the *timing* of dividend risk. The latter is key to reconcile long and short term patterns of equity returns. My paper differs from the above mentioned works also on another dimension. Danthine and Donaldson (1992, 2002), Favilukis and Lin (2012) and Kuehn et al. (2012) propose big macro-finance models and rely on numerical solutions. Instead, I consider a simpler and more parsimonious economy and provide analytical solutions.[9] This allows for a deep investigation of term-structures under recursive preferences and for a direct comparison with the long-run risk model of Bansal and Yaron (2004), a standard benchmark in asset pricing.

Ai, Croce, Diercks, and Li (2012), Belo, Collin-Dufresne, and Goldstein (2014) and Kogan and Papanikolaou (2015) suggest alternative channels which can contribute to explain the term-structure of equity.[10] Ai et al. (2012) proposes a general equilibrium model where heterogeneous investment risk due to vintage capital leads to a downward-sloping term-structure of equity premia. Ai et al. (2012) does not investigate the term-structure of return volatility and requires an extremely high risk aversion to obtain a sizeable equity premium. Belo et al. (2014) shows that fluctuations in financial leverage can shift the risk from EBIT to dividends toward the short horizon. However, at the aggregate level EBIT risk and dividend risk have essentially the same term-structure. This implies that the leverage channel is quantitatively negligible and that the determination of short-term risk takes place earlier than financing decisions. Both Belo et al. (2014) and Kogan and Papanikolaou (2015) are *partial equilibrium* models: a downward-sloping term-structure of equity premia obtains because of the exogenous co-movements between the priced factors and, respectively, financial leverage and investment-specific risk.[11] Labor relations, financial leverage, and investment decisions are three channels which can contribute to the equilibrium explanation of the timing of equity risk. The empirical analysis supports the idea that, at the aggregate level, variations in the labor-share are a more important determinant of the dividend-share and of the timing of dividend risk than financing and investment decisions. Namely, the gap between the

[9] Even if my model is silent on production and workers saving decisions, Danthine, Donaldson, and Siconolfi (2006) have shown that a similar form of distributional risk improves the asset pricing implications of a standard real business cycle model without deteriorating business cycle predictions.

[10] Croce, Lettau, and Ludvigson (2014) and Marfè (2013) consider the term-structure of equity in the context of a long-run risk model. Berrada, Detemple, and Rindisbacher (2013), Curatola (2014) and Marfè (2014) focus on non-standard beliefs formation and state-dependent risk preferences.

[11] A general equilibrium approach would produce economic restrictions on those correlations and offer additional insights into the relationship between either leverage or investment-specific risk and equity term-structures.

flat term-structure of GDP risk and the downward-sloping one of dividend risk should be almost entirely imputed to the timing of wage risk.

This paper is also related to the literature about preference heterogeneity. Under complete markets, Chan and Kogan (2002) show that preference heterogeneity leads to counter-cyclical price of risk as a result of endogenous time-variation in the aggregate relative risk aversion.[12] The same endogenous counter-cyclical dynamics obtains here and is due to variations in dividend volatility. The result is still due to preference heterogeneity but obtains through the income insurance channel.

The paper is organized as follows. Section II provides empirical support to the main model mechanism. Section III describes the economy and labor relations. Section IV and V derive the equilibrium pricing kernel and asset prices. Asset pricing results are in Section VI. Alternative model specifications are discussed in Section VII. Section VIII concludes.

II. Empirical Analysis

This section documents several properties of aggregate dividends, wages and other GDP components in the US data. These properties provide empirical support to the model and drive the main results. First, I show that i) GDP, dividends and wages are co-integrated, ii) wages are smoother and more persistent than dividends, and iii) the labor-share moves counter-cyclically, forecasts dividend growth and is the main driving force of the dividend-share. Second, I document that the term-structure of GDP volatility is approximately flat over long horizons, whereas those associated with wages and dividends are respectively increasing and decreasing. Third, I document that the timing of wage risk and dividend risk is strongly driven by variation in workers' remuneration relative to shareholders' remuneration. In a nutshell, the analysis offers empirical support to the main economic mechanism of the model, that is income insurance from shareholders to workers and its implications for both dividends and equity returns. Online appendix OA.B focuses on the non-financial corporate sector only and provides further robustness.

A. Data

The key variables are the GDP shares of aggregate wages and dividends. The main data source is the National Income and Products Account (NIPA), available through the Bureau of Economic Analysis (BEA) website. Real GDP, aggregate wages (*"Compensation of employees paid"*), dividends (*"Net dividends"*) and investments (*"Gross private domestic investment"*) are from sections 1.10 and 1.1.6. As a second and broader measure of shareholders' compensation I consider corporate profits after tax (*"Profits after tax with inventory valuation and capital consumption adjustments"*) from section 1.10. Data are collected at

[12] Therefore, Chan and Kogan (2002) model can be interpreted as a micro-founded version of nonlinear habit models such as Campbell and Cochrane (1999). See Xiouros and Zapatero (2010) for the empirical shortcomings of such an approach.

yearly frequency since 1932 to 2012.[13] I also consider a measure of the aggregate leverage ratio, computed as in Belo et al. (2014), from the Flow of Funds Accounts of the US (Board of Governors of the Federal Reserve System) table B.102. Data of excess returns on the S&P500 are from Robert Shiller's webpage.

B. Dividend-share and the cyclicality effect of income insurance

Table I summarizes statistics about GDP growth rates as well as both growth rates and GDP shares of wages, investments, corporate profits and net dividends.

Insert Table I and Figure 2 about here

First, the growth rates of wages are smooth like those of GDP, whereas the growth rates of net dividends (as well as corporates profits and investments) are substantially more volatile. Second, the labor-share is larger, smoother and more persistent than the dividend- and investment- shares. Moreover, the labor-share is counter-cyclical: its variations are negatively correlated with GDP growth rates (-.26, similarly to Ríos-Rull and Santaeulàlia-Llopis (2010)). Instead, corporate profits move pro-cyclically and net dividends and investments do not feature a correlation significantly different from zero. Third, the GDP shares are stationary: they feature negative coefficients in the regression of the share differences on their lagged levels.[14] Figure 2 shows the scatter plots of the labor-share with the dividend-share, the profit-share and the GDP growth rates. We can observe a clear negative relation between the labor-share and the dividend-share (their correlation is -.67).

Insert Table II about here

Shareholders' remuneration and can be interpreted as a residual choice of the representative firm with respect to the remuneration of human capital as well as the investment and financing decisions. Therefore, I investigate the intertemporal relationship between the dividend-share (D/Y) and the labor-share (L/Y), investment-share (I/Y) and aggregate leverage ratio (Lev). At first, I regress either the shares of net dividends (D^1) or after tax corporate profits (D^2) on their lags as well as the lags of the labor-share, the investment-share and the leverage ratio:

$$D^i/Y_t = b_0 + \sum_{j=1,2}\left(b_{0j}\mathrm{D}^i/\mathrm{Y}_{t-j} + b_{1j}\mathrm{Lev}_{t-j} + b_{2j}\mathrm{I/Y}_{t-j} + b_{3j}\mathrm{L/Y}_{t-j}\right) + \varepsilon_t, \qquad i \in \{1,2\}.$$

Then I test for Granger causality. Table II reports the estimation results.[15] When I control for the lags of the dependent variable, the leverage-ratio is not significant, whereas the labor-share explains

[13] Quarterly data from 1947 to 2012 are used to compute with higher precision the term-structures of variance ratios, but similar results obtain using annual data.

[14] Consistently, the levels of GDP, wages, investments, corporate profits and net dividends are co-integrated and share a unique stochastic trend: a Johansen tests indicates that there are four co-integrating equations in a vector error-correction model (VECM). Reported results make use of the Johansen's trace statistics but also the maximum eigenvalue statistic and the minimization of information criteria confirm a unique common stochastic trend.

[15] Further results are reported in the online appendix OA.A.

both the future dividend- and profit- shares (the economic significance of the first lag is about -.50). The investment-share shows some explanatory power about corporate profits but its correlation with dividends is essentially zero. As a result, testing for Granger causality, we observe that the labor-share Granger-causes the dividend-share while leverage-ratio and investment-share do not. Corporate profits are Granger-caused by both the labor-share and the investment-share but not by the leverage ratio.

The above results support the model assumption about income insurance commented in the Introduction. Workers benefit of an insurance from shareholders which smoothes wages but makes dividends more volatile. Moreover, such an insurance protects wages in bad times as implied by the strong persistence and counter-cyclical dynamics of the labor-share. Finally, co-integration suggests that the insurance mechanism concerns the transitory component of GDP, whereas both workers and shareholders are subject to permanent shocks. If this interpretation of the data is correct, we should observe a coherent term-structure effect on the riskiness of both wages and dividends, as documented below.

C. Variance ratios and the term-structure effect of income insurance

Now the focus turns on the term-structures of growth rates volatility. Consider at first the GDP: the term-structure of its variance ratios (VR's) is close to one. This implies that GDP volatility over n intervals is about $\sqrt{n}$ times larger than GDP volatility over one interval. The upper panels of Figure 3 reports the term-structures of VR's for GDP, wages, investments, corporate profits and net dividends, computed accounting for heteroskedasticity and overlapping observations.

Insert Figure 3 about here

The VR's of wages are larger than one and increasing with the horizon. Instead the VR's of net dividends –as well as corporate profits and investments– are lower than one and strongly decreasing with the horizon, consistently with Beeler and Campbell (2012) and Belo et al. (2014). These ratios reach a positive long-run limit, consistently with cointegration.

The interpretation is as follows. The term-structure of GDP risk is flat because the upward-sloping effect of the permanent shock (e.g. time-variation in expected growth) is approximately offset by the downward-sloping effect of the transitory shock.[16] Wages load less on the transitory shock such that the upward-sloping effect dominates and the VR's increase with the horizon; whereas dividends (as well as profits and investments) load more on the transitory shock such that the downward-sloping effect dominates and the VR's are decreasing. These term-structure effects are exactly consistent with the main model mechanism. Indeed, income insurance from shareholders to workers concerning the transitory shocks of GDP induces an asymmetry in the short-run risk of wages and dividends. Wages have to be less subject to short-run risk than GDP, such that the long-run risk dominates and the VR's are increasing.

[16]The alternative hypothesis that the term-structure of GDP risk is flat because GDP is indeed a random-walk with i.i.d. increments is not consistent with the upward-sloping term-structure of wage risk, given cointegration and the stationarity of the labor-share.

Vice-versa, dividends should load more short-run risk than GDP, such that the long-run risk is more than offset and decreasing VR's obtain.

The lower panels of Figure 3 show two patterns. First,to quantify the term-structure effect of income insurance I look at the remainder of GDP minus wages. It features downward-sloping VR's similar to those of dividends. This suggests that wages are the main determinant of the timing of dividend risk, whereas alternative channels, such as financial leverage, can have only a second order effect. Second, VR's of consumption are slightly increasing and lie between those of wages and GDP: this obtains because wages weigh more to consumption than to GDP. Approximating consumption with wages plus dividends –as I do in the model– is safe from a term-structure perspective: VR's feature a similar upward-sloping shape.

The main model mechanism is as follows. Given a negative (positive) transitory shock, workers' remuneration increases (decreases) relative to shareholders' remuneration, and the VR's of wages and dividends respectively increase and decrease (decrease and increase) relative to those of GDP. To test this dynamic relation: i) I build time-series of VR's; ii) I construct a measure of the above term-structure effect: $\Delta_{t,\tau} = \frac{VR_L - VR_D}{VR_Y}\Big|_{t,\tau}$, that is the distance between the VR's of wages and dividends at time t for an horizon τ, standardized with the VR of GDP; iii) I regress $\Delta_{t,\tau}$ on the wage-to-dividends ratio:[17]

$$\Delta_{t,\tau} = b_0 + b_1 \log L_t/D_t + \varepsilon_t, \qquad \forall \tau.$$

Estimation results are reported in Table III. The positive relation implied by the model is extremely strong in the data. The adjusted-R^2 ranges from about 80% at short horizons to about 60% at ten years horizon. The economic significance ranges from about 90% to 75%. The left panel of Figure 4 shows the time-series of L_t/D_t and $\Delta_{t,\tau}$ for several τ. The right panel reports the adjusted-R^2 and t-statistics as a function of τ. While in the model L_t/D_t and L_t/Y_t have the same information, this is not necessarily true in the real data. However, when I use the labor-share as independent variable, a positive strongly significant relation obtains. The adjusted-R^2 ranges around 40% and the economic significance ranges around 60%, as shown in panel B. In panel C I use the financial leverage ratio and the investment-share as controls to account for the role of investment and financing decisions. While those decisions seem do not matter, the labor-share is still highly significant: its economic significance and the adjusted-R^2 are essentially unchanged.[18]

Insert Table III and Figure 4 about here

To the best of my knowledge this is the first paper that empirically investigates the determinants of the term-structure of risk of macroeconomics variables. In particular, the above results strongly support the idea that income insurance within the firm is the main driver of the timing of dividend risk.

[17] The VR at time t is computed using a rolling windows of quarterly growth rates centered at time t.

[18] The results of Table III are robust to sub-samples and time trend effects, as documented in the online appendix OA.A.

D. Long-horizon predictability

The previous analysis documents how income insurance determines the dividend policy and its term-structure properties. Now I look at additional testable implications.[19] As long as the labor-share negatively affects the dividend-share, the labor-share should positively forecast dividend growth. Therefore, I consider long-horizon predictability regressions of dividend growth on the leverage-ratio, the investment-share and the labor-share. In accord with Belo et al. (2014), the aggregate leverage-ratio shows a substantial explanatory power of future dividend growth. However, also the investment-share and the labor-share strongly explain future dividends over long horizons. Moreover, standardized coefficients show that the economic significance is quite comparable. When the three independent variables are jointly included in the regression equation, we observe that the labor-share is still significant. Therefore, the channels through which leverage and labor-share predict dividend growth are distinct and do not offset each other. Differently, corporate profits are less predictable than dividends, by the same independent variables.

I also investigate whether the labor-share negatively forecasts excess stock returns. Namely, I regress cumulative excess returns on the leverage-ratio, the investment-share and the labor-share. Consistently with the model and in accord with Santos and Veronesi (2006), labor remuneration negatively predicts excess returns at some horizons. Such a relationship survives when also the leverage-ratio and the investment-share are taken in consideration and large adjusted R^2 can be observed.

III. The Economy

A. Agents

The economy consists of two classes of agents, workers (w) and shareholders (s). Both are equipped with recursive preferences as in Duffie and Epstein (1992). Given consumption C, the utility at each time t is

$$J_t = \mathbb{E}_t \int_{u \geq t} f(C_u, J_u) du, \quad \forall t \geq 0, \tag{1}$$

where $\mathbb{E}_t$ is the expectation operator under full information and $f(c, j)$ is an aggregator function. Under usual technical conditions, the aggregator is given by

$$f(C, J) = \beta \chi J \left(C^{1-1/\psi} ((1-\gamma)J)^{-1/\chi} - 1 \right), \tag{2}$$

where $\chi = \frac{1-\gamma}{1-1/\psi}$, γ is relative risk aversion, ψ is elasticity of intertemporal substitution and β is the subjective time-discount rate. Power utility obtains for $\psi \to \gamma^{-1}$. Hereafter, workers' and shareholders' utilities are denoted by $U(C_{w,t})$ and $V(C_{s,t})$, where $C_{w,t}$ and $C_{s,t}$ stand for their consumption levels. I allow risk attitudes to differ as follows.

[19] Estimation results are reported in the online appendix OA.A.

Assumption 1 – Preference heterogeneity. *Workers are more risk averse than shareholders:* $\gamma_w > \gamma_s$.

It will be shown that preference heterogeneity is crucial to both the endogenous riskiness of dividend distributions as well as the equilibrium pricing of risk. In order to emphasize the workers' need for income smoothing in the context of their employment relationship, I make the following assumption.

Assumption 2 – Limited market participation. *Workers do not participate in the financial markets, inelastically supply labor and consume their wage.*

Such an assumption provides a simplistic and extreme representation of workers but helps to understand the role of labor relations by avoiding alternative channels of income smoothing. Thus, workers solve:

$$\begin{aligned} &\sup_{C_{w,t},n_{w,t}} U(C_{w,0}) = \sup_{C_{w,t},n_{w,t}} \mathbb{E}_0 \int_0^\infty f(C_{w,u}, U(C_{w,u}))du, \\ s.t. \quad & C_{w,t} \leq W_t n_{w,t}, \qquad n_{w,t} \leq 1, \end{aligned} \tag{3}$$

where W_t is the wage and $n_{w,t}$ is the labor supply. The problem has solution: $C_{w,t} = W_t$ and $n_{w,t} = 1$.

Shareholders act as rentiers: they consume dividends and trade securities in the financial markets. Under Assumption 2, shareholders own all traded securities, do not supply labor services and solve:

$$\begin{aligned} &\sup_{C_{s,t},n_{s,t}} V(C_{s,0}) = \sup_{C_{s,t},n_{s,t}} \mathbb{E}_0 \int_0^\infty f(C_{s,u}, V(C_{s,u}))du, \\ s.t. \quad & dQ_{s,t} = Q_{s,t}(r + n_{s,t}(\mu_P - r))dt - C_{s,t}dt + Q_{s,t}n_{s,t}(\sigma_{P,x}dB_{x,t} + \sigma_{P,\mu}dB_{\mu,t} + \sigma_{P,z}dB_{z,t}), \\ & C_{s,t} \leq D_t n_{s,t}, \qquad n_{s,t} \leq 1, \end{aligned} \tag{4}$$

where $Q_{s,t}$ is the shareholders' wealth and $n_{s,t}$ is the wealth proportion invested in the claim on the flow of firm's dividends D_t. The risk-free rate r, the stock expected return μ_P and the volatilities $\sigma_{P,x}$, $\sigma_{P,\mu}$ and $\sigma_{P,z}$ have to be determined in equilibrium. The shocks $B_{x,t}$, $B_{\mu,t}$ and $B_{z,t}$ are defined later. Under standard concavity and differentiability conditions, the above problem has solution $C_{s,t} = D_t$ and $n_{s,t} = 1$.

B. The firm

A representative firm behaves competitively and lives forever. The firm activity is simplified for the sake of tractability and to emphasize the role of labor relations, as the unique channel for income smoothing. A cash-flows process embeds the firm decisions and operations:

$$Y_t = A_t F(K_t, N_t) - I_t - G(K_t, N_t, I_t) = X_t Z_t, \tag{5}$$

where the first equality is a profit identity and the second one captures the reduced form approach with

$$d\log X_t = dx_t = (\mu_t - \sigma_x^2/2)dt + \sigma_x dB_{x,t}, \tag{6}$$

$$d\mu_t = \lambda_\mu(\bar{\mu} - \mu_t)dt + \sigma_\mu dB_{\mu,t}, \tag{7}$$

$$d\log Z_t = dz_t = \lambda_z(\bar{z} - z_t)dt + \sigma_z dB_{z,t}. \tag{8}$$

Aggregate risk is then described by an integrated process X_t and a stationary process Z_t. Long-run growth μ_t is a stationary and mean-reverting process. All the above processes are assumed to be homoscedastic only for the sake of simplicity. The cash-flows process also determines the resources to be distributed to the workers and the shareholders:

$$Y_t = W_t + D_t. \tag{9}$$

This resource constraint captures the endogenous nature of wages and dividends due to labor relations. Permanent and transitory shocks are jointly taken into account to study a flexible and general class of term-structure models. Namely, the simplest case $Y_t = X_t$ with $\mu_t = \bar{\mu}$ implies a flat term-structure for both mean and volatility of $\log Y_t$. Persistent fluctuations in μ_t and z_t lead respectively to upward- and downward-sloping term-structures, whereas their joint dynamics leads to non-monotone term-structures.[20]

C. Labor relations

The standard Walrasian Cobb-Douglas economy leads to a constant labor share and workers paid at their marginal productivity: $W_t = \alpha Y_t$, with $\alpha \in (0,1)$. The opposite extreme scenario would be the case where workers are promised perfect income smoothing: wages are constant and potentially equal to the unconditional expected marginal productivity: $W_t = \bar{W} = \mathbb{E}[\alpha Y_t]$ (provided Y_t stationary).

Here, I postulate that workers and shareholders agree in advance on a risk sharing rule. Namely, a contract $\mathcal{C}$ is designed to effect risk sharing between workers and shareholders: The two agent types arrange an agreement such that wages and dividends share the same long-run dynamics of total resources (i.e. Y_t, W_t and D_t are cointegrated) but the former feature an insurance to transitory shocks, implying a levered exposition of the latter.

Assumption 3 – Income Insurance. *The contingent wage payments and dividend distributions are governed by the following sharing rule:*

$$\mathcal{C} = \Big\{ (W_t, D_t) : \quad W_t = X_t \tilde{W}_t(\phi^*), \quad D_t = X_t \tilde{D}_t(\phi^*), \quad W_t + D_t = Y_t \quad \forall t \Big\}, \tag{10}$$

where

$$\tilde{W}_t(\phi^*) = (\alpha Z_t)^{1-\phi^*} \quad \textit{and} \quad \tilde{D}_t(\phi^*) = Z_t - (\alpha Z_t)^{1-\phi^*}, \tag{11}$$

[20] The sub-cases $Y_t = X_t$ and $Y_t = Z_t$ are the exogenous processes usually considered respectively in finance literature, such as in Bansal and Yaron (2004), and real business cycle literature, such as in Danthine and Donaldson (1992, 2002).

$\alpha \in (0,1)$ *and* $\phi^* \in (0,1)$ *denotes the optimal degree of income insurance. Hence, under Assumption 2 in equilibrium,*

$$C_{w,t} = X_t \tilde{W}_t(\phi^*), \quad \textit{and} \quad C_{s,t} = X_t \tilde{D}_t(\phi^*). \tag{12}$$

The optimal degree of income insurance ϕ^* represents the smoothing effect of labor relations on wages and the corresponding leverage effect on dividend distributions. Namely, wages are a concave function of Z_t for $\phi^* > 0$, implying a countercyclical labor share (W/Y), as suggested by real data. Instead, wages reduce to the standard Walrasian case with constant labor-share for $\phi^* \to 0$. The degree of income insurance is constrained in the interval $(0,1)$ because $\phi^* < 0$ implies an income insurance from workers to shareholders and a pro-cyclical labor-share, whereas $\phi^* > 1$ implies wages decreasing in the aggregate shock Z_t. Thus $\phi^* \neq (0,1)$ is implausible.

Lemma 1 – Contract rule and bargaining power.
The optimal degree of income insurance from shareholders to workers satisfies the following objective:

$$\phi^* = \underset{\phi \in (0,1)}{\arg\max}\, \mathcal{J}(\phi) = \underset{\phi \in (0,1)}{\arg\max} \left(\mathbb{E}_z[\tilde{W}(\phi)]\right)^{\vartheta} \left(\mathbb{E}_z[\tilde{D}(\phi)]\right)^{1-\vartheta} = \max(0, \min(1, \tilde{\phi})), \tag{13}$$

where $\tilde{\phi} = 1 + \frac{2\lambda_z(\bar{z}+\log\alpha)}{\sigma_z^2} + \frac{\sqrt{2}}{\sigma_z^2}\sqrt{q_0 + q_1 - q_2}$, *with* q_0, q_1 *and* q_2 *derived in Appendix A.*

The objective $\mathcal{J}(\phi)$ is intended to reflect the relative bargaining power of the two parties and, jointly with the contract rule of Eq. (10), determines simultaneously both their consumption shares. Namely, Eq. (13) can be interpreted as the aggregate outcome of bargaining in labor negotiations –both individual ones and those accomplished by unions.[21] The bargaining parameter ϑ is set:

$$\vartheta = \frac{\alpha\gamma_w}{\alpha\gamma_w + (1-\alpha)\gamma_s} = \frac{\alpha h}{1+\alpha(h-1)}, \tag{14}$$

and is determined by both heterogeneity in risk aversion, $h = \gamma_w/\gamma_s$, and the consumption shares in the Walrasian benchmark, α (i.e. the model with constant labor-share W/Y, which obtains for $h = 1$). Namely, ϑ is increasing in h, in spirit of Danthine and Donaldson (1992, 2002): $h > 1$ leads to an insurance wage component upon the labor productivity level αY. The optimal degree of income insurance is increasing

[21] Assuming workers and shareholders negotiate directly with each other, a proper formulation of a Nash bargaining solution would require to define ϕ^* as

$$\phi_t^* = \underset{\phi \in (0,1)}{\arg\max} \left(U(C_{w,t})\right)^{\tilde{\vartheta}} \left(V(C_{s,t})\right)^{1-\tilde{\vartheta}}$$

where $\tilde{\vartheta} \in (0,1)$ is a bargaining power parameter and heterogeneity in risk aversion induces the incentive to bargain, given an outside option standardized to zero (i.e. bargaining on the concavity of the wage schedule obtains in exchange of long-term contract arrangements and full employment). Unfortunately, such a solution is neither tractable nor stationary. Instead, the objective in Eq. (13) guarantees tractability and provides a similar result. Lifetime utilities are replaced by expectations (over the stationary distribution of z_t) of the components of workers' and shareholders' consumptions affected by income insurance (i.e. $\tilde{W}(\phi)$ and $\tilde{D}(\phi)$) and the heterogeneity in preferences is embedded in the parameter ϑ which, with a slight abuse of terminology, can be interpreted as the bargaining power of workers.

in the heterogeneity in risk aversion, h, and in the Walrasian benchmark, α:

$$\partial_h \phi^* > 0, \qquad \partial_\alpha \phi^* > 0, \qquad \text{if } \gamma_w > \gamma_s.$$

Consequently, income insurance described by Assumption 3 and Lemma 1 leads to smooth wages and risky dividends with the additional feature of counter-cyclical labor-share. However, the endogenous cointegration among Y, W and D implies that distributional risk due to income insurance is a short-term risk and provides a rationale for downward sloping term-structures of both dividends and equity.

This stylized model of income insurance is intended to capture the main stylized facts needed to generate the dynamics of dividends which matter from an asset pricing perspective. The smoothing effect on wages due to income insurance, as in Eq. (12), can obtain also in an indirect way. For instance, recently Shimer (2005), Gertler and Trigari (2009) and Favilukis and Lin (2012) study the importance of wage rigidity in real business cycle models. Also other deviations from the standard Walrasian labor markets, which account for the dynamics of employment, can lead to similar implications.[22] This paper empirically supports and explicitly models income insurance since it represents the general and intuitive economic mechanism which explains the dynamics of dividends, despite the exact friction or combination of frictions in labor markets. As it will be shown, dividends implied by Assumption 3 reconcile the traditional asset pricing facts with the recent evidence about the term-structures.

Finally, the model is kept simple and parsimonious such that the effect of income insurance on asset prices is fully captured by a unique endogenous parameter, ϕ^*. If such a parameter is interpreted as exogenous, all equilibrium asset pricing implications characterized in the paper continue to hold even if alternative or complementary explanations for the dividend dynamics are taken into consideration –such as financial leverage (Belo et al. (2014)) and heterogeneous investment risk (Ai et al. (2012)).

IV. The Equilibrium

The equilibrium is a pair D_t and W_t which jointly satisfies Eq. (5), (9) and (12). Dividends are optimal to shareholders, given the contract rule and the resources constraint.

A. Value function and state price density

To preserve both analytic tractability and the economic meaning of primitive parameters, consider a standard log-linearization of the dividend process:

$$\log D_t \approx \log X_t + \log \bar{D} + \partial_z \log D\big|_{z=\bar{z}} (z_t - \bar{z}) = x_t + d_0 + d_z z_t, \tag{15}$$

[22] See the recent works by Kuehn, Petrosky-Nadeau, and Zhang (2012) and Donangelo (2014).

where $\log \bar{D} = d_0 + d_z \bar{z} = \log\left(e^{\bar{z}} - \alpha^{1-\phi^*} e^{(1-\phi^*)\bar{z}}\right)$ captures the steady-state of $\tilde{D}(\phi^*)$, and d_z satisfies:

$$d_z = 1 + \frac{\phi^* \alpha}{\alpha^{\phi^*} e^{\bar{z}} - \alpha}. \tag{16}$$

Notice that, as usual, the dividend process is still increasing in the states x_t and z_t but inherits their homoscedasticity.

Preferences in Eq. (1)-(2) and the log-linearized dividend dynamics of Eq. (15) guarantee a model solution which emphasizes the role of the state-variables μ_t and z_t in the formation of prices. A first order approximation of the shareholders' consumption-wealth ratio around its (endogenous) steady state provides closed form solutions for prices and return moments up to such approximation (Benzoni, Collin-Dufresne, and Goldstein (2011)).

Proposition 1 *The shareholders' utility process in Eq. (1) is given by*

$$J(X_t, \mu_t, z_t) = \frac{1}{1-\gamma_s} X_t^{1-\gamma_s} \exp\left(u_0 + (1-\gamma_s) d_0 + u_\mu \mu_t + (u_z + (1-\gamma_s) d_z) z_t\right), \tag{17}$$

where u_0,

$$u_\mu = \frac{1-\gamma_s}{e^{cq} + \lambda_\mu}, \qquad u_z = \frac{\lambda_z(\gamma_s - 1)}{e^{cq} + \lambda_z}\left(1 + \frac{\phi^* \alpha}{\alpha^{\phi^*} e^{\phi^* \bar{z}} - \alpha}\right),$$

and $cq = \mathbb{E}[cq_t]$ are endogenous constants depending on the primitive parameters derived in Appendix B. The shareholders' consumption-wealth ratio is equal to

$$cq_t = \log C_{s,t}/Q_{s,t} = \log \beta - \chi^{-1}(u_0 + u_\mu \mu_t + u_z z_t). \tag{18}$$

As usual, the consumption-wealth ratio reduces to β when $\psi \to 1$ and equals the market dividend-price ratio under the limited market participation of Assumption 2. Coefficients u_0 and $\{u_\mu, u_z\}$ determine respectively the unconditional level of the consumption-wealth ratio and both the prices of risk and the growth rate of wealth.

From the shareholders' perspective, that is the point of view of market participants, the marginal utility evaluated at optimal consumption is the valid state-price density (Duffie and Epstein (1992)):

$$\xi_{t,u} = e^{\int_t^u f_V(C_{s,\tau}, V(C_{s,\tau}))d\tau} \frac{f_C(C_{s,u}, V(C_{s,u}))}{f_C(C_{s,t}, V(C_{s,t}))}, \quad \forall u \geq t. \tag{19}$$

Hence, the price of an arbitrary payoff stream $\{F_u, u \in (t, \infty)\}$ is given by $\mathbb{E}_t[\int_t^\infty \xi_{t,u} F_u du]$.

Proposition 2 *The equilibrium state price density $\xi_{0,t}$ satisfies:*

$$Y_t = I_C[\xi_{0,t}]\, e^{-d_0 - (d_z - 1) z_t}, \tag{20}$$

where $I_C[\xi_{0,t}] = \left\{C_{s,t} : \xi_{0,t} = e^{\int_0^t f_J(s)ds} f_C(t) \,\middle|\, X_t, \mu_t, z_t\right\}$ denotes the time t shareholders' optimal consump-

tion implied by $\xi_{0,t}$. In absence of income insurance ($\phi^* \to 0$), $\xi_{0,t}$ satisfies $Y_t = I_C[\xi_{0,t}](1-\alpha)^{-1}$.

Although the state price density equals the first order condition of shareholders, it depends on the risk attitudes of both agent types. Eq. (20) represents the equilibrium state-price density by means of the resource constraint of Eq. (9). The left hand side is given by the exogenous flows of total resources produced by the firm. The right hand side is given by the product of two terms: the former is the optimality condition for market participants, that is the shareholders; the latter is a time-varying term which captures the distributional risk due to income insurance. Namely, such a term equals the inverse of the dividend share, i.e. $Y_t, /D_t$, and its variability increases with ϕ^*. Instead, in absence of income insurance ($\phi^* \to 0$), the model reduces to the Walrasian benchmark and, hence, the labor-share is constant and equal to α: only shareholders' preferences matter and the second term on the right hand side of Eq. (20) reduces to the constant $(1-\alpha)^{-1} = Y_t/D_t$.

Under CRRA preferences ($\psi \to \gamma_s^{-1}$), the optimality condition for the shareholders takes the usual power form: $I_C[\xi_{0,t}] = (\xi_{0,t}e^{\beta t})^{-1/\gamma_s}$ and the term capturing the distributional risk is unchanged. Instead, for $\phi^* \to 0$, the state-price density reduces to $\xi_{0,t} = e^{-\beta t}((1-\alpha)Y_t)^{-\gamma_s}$, as in a Lucas economy.

Notice that, even if the equilibrium state-price density is an involved function of the integrated process Y_t, the economy is characterized by a stationary equilibrium: this is a necessary condition to produce realistic testable implications but usually fails to hold in models of preference heterogeneity. Many real business cycle models circumvent the problem by excluding permanent shocks (e.g. $Y_t = Z_t$). Here, allowing for both permanent and transitory shocks (e.g. $Y_t = X_t Z_t$ as in Eq. (5)) and still obtaining a stationary equilibrium is of crucial importance in order to study the equilibrium implications for the term-structure of equity.

Proposition 3 *The equilibrium state price density has dynamics given by*

$$d\xi_{0,t} = \xi_{0,t}\frac{df_C}{f_C} + \xi_{0,t}f_J dt = -r(t)\xi_{0,t}dt - \theta_x(t)\xi_{0,t}dB_{x,t} - \theta_\mu(t)\xi_{0,t}dB_{\mu,t} - \theta_z(t)\xi_{0,t}dB_{z,t}, \tag{21}$$

where the instantaneous risk-free rate satisfies

$$r(t) = r_0 + r_\mu \mu_t + r_z z_t, \tag{22}$$

with r_0 derived in Appendix B, $r_\mu = \frac{1}{\psi}$, $r_z = -\frac{\lambda_z}{\psi}d_z$ and the instantaneous prices of risk are given by

$$\theta_x(t) = -\frac{\partial_X f_C}{f_C}X_t\sigma_x = \gamma_s\sigma_x, \tag{23}$$

$$\theta_\mu(t) = -\frac{\partial_\mu f_C}{f_C}\sigma_\mu = \frac{\gamma_s - 1/\psi}{e^{cq}+\lambda_\mu}\sigma_\mu, \tag{24}$$

$$\theta_z(t) = -\frac{\partial_z f_C}{f_C}\sigma_z = \left(\gamma_s - \frac{\lambda_z(\gamma_s - 1/\psi)}{e^{cq}+\lambda_z}\right)d_z\sigma_z. \tag{25}$$

The risk-free rate is affine in μ_t and z_t. The coefficients r_μ and r_z are respectively positive and negative

and both decrease in magnitude with ψ, as usual under recursive preferences. Moreover, r_z increases in magnitude with the degree of income insurance ϕ^*.

The permanent shock commands a price of risk $\theta_x(t)$, which has the usual form and is a price for transient risk, that is for the contribution of x_t to the instantaneous volatility of shareholders' consumption. Long-run growth commands a price of risk $\theta_\mu(t)$, which has the usual form in long-run risk models. This is a price for non-transient risk, that is a price for the contribution of μ_t to the variation in the continuation utility value of shareholders. Hence, $\theta_\mu(t)$ disappears under power utility ($\psi \to \gamma_s^{-1}$). Such a price of risk is increasing with the degree of early resolution of uncertainty $\gamma_s - 1/\psi$ and decreasing in magnitude with rate of reversion in long-run growth. The transitory shock leads to a price of risk $\theta_z(t)$, which has two components. The first, $\gamma_s d_z \sigma_z$, is a positive price for transient risk, that is the contribution of z_t to the instantaneous volatility of shareholders' consumption. The second, $\theta_z(t) - \gamma_s d_z \sigma_z$, is a price for non-transient risk, that is a price for the contribution of z_t to the variation in the continuation utility value of shareholders. Namely, at the opposite than θ_μ, the latter is negative and increasing in the rate of reversion of z_t under preferences for the early resolution of uncertainty. Such a term disappears under power utility. Interestingly, both the components of $\theta_z(t)$ are proportional to the coefficient $d_z > 1$, which is increasing in ϕ^*. Therefore, for $\gamma_s > \psi > 1$, income insurance leads to a positive price for its effect on the current dividend volatility and a negative price for its effect on the evolution of shareholders' utility.

B. Endogenous dividends and wages

Dividends and wages evolve endogenously at equilibrium. Their dynamics is characterized by the resource constraint and the contract rule, which lead to[23]

$$D_t = X_t Z_t (1 - \omega(z_t)), \qquad \text{and} \qquad W_t = X_t Z_t \omega(z_t), \tag{26}$$

where Y_t, D_t and W_t share the integrated factor X_t and, hence, are cointegrated in equilibrium. The labor-share $\omega(z_t) = W_t/Y_t$:

$$\omega(z_t) = \alpha(\alpha e^{z_t})^{-\phi^*}, \tag{27}$$

is the Walrasian benchmark α times an adjustment factor decreasing and convex in z_t. The latter captures the effect of labor relations and reduces to one if $\phi^* \to 0$. Since $\phi^* \in (0,1)$, it follows that dividends and wages are both increasing in z_t, but the former are convex and the latter are concave in z_t.

Let denote with $d\log I = \mu^I dt + \sigma_x^I dB_x + \sigma_\mu^I dB_\mu + \sigma_z^I dB_z$, $I = \{Y, W, D\}$ the dynamics of total consumption, wages and dividends.

[23] Notice that for low values of Z_t, $\tilde{D}_t(\phi^*)$ could become negative: this can be avoided by setting $Z_t > 1 \forall t$ (e.g. if $z_t > 0$ is a square-root process). However, for reasonable parameters, the Ornstein-Uhlenbeck dynamics for z_t in Eq. (8) guarantees that the probability of $D_t < 0$ is negligible ($\approx .001\%$) and provides tractability. Hence, throughout the paper I assume $\tilde{D}_t(\phi^*) > 0$. Morover, such an issue does not arise in the equilibrium solution of the previous section, which makes use of the approximation in Eq. (15), provided $\bar{z} \geq 0$.

Lemma 2 – Cash-flows persistence, smoothing and leverage.
i) The sensitivity of the expected growth rates satisfy:[24]

$$\begin{aligned}
&\partial_\mu \mu^W = \partial_\mu \mu^Y = \partial_\mu \mu^D = 1, \\
&|\partial_z \mu^W| < |\partial_z \mu^Y| = |\partial_z \mathbb{E}[dz_t]/dt| < |\partial_z \mu^D|, \qquad (\textit{with equalities for } \phi^* \to 0).
\end{aligned} \tag{28}$$

ii) The volatilities of growth rates associated to permanent and transitory shocks satisfy

$$\begin{aligned}
&\sigma_x^W = \sigma_x^Y = \sigma_x^D = \sigma_x, \\
&\sigma_z^W < \sigma_z^Y = \sigma_z < \sigma_z^D, \qquad (\textit{with equalities for } \phi^* \to 0).
\end{aligned} \tag{29}$$

iii) The sensitivity of growth rates volatilities with respect to the degree of income insurance satisfy

$$\partial_{\phi^*} \sigma_z^W < 0, \qquad \partial_{\phi^*} \sigma_z^D > 0.$$

Total consumption, dividends and wages share the same exposition to long-run expected growth since labor relations do not alter the impact of permanent shocks on either workers or shareholders consumption. Expected consumption growth also depends on z_t: $|\partial_z \mu^Y| = \lambda_z$. The *persistence* of expected growth of wages and dividends is altered by income insurance: the larger ϕ^*, the stronger the persistence of wages (i.e. $|\partial_z \mu^W| < \lambda_z$) and the weaker that of dividends (i.e. $|\partial_z \mu^D| > \lambda_z$).

Total consumption, wags and dividends share the same exposition to permanent shocks, σ_x. Instead, the exposition of dividends and wages to transitory shocks is affected by income insurance. Namely, σ_z^W is constant, lower than σ_z and decreasing with ϕ^*. Therefore, income insurance induces a *smoothing* effect on wages at the cost of more volatile dividends. Indeed, σ_z^D is larger than σ_z and endogenously heteroscedastic:[25]

$$\sigma_z < \sigma_z^D = \left(1 + \phi^* \tfrac{\omega(z_t)}{1-\omega(z_t)}\right) \sigma_z. \tag{30}$$

Such an excess volatility or *leverage*, $\sigma_z^D - \sigma_z$, is: i) proportional and increasing with both ϕ^* and the wages-to-dividends ratio: $\omega(z_t)/(1-\omega(z_t))$; ii) increasing with workers' bargaining power; iii) endogenously counter-cyclical (i.e. $\partial_z \sigma_z^D < 0$).[26]

Insert Figure 5 about here.

[24]The second inequality in Eq. (28) holds for σ_z^2 not too large. Since the labor-share $\omega(z_t)$ is very smooth in the real data, Eq. (28) is satisfied for reasonable parameters.

[25]The state-dependent dynamics of σ_z^D is ruled out by the log-linearization of Eq. (15). Section VII.B solves analytically for the equilibrium under power utility ($\psi \to \gamma_s^{-1}$): in such a case the log-linearization of Eq. (15) is not needed and the endogenous heteroscedasticity is preserved for both dividends and equity returns dynamics.

[26]This mechanism is similar to that of an economy with preference heterogeneity but without limited market participation: in good times wealth shifts from high risk averse agents towards low risk averse agents. Here, labor relations would lead to a similar countercyclical dynamics through the endogenous dynamics of dividends instead of the aggregate risk aversion.

In the left panel of Figure 5, volatilities are plotted as a function of z_t. We can observe the smoothing effect on σ_z^W and the leverage effect on σ_z^D as well as the counter-cyclical dynamics of the latter. The middle panel shows the volatilities as a function of the labor-share and, hence, we observe the increasing dynamics of σ_z^D. The right panel plots the volatilities as a function of ϕ^* at the steady-state ($z_t = \bar{z}$): the stronger income insurance, the stronger both the smoothing and the leverage effects respectively on wages and dividends.

The focus now turns on the term structure of growth rates volatility. For the sake of tractability I make use of the log-linearized dynamics of dividends in Eq. (15).[27] The main quantity of interest is the moment generating function of the logarithm of dividends. We need to compute the following expectation.

Corollary 1 *The moment generating function of the logarithm of dividends has the following approximation:*

$$\mathbb{D}_t(\tau, n) = \mathbb{E}_t[D_{t+\tau}^n] \approx e^{nx_t + B_0(n,\tau) + B_\mu(n,\tau)\mu_t + B_z(n,\tau)z_t}, \tag{31}$$

where n and model parameters are such that the expectation exists finite, the approximation makes use of Eq. (15) and B_0, B_μ and B_z are deterministic functions of time derived in the Appendix B.

With this result in hand, the term structures of expected dividend growth and volatility are computed as[28]

$$g_D(t,\tau) = \frac{1}{\tau}\log\left(\frac{\mathbb{D}_t(\tau,1)}{\mathbb{D}_t(0,1)}\right) = \bar{g}_D(\tau) + g_{D,\mu}(\tau)\mu_t + g_{D,z}(\tau)z_t \tag{32}$$

$$\sigma_D^2(t,\tau) = \frac{1}{\tau}\log\left(\frac{\mathbb{D}_t(\tau,2)}{\mathbb{D}_t(\tau,1)^2}\right) = v_{D,x}(\tau)\sigma_x^2 + v_{D,\mu}(\tau)\sigma_\mu^2 + v_{D,z}(\tau)\sigma_z^2 \tag{33}$$

with $g_{D,\mu}(\tau) > 0$, $g_{D,z}(\tau) < 0$, $\partial_\tau v_{D,x}(\tau) = 0$, $\partial_\tau v_{D,\mu}(\tau) > 0$ and $\partial_\tau v_{D,z}(\tau) < 0$. First, $g_D(t,\tau)$ is increasing and decreasing respectively in μ_t and z_t. Second, μ_t and z_t lead respectively to an upward-sloping and a downward-sloping effect on the growth rates volatility. Interestingly, the stronger income insurance, the more pronounced the downward-sloping effect: $\partial_{\phi^*}|\partial_\tau v_{D,z}(\tau)| > 0$. Therefore, income insurance enhances the strength of the transitory shock and leads to an excess of short-run risk in dividends distributions with respect to total consumption. Such an effect is the model counterpart of the distance between the term-structures of GDP and dividend risk documented in Section II and in Figure 3 and 4.[29] The term-structure of total consumption and dividend correlation is:

$$\rho_{Y,D}(t,\tau) = \frac{1}{\tau\sigma_Y(t,\tau)\sigma_D(t,\tau)}\log\left(\frac{\mathbb{YD}_t(\tau,1,1)}{\mathbb{Y}_t(\tau,1)\mathbb{D}_t(\tau,1)}\right),$$

[27] The term structures of dividend expected growth rates and volatility can be derived in closed-form even under the exact dynamics of Eq. (26), as shown in Appendix B; however, such results are more involved and less intuitive.

[28] Also, expectations $\mathbb{W}_t(\tau,n)$ and $\mathbb{Y}_t(\tau,n)$ and term-structures $\{g_W(t,\tau), \sigma_W(t,\tau)\}$ and $\{g_Y(t,\tau), \sigma_Y(t,\tau)\}$ can be computed respectively for wages and total consumption.

[29] The transitory shock affects the level of the short-run limit ($\tau \to 0$) of $\sigma_D^2(t,\tau)$, whereas long-run growth affects the level of the long-run limit ($\tau \to \infty$). The term-spread (at the steady state) $\sigma_D^2(t,\infty) - \sigma_D^2(t,0) = \sigma_\mu^2/\lambda_\mu^2 - d_z^2\sigma_z^2$, can be either positive or negative depending on the model parameters. A negative spread obtains if the leverage effect d_z is large enough.

where $\mathbb{YD}_t(\tau,n,m) = \mathbb{E}_t[Y_{t+\tau}^n D_{t+\tau}^m]$. Such a correlation is equal to one for any τ when $\phi^* \to 0$, whereas it is U-shaped when $\phi^* > 0$ but converges to one in the long-run, due cointegration. Income insurance also alters the volatility and cyclicality of the dividend- and wage-shares (i.e. $D_t/Y_t = 1-\omega(z_t)$ and $W_t/Y_t = \omega(z_t)$):

$$\sigma_{1-\omega}(t,\tau) = \sqrt{\frac{1}{\tau}\log\left(\frac{\mathbb{E}_t[(1-\omega(z_{t+\tau}))^2]}{\mathbb{E}_t[1-\omega(z_{t+\tau})]^2}\right)}, \qquad \rho_{Y,(1-\omega)}(t,\tau) = \frac{1}{\tau\sigma_Y(t,\tau)\sigma_{1-\omega}(t,\tau)}\log\left(\frac{\mathbb{E}_t[Y_{t+\tau}(1-\omega(z_{t+\tau}))]}{\mathbb{Y}_t(\tau,1)\mathbb{E}_t[1-\omega(z_{t+\tau})]}\right),$$

$$\sigma_{\omega}(t,\tau) = \sqrt{\frac{1}{\tau}\log\left(\frac{\mathbb{E}_t[\omega(z_{t+\tau})^2]}{\mathbb{E}_t[\omega(z_{t+\tau})]^2}\right)}, \qquad \rho_{Y,\omega}(t,\tau) = \frac{1}{\tau\sigma_Y(t,\tau)\sigma_{\omega}(t,\tau)}\log\left(\frac{\mathbb{E}_t[Y_{t+\tau}\omega(z_{t+\tau})]}{\mathbb{Y}_t(\tau,1)\mathbb{E}_t[\omega(z_{t+\tau})]}\right).$$

In the Walrasian benchmark ($\phi^* \to 0$), both volatilities and correlations are equal to zero at any horizon. Instead, under income insurance ($\phi^* > 0$), we have the following results. Wage- and dividend-shares are not constant with

$$0 < \sigma_{\omega}(t,\tau) < \sigma_{1-\omega}(t,\tau),$$

and zero long-run limits: $\sigma_{\omega}(t,\infty) = \sigma_{1-\omega}(t,\infty) = 0$. The shares are respectively counter-cyclical and pro-cyclical:

$$\rho_{Y,\omega}(t,\tau) < 0 < \rho_{Y,(1-\omega)}(t,\tau),$$

monotonically decreasing with τ in magnitude and with zero long-run limits: $\rho_{Y,\omega}(t,\infty) = \rho_{Y,(1-\omega)}(t,\infty) = 0$. The sign of such correlations is an endogenous result of income insurance, but also an important feature of the data, as shown in Section II.

V. Equilibrium Asset Prices

A. Equilibrium dividend strips

Proposition 4 *The equilibrium price of the market dividend strip with maturity τ is given by*

$$P_{t,\tau} = \mathbb{E}_t\left[\xi_{t,t+\tau}D_{t+\tau}\right] \approx X_t e^{A_0(\tau)+A_\mu(\tau)\mu_t+A_z(\tau)z_t}, \tag{34}$$

where model parameters are such that the expectation exists finite, the approximation makes use of Eq. (15) and A_0, A_μ and A_z are deterministic functions of time derived in the Appendix B. The instantaneous volatility and premium on the dividend strip with maturity τ are given by

$$\sigma_P(t,\tau) = \sqrt{\sigma_x^2 + \frac{(1-e^{-\lambda_\mu\tau})^2(\psi-1)^2}{\lambda_\mu^2\psi^2}\sigma_\mu^2 + \frac{e^{-2\lambda_z\tau}(\psi+e^{\lambda_z\tau}-1)^2}{\psi^2}d_z^2\sigma_z^2}, \tag{35}$$

$$(\mu_P - r)(t,\tau) = \gamma_s\sigma_x^2 + \frac{(1-e^{-\lambda_\mu\tau})(1-1/\psi)(\gamma_s-1/\psi)}{\lambda_\mu(e^{cq}+\lambda_\mu)}\sigma_\mu^2 + \frac{\left(\frac{1}{\psi}(1-e^{-\lambda_z\tau})+e^{-\lambda_z\tau}\right)(\lambda_z+\psi\gamma_s e^{cq})}{\psi(e^{cq}+\lambda_z)}d_z^2\sigma_z^2. \tag{36}$$

The price of the dividend strip is exponential affine in x_t, μ_t and z_t. Hence, the strip's price-dividend

ratio is a stationary. The functions $A_\mu(\tau)$ and $A_z(\tau)$ are respectively the semi-elasticity of the price with respect to μ_t and z_t:

$$A_\mu(\tau) = \partial_\mu \log P_{t,\tau} = \frac{(1-e^{-\lambda_\mu \tau})(1-1/\psi)}{\lambda_\mu}, \tag{37}$$

$$A_z(\tau) = \partial_z \log P_{t,\tau} = \left(\frac{1}{\psi}(1-e^{-\lambda_z \tau}) + e^{-\lambda_z \tau}\right)\left(1 + \frac{\phi^* \alpha}{\alpha^{\phi^*} e^{\phi^* \bar{z}} - \alpha}\right). \tag{38}$$

First, $A_\mu(\tau)$ increases with ψ, whereas $A_z(\tau)$ decreases with ψ. Second, $A_z(\tau)$ increases with the degree of income insurance ϕ^*. Therefore, the leverage effect on dividends due to income insurance also affects prices: namely, such an effect is amplified for $\psi < 1$ and vice-versa. Finally, for $\psi \to 1$, $A_\mu(\tau) = 0$ and $A_z(\tau) = d_z$ and, hence, the strip's price-dividend ratio reduces to a state-independent function of the horizon τ. The permanent shock as well as the states μ_t and z_t contribute to the return volatility and command a premium. Permanent shocks do not lead to excess volatility. Instead, the expositions to $B_{\mu,t}$ and $B_{z,t}$, are proportional to the fundamentals' volatilities σ_μ and σ_z, but also depend on the horizon τ, the elasticity of intertemporal substitution and the persistence of the states. Namely, the exposition to long-run growth is increasing in ψ and decreasing in the rate of reversion λ_μ. Instead the exposition to the transitory shock is decreasing in ψ and increasing in λ_z. The latter is also amplified by the leverage effect due to income insurance, d_z.

The premium on the dividend strip is given by the sum of the compensations to the three shocks of the model. The compensation for the permanent shock is positive and has the usual form: $\gamma_s \sigma_x^2$. Such a premium is a compensation for transient risk. Instead, the compensations associated to the states μ_t and z_t depend also on the horizon τ, the elasticity of intertemporal substitution and the persistence of the states. Long-run growth commands a premium which is increasing in ψ and decreasing in the rate of reversion λ_μ:

$$\frac{\gamma_s - 1/\psi}{\lambda_\mu (e^{cq} + \lambda_\mu)} A_\mu(\tau) \sigma_\mu^2. \tag{39}$$

Such a term is compensation for non-transient *long-run* risk, that is the contribution of μ_t to the variability of the continuation utility value of shareholders. If the intertemporal substitution effect dominates the wealth effect and shareholders have preferences for the early resolution of uncertainty –i.e. the usual parametrization $\gamma_s > \psi > 1$– long-run growth leads to a positive premium on the dividend strip. The premium for the exposition to the transitory shock z_t is given by the sum of two terms:

$$\gamma_s A_z(\tau) d_z \sigma_z^2, \qquad \text{and} \qquad -\frac{\lambda_z(\gamma_s - 1/\psi)}{e^{cq} + \lambda_z} A_z(\tau) d_z \sigma_z^2. \tag{40}$$

The former term represents the compensation for transient risk. Such a premium is always positive and decreases with ψ. Instead, the latter term is a compensation for non-transient *short-run* risk. It is decreasing in ψ and is negative (positive) under the shareholders' preference for the early (late) resolution of uncertainty. Moreover, both terms depend on the horizon τ and increase in magnitude with income

insurance, d_z.

Insert Table IV about here.

Panel A of Table IV shows the signs of transient and non-transient components of dividend strips premia under several scenarios, which are all subcases of exogenous uncertainty in Eq. (5). Namely, I consider the cases of a) an i.i.d. economy ($Y_t = X_t, \mu_t = \bar{\mu}$), b) an economy with only permanent shocks and time-varying long-run growth ($Y_t = X_t$) in spirit of long-run risk literature, c) an economy with only transitory shocks ($Y_t = Z_t$) in spirit of real business cycle literature, and finally d) and e) the economies with both permanent and transitory shocks ($Y_t = X_t Z_t$) with and without time-varying long-run growth. Transient risk always requires a positive compensation, whereas non-transient long-run risk can command a positive premium under preference for both early or late resolution of uncertainty depending on ψ. Non-transient short-run risk leads to a positive or negative premium under preference for respectively late or early resolution of uncertainty. Finally, the whole premium is not necessarily monotone in γ_s and ψ.

Corollary 2 *The slopes of the term-structures of dividend strips' volatility and premia are given by*

$$\partial_\tau \sigma_P^2(t,\tau) = \tfrac{2}{\psi^2}\big(e^{-2\lambda_\mu \tau}(e^{\lambda_\mu \tau}-1)(\psi-1)^2\lambda_\mu^{-1}\sigma_\mu^2 \;-\; e^{-2\lambda_z \tau}(\psi+e^{\lambda_z \tau}-1)(\psi-1)\lambda_z d_z^2 \sigma_z^2\big), \tag{41}$$

$$\partial_\tau (\mu_P - r)(t,\tau) = \tfrac{\psi-1}{\psi^2}\big(\tfrac{(\gamma_s - 1/\psi)e^{-\lambda_\mu \tau}\sigma_\mu^2}{e^{cq}+\lambda_\mu} \;-\; \tfrac{(\lambda_z+\gamma_s\psi e^{cq})e^{-\lambda_z \tau}\lambda_z d_z^2 \sigma_z^2}{e^{cq}+\lambda_z}\big). \tag{42}$$

Moreover, the instantaneous volatility and premium of the dividend strip satisfy

$$\sigma_P^2(t,\infty) - \sigma_P^2(t,0) = \tfrac{(\psi-1)^2\sigma_\mu^2}{\lambda_\mu^2\psi^2} - \tfrac{(\psi^2-1)d_z^2\sigma_z^2}{\psi^2}, \tag{43}$$

$$(\mu_P - r)(t,\infty) - (\mu_P - r)(t,0) = (\psi-1)\big(\tfrac{(\gamma_s - 1/\psi)\sigma_\mu^2}{\psi\lambda_\mu(e^{cq}+\lambda_\mu)} - \tfrac{(\lambda_z+\gamma_s\psi e^{cq})d_z^2\sigma_z^2}{(e^{cq}+\lambda_z)\psi^2}\big). \tag{44}$$

Corollary 2 provides a number of new results and sheds light on the role of preferences on the equilibrium term-structure of equity risk and premia. The slope of the term-structure of volatility depends on two terms, due respectively to μ_t and z_t. The former is always positive and, hence, implies an upward sloping effect. Instead, the latter term is negative if the intertemporal substitution effect dominates the wealth effect and vice-versa. Therefore, the term-structure of volatility is monotone upward sloping if $\psi < 1$, whereas it is not necessarily monotone if $\psi > 1$. A non-monotone (e.g. U-shaped) term-structure of risk obtains if the leverage effect due to income insurance, d_z, outweighs the upward sloping effect due to long-run growth for some horizons τ.

Also the slope of the term-structure of premia depends on two terms, due to μ_t and z_t. The former is positive if the intertemporal substitution effect dominates the wealth effect and shareholders have preferences for the early resolution of uncertainty. The latter term is negative if the intertemporal substitution effect dominates the wealth effect and vice-versa. Under the usual parametrization $\gamma_s > \psi > 1$, variation in long-run growth leads to an upward-sloping effect, whereas transitory shocks lead to a

downward-sloping effect. Therefore, the term-structure of equity premia is not necessarily monotone as long as both permanent and transitory shocks enter the model. Such an analytical result clearly explains why the standard long-run risk model cannot capture the recent evidence about dividend strips. Indeed, the model of Bansal and Yaron (2004) (i.e. $Y_t = X_t$) rules out transitory shock z_t and does a good job at matching a number of asset pricing moments as long as $\psi > 1$. Both the term-structures of volatility and premia are monotone increasing. The alternative scenarios, in which either only the transitory shock enters the model ($Y_t = Z_t$) or long-run growth is constant ($Y_t = X_t Z_t$ with $\mu_t = \bar{\mu}$), monotone decreasing term-structures of risk and premia obtain for $\psi > 1$. However, in such a case the non-transient component of the equity premium is decreasing in ψ and the model cannot command a sizeable premium. Instead, when we account for both μ_t and z_t, the model can accommodate for both a high equity premium and downward sloping term-structures of equity risk and premia in the short-run. Therefore, the endogenous dynamics of dividends due to income insurance allows to reconcile the above stylized facts. Panel B of Table IV summarizes the model implications about the slope of the term-structure of equity premia under several scenarios.

The volatility term spread in Eq. (43) and premium term spread in Eq. (44) are both given by two terms. One is due to long-run growth and the other is due to transitory shocks. The latter is negative for $\psi > 1$ and increasing in magnitude with d_z. Hence negative spreads obtain if the leverage effect of dividends due to income insurance is large enough.

B. Equilibrium market asset

Proposition 5 *The equilibrium price of the market asset is given by*

$$P_t = \mathbb{E}_t\left[\int_t^\infty \xi_{t,u} D_u du\right] \approx X_t \beta^{-1} e^{u_0 \chi^{-1} + d_0 + u_\mu \chi^{-1} \mu_t + \left(u_z \chi^{-1} + d_z\right) z_t} \tag{45}$$

where model parameters are such that the expectation exists finite, the approximation makes use of Eq. (15), and u_0, u_μ and u_z are endogenous constants depending on the primitive parameters derived in the Appendix B. The instantaneous volatility and premium on the market asset are given by

$$\sigma_P(t) = \sqrt{\sigma_x^2 + \left(\frac{1-1/\psi}{e^{cq}+\lambda_\mu}\right)^2 \sigma_\mu^2 + \left(1 - \frac{(1-1/\psi)\lambda_z}{e^{cq}+\lambda_z}\right)^2 d_z^2 \sigma_z^2}, \tag{46}$$

$$(\mu_P - r)(t) = \gamma_s \sigma_x^2 + \frac{(1-1/\psi)(\gamma_s - 1/\psi)}{(e^{cq}+\lambda_\mu)^2}\sigma_\mu^2 + \frac{(\gamma_s \psi e^{cq} + \lambda_z)(\psi e^{cq} + \lambda_z)}{(e^{cq}+\lambda_z)^2 \psi^2} d_z^2 \sigma_z^2. \tag{47}$$

The market price is given by the time integral of the dividend strip price over the infinite horizon: $P_t = \int_0^\infty P_{t,\tau} d\tau$. The market price dividend ratio is a stationary function of μ_t and z_t and equals the wealth-consumption ratio of shareholders under Assumption 2. Prices increase with μ_t as long as the intertemporal substitution effect dominates the wealth effect and increase with z_t despite preferences: $\partial_\mu P_t \gtrless 0$ if $\psi \gtrless 1, \forall \gamma_s$ and $\partial_z P_t > 0, \forall \psi, \gamma_s$.

The permanent shock x_t enters the return dynamics exactly as the dividend dynamics, σ_x. Instead, the price expositions to the states μ_t and z_t depend on the preference parameters and are respectively increasing and decreasing in ψ. The exposition to μ_t always leads to an excess-volatility of market returns over dividends, whereas the exposition to z_t generates excess-volatility for $\psi < 1$ and vice-versa. While expected growth can lead to "long-run" excess volatility of dividends over market returns, the transitory shock can lead to "long-run" excess-volatility of returns over dividends even for $\psi > 1$. In particular the latter effect is enhanced by income insurance: the larger ϕ^*, the steeper the downward-sloping effect due to z_t on the term-structure of dividend volatility.

The equity premium is given by three components associated to the three shocks of the model. The permanent shock x_t requires the usual positive compensation $\gamma_s \sigma_x^2$ for transient risk. Long-run growth leads to a premium, which is positive if the intertemporal substitution effect dominates the wealth effect and shareholders have preference for the early resolution of uncertainty. Instead, z_t commands a premium which is always positive, decreasing with ψ and increasing with γ_s. Furthermore, such a compensation term is increasing in the degree of income insurance ϕ^*, captured by d_z.

Finally, provided $\gamma_s > \psi > 1$, the whole equity premium is increasing in γ_s but non-monotone in ψ: this is due to the fact that under such a parametrization, the equilibrium pricing of μ_t leads to compensations increasing with the horizon, whereas short-run but persistent uncertainty due z_t requires compensations decreasing with the horizon. Once the model is calibrated with realistic parameters, such a term-structure perspective sheds lights about whence priced risk obtains in equilibrium. In order to answer such a question, it is useful to analyze the relative contribution of each time-horizon to the whole equity premium. Using the definition of the market asset price as the time integral of the dividend strip prices, it is possible to write the equity premium as a time integral and, hence, to derive an equilibrium density which describes the contribution of future discounted cash-flows at each time-horizon.

Proposition 6 *The relative contribution of the horizon τ to the equity premium is defined as*

$$\mathcal{H}(t,\tau) = \frac{\Pi(t,\tau)}{(\mu_P - r)(t)}. \tag{48}$$

where the density $\Pi(t,\tau)$ satisfies $(\mu_P - r)(t) = \int_0^\infty \Pi(t,\tau) d\tau$ and is derived in Appendix B.

The shape of the equilibrium density $\mathcal{H}(t,\tau)$ tells us whence the equity premium comes from. The more the mass of probability concentrates on either short or long horizons, the more the riskiness associated to such horizons deserves a compensation and contributes to the whole equity premium. Therefore, $\mathcal{H}(t,\tau)$ is natural metric to understand the effect of the equilibrium term-structures of equity on an important asset pricing equilibrium outcome, such as the equity premium.[30]

[30] Standardizing $\Pi(t,\tau)$ by $(\mu_P - r)(t)$ allows to compare the timing decomposition of the equity premium among models and parameter settings even if they produce different magnitudes for the premium.

C. Equilibrium bond and equity yields

Proposition 7 *The equilibrium price of the zero-coupon bond with maturity τ is given by*

$$B_{t,\tau} = \mathbb{E}_t\left[\xi_{t,t+\tau}\right] \approx e^{K_0(\tau)+K_\mu(\tau)\mu_t+K_z(\tau)z_t} \tag{49}$$

where model parameters are such that the expectation exists finite, the approximation makes use of Eq. (15), and K_0, K_μ and K_z are deterministic functions of time derived in the Appendix B.

The bond price is stationary and exponential affine in μ_t and z_t . Hence, the bond yield is state-dependent but its volatility inherits the homoscedasticity of the states:

$$\varepsilon(t,\tau) = \frac{1}{\tau}\Big(-K_0(\tau) + (1-e^{-\lambda_\mu\tau})r_\mu\lambda_\mu^{-1}\mu_t + (1-e^{-\lambda_z\tau})r_z\lambda_z^{-1}z_t\Big). \tag{50}$$

The short- and long-run limits of the term-structure of real yields lead to the steady state term-spread:

$$\varepsilon(t,\infty) - \varepsilon(t,0) = -\frac{r_\mu\theta_\mu\sigma_\mu}{\lambda_\mu} - \frac{r_\mu^2\sigma_\mu^2}{2\lambda_\mu^2} - \frac{r_z\theta_z\sigma_z}{\lambda_z} - \frac{r_z^2\sigma_z^2}{2\lambda_z^2}, \tag{51}$$

which can be either positive or negative for $\gamma_s > \psi > 1$: indeed, $r_\mu\theta_\mu > 0$ and $r_z\theta_z < 0$.[31]

Armed with these results, the focus turns on the equity yields as introduced by van Binsbergen et al. (2013). The model equity yield is defined as

$$p(t,\tau) = -\frac{1}{\tau}\log\left(\frac{P_{t,\tau}}{D_t}\right) = \frac{1}{\tau}\left(d_0 - A_0(\tau) + \frac{(1-e^{-\lambda_\mu\tau})(1/\psi-1)}{\lambda_\mu}\mu_t + (1-e^{-\lambda_z\tau})(1-1/\psi)d_z z_t\right) \tag{52}$$

and, hence, is a stationary function of the states and the maturity. Moreover, it can be decomposed as:

$$p(t,\tau) = \varepsilon(t,\tau) - g_D(t,\tau) + \rho(t,\tau). \tag{53}$$

The equity yield is given by the difference among the yield on the risk-less bond, $\varepsilon(t,\tau)$, and the dividend expected growth, $g_D(t,\tau)$, plus a premium, $\rho(t,\tau)$. The latter is implicitly defined by Eq. (32)-(50)-(52) and is a state-independent function of the maturity: $\rho(t,\tau) = (K_0(\tau) + B_0(1,\tau) - A_0(\tau))/\tau$. The transitory shock determines the level of the short-run limit, whereas long-run growth determines the level of the long-run limit. Therefore, the term-spread

$$\rho(t,\infty) - \rho(t,0) \;=\; \frac{\gamma_s\psi\lambda_\mu + e^{cq}}{(\lambda_\mu + e^{cq})\lambda_\mu^2\psi}\sigma_\mu^2 - \frac{\lambda_z + \gamma_s\psi e^{cq}}{(\lambda_z + e^{cq})\psi}d_z^2\sigma_z^2,$$

can be either positive or negative depending on the model parameters.

[31] A monotone downward sloping term-structure of real yields obtains in most of long-run risk literature. In such models, a positive nominal term-spread can obtain as a result of inflation risk only.

D. Equilibrium cross-sectional returns

This section characterizes the equilibrium returns of the dividend claims of single firms. In spirit of Lynch (2003) and Menzly, Santos, and Veronesi (2004), I define a share process for the firm cash-flows, that is the fraction of aggregate dividends paid by a single firm.

The share process is such that dividends paid by existing firms sum up to aggregate dividends at each point in time. As in Lettau and Wachter (2007, 2011), the share process is deterministic for the sake of simplicity. The distribution of firms is stable over time: there exists a continuum of firms defined by their residual life $T \in (0, T^{\max})$ and at each point in time the firm with zero residual life is replaced by a new one with maximal residual life $T^{\max}$. The share process $s(T)$ is a hump-shaped function of residual life which peaks at $T^{\max}/2$, similarly to Lettau and Wachter (2007):

$$s(T) = \frac{\sin(\pi T/T^{\max})}{\int_0^{T^{\max}} \sin(\pi T/T^{\max}) dT}, \qquad \forall T \in (0, T^{\max}). \tag{54}$$

The dividends of the firm with residual life T are simply given by $D_t^T = s(T) D_t$.

Proposition 8 *The equilibrium price of the dividend claim of the firm with residual life T is given by*

$$P_t^T = \mathbb{E}_t \left[\int_0^T \xi_{t,t+\tau} D_{t+\tau}^{T-\tau} d\tau \right] \approx X_t \int_0^T e^{H_0(\tau,T) + H_\mu(\tau)\mu_t + H_z(\tau) z_t} d\tau \tag{55}$$

where model parameters are such that the expectation exists finite, the approximation makes use of Eq. (15), and H_0, H_μ and H_z are deterministic functions of time derived in the Appendix B. The instantaneous return volatility and premium are given by

$$\sigma_P^T(t) = \sqrt{\sigma_x^2 + \left(\partial_\mu \log P_t^T\right)^2 \sigma_\mu^2 + \left(\partial_z \log P_t^T\right)^2 \sigma_z^2}, \tag{56}$$

$$(\mu_P^T - r)(t) = \theta_x(t)\sigma_x + \theta_\mu(t)\left(\partial_\mu \log P_t^T\right)\sigma_\mu + \theta_z(t)\left(\partial_z \log P_t^T\right)\sigma_z. \tag{57}$$

The single-firm price is given by the time integral of the market dividend strip prices over the residual life weighted by dividend shares: $P_t^T = \int_0^T s(T-\tau) P_{t,\tau} d\tau$. Prices increase with μ_t as long as the intertemporal substitution effect dominates the wealth effect and increase with z_t despite preferences. The single-firm price relative to the current dividend level is a stationary nonlinear function of the long-run growth and the transitory shock. Hence, the approximation of Eq. (15) does not rules out the endogenous state-dependence of return moments. Similarly to Lettau and Wachter (2011), I interpret value firms as shorter-horizon equity than growth firms, because the cash-flows of the former are weighted more toward the present and those of the latter are weighted more toward the future. Therefore, the value premium is consistent with downward-sloping term-structure of equity premia: if priced risk concentrates in the short-run (i.e. $\mathcal{H}(t,\tau)$ is monotone decreasing with the horizon, as the red line in Figure 1), the compensation $(\mu_P^T - r)(t)$ is decreasing with residual life T. That is, value firms deserve a higher

premium than growth firms. The opposite holds if priced risk concentrates far in the future (i.e. $\mathcal{H}(t,\tau)$ is a hump-shaped function of the horizon, as the blue line in Figure 1). Since income insurance ($\phi^* > 0$) shifts priced risk towards the short-run, it also helps the model to explain the value premium. Such a mechanism obtains as an endogenous result in general equilibrium and provides an economic foundation of the partial equilibrium framework of Lettau and Wachter (2011).

VI. Asset Pricing Results

A. Model calibration

Model calibration is made setting parameters about exogenous uncertainty and labor relations by minimizing a number of moments of cash-flows growth rates. Then, preference parameters are set in the usual range of values in the literature and moments of equity returns are investigated. A peculiarity of the model calibration consists of exploiting the information from the term-structures of wages and dividend risk. This is important because they provide information about the persistence of both transitory and permanent shocks –a main issue in many asset pricing models.

The cash-flows dynamics involves eight parameters: $\theta = \{\bar{\mu}, \sigma_x, \lambda_\mu, \sigma_\mu, \lambda_z, \sigma_z, \alpha, h\}$. Six parameters are from the dynamics of x_t, μ_t and z_t (with $\bar{z} = 0$ for simplicity) and two are from the income insurance mechanism. Therefore, I choose eight moments conditions $m_i(\theta)$: the yearly steady-state growth rate of total resources Y_t, the yearly steady-state volatilities of Y_t, W_t and D_t and other four moments which capture the timing of risk. Namely, I consider the variance ratios (VR's) at 2 and 15 years of both wages and dividends in order to capture both the short- and long-run properties of their term-structure of risk:

$$m(\theta) = \left\{g_Y(t,1), \sigma_Y(t,1), \sigma_W(t,1), \sigma_D(t,1), VR_W(t,2), VR_D(t,2), VR_W(t,15), VR_D(t,15)\right\},$$

where $VR_x(t,\tau) = \frac{\sigma_x^2(t,\tau)}{\sigma_x^2(t,0.25)}$. Finally, I find the parameter values by minimizing the root-mean-square-error:

$$\Theta = \arg\min_{\theta} \sqrt{\frac{1}{8}\sum_{i=1}^{8}(m_i(\theta) - m_i^{\text{empirical}})^2},$$

where the empirical moments are from the analysis of Section II.[32] Table V and VI report respectively the model parameters and the implied moments of cash-flows.

Insert Table V and VI about here.

The permanent shock has instantaneous volatility $\sigma_x = 4.5\%$, whereas long-run growth and the transitory shock have volatilities $\sigma_\mu = 2.7\%$ and $\sigma_z = 3.7\%$ and rate of reversion $\lambda_\mu = 64.4\%$ and $\lambda_z = 27.9\%$.

[32] Empirical moments about dividends are computed as the average of those of net dividends and after tax corporate profits.

The Walrasian benchmark and the degree of preference heterogeneity are $\alpha = 85.7\%$ and $h = 1.784$, which lead to an optimal degree of income insurance $\phi^* = .425$. Since the model is very parsimonious, the fit is not perfect (the average relative error, that is $\left|\frac{m_i(\theta)-m_i^{\text{empirical}}}{m_i^{\text{empirical}}}\right|$, is about 10%), but is accurate enough to match the main stylized facts related to cash-flows and their term-structures of risk, as commented below. These stylized facts are the ingredients needed to understand the impact of the income insurance channel on asset prices. The co-integrating relationship among GDP, wages and dividends is closely related to the timing of risk. Therefore, for the sake of robustness, I verify that the model captures additional moments, such as the VR's of Y_t and the moments of the dividend-share D_t/Y_t.[33]

Remaining parameters concern the shareholders' preferences. I consider the values 5, 7.5 and 10 for the relative risk aversion: $\gamma_s = 5$ is suggested by the empirical investigation by Kimball, Sahm, and Shapiro (2009), whereas the other two values are usually considered in the long-run risk literature. Hence, given the heterogeneity parameter $h = 1.784$, the implied risk aversion of workers γ_w are respectively 8.92, 13.38 and 17.84. The elasticity of intertemporal substitution ψ is set to values between one and two, in line with the literature. Therefore, several cases satisfying $\gamma_s > \psi > 1$ are investigated. Finally, the time discount rate β is set to generate a 1% risk-free rate (if possible), given all other parameters.

B. The term structure of cash flows

Income insurance leads to smooth wages and highly risky dividends. Under the choice of parameters commented above, on the one hand, wages feature a yearly volatility of about $\sigma_W = 5\%$ and, hence, lower than that of total consumption ($\sigma_Y = 5.7\%$). On the other hand, dividends are significantly riskier at yearly frequency: $\sigma_D = 18.5\%$. Table VI shows how dividend and wage volatilities change with the degree of income insurance, ϕ. Anything else equal, the larger preference heterogeneity, h, the smoother wages and the riskier dividends.

The information exploited by the term-structures of VR's leads to a co-integrating relationship between Y_t, W_t and D_t quite in line with the data. The model captures the dynamics of W_t/Y_t, which is large in average, smooth and persistent. Moreover, it negatively correlates with changes in $\log Y_t$. Income insurance endogenously produces the counter-cyclical dynamics of the labor-share. Such a result fails to obtain in most of production-based models but is crucial to the modeling of the term-structures of both cash-flows and equity returns.

Insert Figure 6 about here.

The term structures of VR's and volatilities of Y_t, W_t and D_t are shown in Figure 6. The distance between the term structure of Y_t and those of W_t and D_t reflects the distributional risk due to income insurance. The left upper panel shows the term-structures of the model implied VR's (solid lines) and their empirical counterparts (dashed lines). The model matches the upward- and downward-sloping shapes of wage and

[33] Consistently with the model, I compute the empirical dividend-share as dividends over dividends plus wages.

dividend risk. In addition, Y_t features VR's close to one, in line with the flat term-structure of GDP risk. The left lower panel of Figure 6 shows the model implied volatilities of Y_t, W_t and D_t: income insurance leads to a smoothing and a leverage effect respectively on wages and dividends. The right upper and lower panels of Figure 6 show the VR's and the volatilities in absence of income insurance ($\phi = 0$): wage and dividend risk collapse to that of Y_t and no term-structure effects are observable.

Insert Table VII about here.

Table VII reports the values of the volatilities of cash-flows associated with several horizons and their spread over the short-run limit ($\tau \to 0$) in both the cases $\phi = \phi^*$ and $\phi = 0$. The slope of the term-structure of dividend risk becomes negative once income insurance is taken into account.

C. The term structure of equity

Shareholders have preference for the early resolution of uncertainty ($\gamma_s > 1/\psi$) and from their perspective the intertemporal substitution effect dominates the wealth effect ($\psi > 1$). The term structures of both premia and return volatility are decreasing with the maturity at short and medium horizons –in which the downward-sloping effect due to the transitory shock dominates the upward-sloping effect due to long-run growth– and slightly increasing at long horizons –in which the two effects barely offset each other. Figure 7 shows the term structures of equity premia and volatility as a function of the horizon. Numerical results are reported in the Panel C of Table VII.

Insert Figure 7 and 8 about here.

By changing preference heterogeneity, h, and, hence, the degree of income insurance, ϕ^*, it is possible to alter the slope of the term structures. A larger parameter ϕ^* increases the leverage effect on dividends and, in turn, the price associated to transitory risk. Consequently, for $\psi > 1$ the term-structures of equity volatility and premia are decreasing over a longer horizon and slopes are larger in magnitude. Equity features excess volatility over dividends at any horizon and approaches a value above fundamentals' risk in the long-run, as shown in Figure 8. Such a result obtains endogenously and although homoscedastic exogenous uncertainty. In particular, the "long-run" excess volatility is given by:

$$\sigma_P^2(t,\infty) - \sigma_D^2(t,\infty) = \frac{(1-2\psi)\sigma_\mu^2}{\lambda_\mu^2\psi^2} + \frac{d_z^2\sigma_z^2}{\psi^2}.$$

First, long-run excess-volatility does not depend on risk aversion. Second, μ_t contributes negatively to the long-run excess-volatility for $\psi > 1/2$ and vice-versa. Third, z_t always contributes positively to the long-run excess-volatility. For ϕ^* large enough, such an effect can lead to a total excess-volatility even if $\psi > 1/2$. The middle and left panels show the long-run excess-volatility as a function of ψ and ϕ^*. Excess-volatility increases with ϕ^* and decreases with ψ. The role of ψ has the following rationale:

$$\partial_\psi \left(\sigma_P^2(t,\infty) - \sigma_D^2(t,\infty)\right) \gtrless 0 \quad \text{if} \quad \psi - 1 \gtrless \frac{d_z^2\sigma_z^2}{\sigma_\mu^2/\lambda_\mu^2}.$$

On the left hand side $\psi - 1$ is the magnitude of either the intertemporal substitution effect ($\psi > 1$) or the wealth effect ($\psi < 1$). On the right hand side, we observe a value that is larger than one if $\sigma_D^2(t,0) > \sigma_D^2(t,\infty)$ and vice-versa. As a consequence, the more decreasing the term-structure of dividends volatility, the larger ψ needed to obtain excess-volatility of equity returns.

The model generates a long-run excess volatility of equity over dividends in line with the recent empirical evidence about the decreasing variance ratios of dividends, documented by Belo et al. (2014). Instead long-run risk models imply payouts to shareholders riskier than equity returns, as pointed out by Beeler and Campbell (2012). Namely, a long-run risk model ($Y_t = X_t$) produces "long-run" excess volatility only for $\psi < 1/2$ –a parametrization under which standard asset pricing implications would fail to obtain.

D. The market asset and other asset pricing implications

The model reconciles standard asset pricing facts with the evidence about the term-structure of both cash-flows and equity. The baseline calibration ($\gamma_s = 10, \psi = 1.5$) allows to match the unconditional level of the risk-free rate (about 1%) and a low volatility (about 4%) in line with the real data. The model leads to an equity premium of about 7% as in the real data: such a result is particularly remarkable since neither stochastic volatility and jumps nor unrealistically high and time-varying risk aversion are required. The return volatility is about 15%, which is somewhat lower than in the real data but implies a large excess-volatility over total consumption and dividends on the whole term-structure, including the long-run limit. As a consequence, the model produces a Sharpe ratio (about 48%), which is somewhat larger than in the real data. Such a result is peculiar and due to short-run but persistent risk. The model also captures quite well the level and the volatility of the price-dividend ratio (about $\exp(3.50)$ and 35%). Table VIII summarizes the model-implied moments.

Insert Table VIII about here

Table VIII reports the asset pricing moments for many pairs (γ_s, ψ). A good fit of the risk-free rate level and of the first two return moments of the market asset obtains for ($\gamma_s = 10, \psi = 1.5$), ($\gamma_s = 7.5, \psi = 1.25$) and ($\gamma_s = 5, \psi = 1.05$). However, decreasing the elasticity of intertemporal substitution leads to a risk-free rate and a price-dividend ratio which are respectively too volatile and too smooth in comparison with the data. Hence, the choice ($\gamma_s = 10, \psi = 1.5$) seems preferable. The model significantly improves over the Walrasian benchmark without income insurance ($\phi = 0$). The leverage effect due to income insurance not only is crucial to the modeling of the term-structures but also helps to match the standard asset pricing moments. For none of the 12 pairs (γ_s, ψ) a sizeable equity premium obtains in absence of income insurance. Figure 9 shows the equity premium as a function of ψ and ϕ^*(upper panels).

Insert Figure 9 about here

The premium is decreasing in ψ and increasing in ϕ^*, all else being equal. The former relation is due to the fact that, in order to describe the term-structures in equilibrium, we need that the pricing of

short-run but persistent risk dominates the pricing of long-run variations in expected growth.

In order to better understand the nature of the equity premium, it is instructive to adopt a term-structure perspective. The equilibrium density $\mathcal{H}(t,\tau)$ of Eq. (48) represents the relative contribution of each horizon τ to the whole premium. As shown in Figure 1, the model leads to a shift towards the short-run of the equilibrium density in comparison with a long-run risk model. The equilibrium density is monotone decreasing with the horizon, whereas the density associated to a long-run risk model ($Y_t = X_t$) is hump-shaped. Up to an horizon of about 5 years, the former stays above the latter; while after 5 years the relation reverses, implying a larger weight on the far future for the long-run risk model.

The timing decomposition of the premium does not depend only on the transitory shock z_t, but it also depends on how such a shock is priced in equilibrium. The right panel of Figure 9 shows how the equilibrium density $\mathcal{H}(t,\tau)$ changes for several values of the degree of income insurance ϕ^*. Indeed, income insurance leads not only to riskier dividends but also to a shift of the priced risk towards the short-run. The larger ϕ^*, the more important the contribution of the short horizons to the whole equity premium. This timing decomposition sheds lights on the nature of priced risk in equilibrium. Indeed, the model shows that a short-run explanation of market compensations is needed in order to simultaneously describe both the standard asset pricing moments and the downward sloping term-structures of both cash-flows and equity returns (given standard preferences: $\gamma_s > \psi > 1$).

E. Bond and equity yields

Now I consider the effect of income insurance on the equilibrium equity yields and their components, that is the yields on the risk-less bond, the expected dividend growth and the premium on the equity yields. All these quantities at the steady-state are plotted in Figure 10 as a function of the horizon both in presence ($\phi = \phi^*$, red solid lines) and absence ($\phi = 0$, blue dashed lines) of income insurance. Numerical results are reported in the Panel D of Table VII.

Insert Figure 10 about here

The left upper panel of Figure 10 shows that the equity yield is slightly decreasing with the horizon under income insurance, whereas it is strongly increasing in the Walrasian benchmark. For $\psi > 1$, z_t induces a downward-sloping effect, which is enhanced by the degree of income insurance and more than offset the upward-sloping effect due to μ_t. The right upper panel of Figure 10 shows the term-structure of the yield on the risk-less bond. In absence of income insurance, μ_t is the main determinant of slope of the term-structure of interest rates. Similarly to the long-run risk literature, a downward-sloping term-structure obtain for $\psi > 1$ and, hence, a positive nominal term-spread can only be imputed to inflation risk. Instead, in presence of income insurance, z_t induces a stronger upward-sloping effect which leads to a positive slope for the bond yields, in accord with the real data. Intuitively, income insurance makes dividends riskier at short horizons: such a short-run risk is priced in equilibrium and bonds with short

maturities are a better hedge than equity. Therefore, increasing bond yields obtain, although $\psi > 1$. The left lower panel of Figure 10 shows the term-structure of the expected dividend growth rates. Similarly to the equity yields, under income insurance the slope switches from slightly positive to negative. Finally, the right lower panel of Figure 10 reports the term-structure of the premia on the equity yields. Such premia increase and decrease with horizon respectively in absence and in presence of income insurance.

In summary, the analysis of the decomposition of equity yields provides two noteworthy results. First, income insurance helps to understand the term-structures of the equity yield premia, in line with the findings of van Binsbergen et al. (2013). Second, at the same time, income insurance helps to describe the term-structure of interest rates.

F. Cross-sectional returns and the value premium

Firm dividends are as in Section V.F and in spirit of Lettau and Wachter (2011). The share process and, equivalently, the (stable) distribution of firms is plotted in the upper left panel of Figure 11.

Insert Figure 11 about here

Firms with short residual life are interpreted as value firms, whereas firms with long residual life are interpreted as growth firms. Equilibrium price-dividend ratios are increasing with residual life in both the Walrasian benchmark ($\phi = 0$, blue dashed lines) and in case of income insurance ($\phi = \phi^*$, red solid lines). As shown in the upper right panel of Figure 11, price-dividend ratios are larger for growth firms than for value firms, as in the real data, but these levels are barely affected by income insurance.

Instead, income insurance is important in the determination of equilibrium volatility and premia. Steady state return volatility and premia are plotted as a function of the firm residual life in the lower panels of Figure 11. For $\gamma_s > \psi > 1$ and in absence on income insurance ($\phi = 0$, blue dashed lines), long-run growth is the main determinant of priced risk and, hence, long-run cash-flows require a substantial compensation. As a consequence, premia slightly increase with residual life. Therefore, the model produces a growth premium. Vice-versa, in presence of income insurance ($\phi = \phi^*$, red solid lines), priced risk shifts towards the short-run and, hence, short-run cash-flows deserve a substantial compensation. Then, equilibrium premia and volatilities decrease with the firm residual life: in line with the real data, value firms require larger premia than growth firms. Hence, income insurance helps to jointly explain in general equilibrium the value premium, the term-structure of cash-flows and equity as well as standard asset pricing facts.[34]

The rationale for the value premium differs from Lettau and Wachter (2011). In their paper the state-price density is exogenously specified, featuring a time-varying price of risk uncorrelated with fundamentals –which is an assumption difficult to obtain in general equilibrium. Here, instead, the value

[34]The range of variation of premia over the firm life-cycle is relatively small. This can be due to the fact that the share-process is assumed to be deterministic and the state-variables are homoscedastic. However, such assumptions are made for the sake of simplicity and exposition and can be relaxed without altering the qualitative implications of the model.

premium, as well as positive and negative slopes for the term-structures of bond yields and equity premia, obtain for a state-price density derived in general equilibrium.

VII. Alternative Specifications

A. Countercyclical heteroscedasticity and time-varying slope of equity premia

While the downward-sloping term-structure of dividend risk is a very robust feature of the data, the empirical evidence about the term structures of equity is under debate.[35] On the one hand, equity risk inherits the negative slope of dividend risk; on the other hand, the slope of equity premia depends on how and how much the sources of long-run and short-run risks are priced in equilibrium.

Downward-sloping equity premia suggested by van Binsbergen et al. (2012) appear as a stylized fact concerning the unconditional distribution of equity returns. Indeed, van Binsbergen et al. (2013) and Muir (2014) provide some evidence supporting the idea that the term-structure of equity premia is time-varying and switches from flat or slightly increasing in good times to strongly decreasing in bad times.

While I am agnostic about the exact properties we can infer from available data, this section shows that a minimal modification of the model allows for the above dynamics of the term-structure of equity premia. I slightly modify the dynamics of the transitory shock to include countercyclical heteroscedasticity:

$$Z_t = e^{z_t} = e^{\bar{z}-\hat{z}_t}, \quad \text{with} \quad d\hat{z}_t = \lambda_z(\bar{z}-\hat{z}_t)dt + \hat{\sigma}_z\sqrt{\hat{z}_t}\,dB_{z,t},$$

with $\hat{\sigma}_z = \sigma_z/\sqrt{\bar{z}}$ and $\bar{z} > 0$ such that z_t is still a zero-mean reverting process but its volatility is decreasing in z_t. Given the affine specification of the transitory shock, the model can be solved with the same methodology of the previous sections and all results are derived in the online appendix OA.C.

Countercyclical heteroscedasticity in fundamentals leads to two results. First, under income insurance the labor-share is decreasing in z_t and, hence, dividends load more on the transitory shock when the latter is high volatile. Second, the price of risk associated to z_t is no more a constant but is decreasing with z_t and, hence, higher compensations obtain in bad times.

These two facts lead to a slope of the term-structure of equity premia which is time-varying. Indeed, long-run growth leads to an upwards-sloping effect and the transitory shock leads to a downward-sloping effect but the latter depends on the level of the volatility of z_t. Therefore, the downward-sloping effect is countercyclical. As a result, the term-structure of equity premia can be flat or slightly upward-sloping in good times and strongly downward-sloping in bad times.

To provide an illustration of the equilibrium results, I set all parameters as in the baseline calibration of Table V apart $\bar{z} = .1$. The model generates steady state moments in line with the data: 0.6% risk-free rate with 3% volatility, an equity premium of 7.6% with return volatility of 15.2% and Sharpe ratio of 50%

[35] See Boguth, Carlson, Fisher, and Simutin (2012) and van Binsbergen and Koijen (2012b).

and log price-dividend ratio of 3.48 with 34% volatility.

Insert Figure 12 about here

The left upper panel of Figure 12 shows the term-structures of the model implied VR's (solid lines) and their empirical counterparts (dashed lines). Similarly to Figure 6, the model matches the upward- and downward-sloping shapes of wage and dividend risk and the flat term-structure of GDP risk. The right upper panel of Figure 12 shows the model implied volatility of dividends at the steady-state (solid line) and the 5-95% probability interval (dot-dashed and dashed lines). Dividend volatility is strongly decreasing with z_t at short horizons, whereas the effect of the transitory shock disappears in the long-run. The lower panels of Figure 12 shows the model implied volatility (left) and premia (right) of the dividend strip returns at the steady-state (solid line) and the 5-95% probability interval (dot-dashed and dashed lines). Under standard preferences ($\gamma_s > \psi > 1$), equity risk inherits the negative slope of dividend risk. The transitory shock affects the level of the term-structure, which is countercyclical, but does not alter the sign of the slope. Instead, the transitory shock affects both the level and the slope of the term-structure of equity premia. Equity compensations are low and barely flat or slightly increasing in normal and good times. However, compensations increase in size and become markedly downward-sloping in bad times.

Countercyclical heteroscedasticity helps the model to match the conditional dynamics of the term-structure of equity premia and, hence, to provide further support to the main model mechanism of income insurance.

B. Power utility and endogenously time-varying risk premia

This section characterizes asset prices in the case of power utility ($\psi \to \gamma_s^{-1}$). Analytical solutions obtain even for the true dynamics of dividends in Eq. (26). All results are derived in the online appendix OA.D.

Both the price of risk as well as return moments are endogenously time-varying. The state-price density is $\xi_{0,t} = e^{-\beta t} Y_t^{-\gamma_s} (1-\omega(z_t))^{-\gamma_s}$. The usual power function of total consumption is multiplied by a power function of the dividend-share $1-\omega(z_t)$. The latter depends on the transitory shock and embeds the effect of income insurance. Since, the dividend-share is not an exponential affine functional of z_t, as an endogenous result, the logarithm of the state-price density is convex in z_t and, hence, the price of risk is state-dependent.

Under power utility only the transient risk of shareholders' consumption is priced in equilibrium. Therefore, μ_t does not deserve a premium and z_t is priced only for its contribution to the instantaneous volatility of dividends. Such a contribution is affected by income insurance: $\theta_z(t) = \gamma_s \sigma_z \big(1 + \phi^* \omega(z_t)/(1-\omega(z_t))\big)$. On the one hand, the stronger the degree of income insurance the larger the price of risk in magnitude; on the other hand, a negative (positive) transitory shock leads to a decrease (increase) of the fraction of total consumption devoted to shareholders' remuneration and, hence, to an increase (decrease) in the price of risk. Such a counter-cyclical dynamics of the price of risk is a robust feature of the real data and obtains in the model as an endogenous result of income insurance, whereas it disappears for $\phi \to 0$.

The strip price-dividend ratio is a stationary function of μ_t and z_t. However, income insurance leads to non-linearities in asset prices and, in turn, to endogenous time-variation in the return moments. Instead, for $\phi \to 0$, the price reduces to the exponential affine case and return moments are constant. Interestingly, even if the effect of income insurance on dividends only concerns the transitory shock, both μ_t and z_t enter the return dynamics. Therefore, the shape of the term structure of equity risk and premia is state-dependent and moves with both μ_t and z_t. The premium on the dividend strip is a compensation for transient risk only:

$$(\mu_P - r)(t,\tau) = \gamma_s \sigma_x^2 + \gamma_s \mathcal{P}_z(\tau, \mu_t, z_t) \left(1 + \phi^* \tfrac{\omega(z_t)}{1-\omega(z_t)}\right) \sigma_z^2.$$

The first term has the usual form $\gamma_s \sigma_x^2$. Instead, the second term has the following rationale. The volatility of the transitory shock σ_z is multiplied by the price of risk $\theta_z(t)$ and by the semi-elasticity $\mathcal{P}_z$ of the price with respect to z_t. Notice that such a term is the only one involving the maturity τ and, hence, determines the sign of the term-structure of equity premia. To gain intuition consider again the log-linearization of Eq. (15). Specializing Corollary 2 to the power utility case ($\psi \to \gamma_s^{-1}$), we obtain the following approximation results:

$$\begin{aligned}
\partial_\tau \sigma_P^2(t,\tau) &= 2\left(e^{-2\lambda_\mu \tau}(e^{\lambda_\mu \tau} - 1)(\gamma_s - 1)^2 \lambda_\mu^{-1} \sigma_\mu^2 \; - \; e^{-2\tau\lambda_z}(\gamma_s - 1)(1 + (e^{\tau\lambda_z} - 1)\gamma_s)\lambda_z d_z^2 \sigma_z^2\right), \\
\partial_\tau (\mu_P - r)(t,\tau) &= e^{-\tau\lambda_z}(\gamma_s - 1)\lambda_z d_z^2 \sigma_z^2.
\end{aligned}$$

The term-structure of equity risk is monotone increasing for $\gamma_s > 1$ and non-monotone for $\gamma_s < 1$, whereas the term-structure of equity premia is always monotone but the slope has the same sign of $\gamma_s - 1$ (such a restriction is due to power utility).

The equilibrium price of the market asset is given by the time integral of the strip prices. Then, the logarithm of the price dividend ratio is a stationary but nonlinear function of μ_t and z_t. Consequently, the equity premium and the market volatility are endogenously time-varying as long as $\phi^* > 0$.

Since shareholders' risk aversion has to be lower than unit in order to generate negative slopes for equity premia, the model cannot simultaneously generate a sizeable equity premium under power utility. Nevertheless, it is instructive to examine the model predictions. I preserve all parameter values from the calibration of Section VI.A (see Table V), but the shareholders risk aversion is set to $\gamma_s = 0.75$. The optimal degree of income insurance is unchanged and, in turn, cash-flows unconditional moments are also unchanged. Instead, the unconditional equity premium reduces to about 3% but $\gamma_s < 1$ guarantees that the term-structure of premia on the dividend strip is still downward sloping.

Insert Figure 13 about here

The upper left panel of Figure 13 shows that the volatility of dividends decreases with both the maturity τ and z_t. Consequently, a steeper downward sloping term structure obtains when z_t is low or equivalently

when the labor share $\omega(z_t)$ or the wage-to-dividends ratio $\frac{\omega(z_t)}{1-\omega(z_t)}$ are large. This result is consistent with the empirical findings of Table III and Figure 4. The other two panels in the first row of Figure 13 show that both the premium and the return volatility of the dividend strip are decreasing with the maturity. Moreover, the model generates an excess of volatility over both dividends and consumption, although only transient risk is priced under power utility. The second row of Figure 13 shows that both the premium and the return volatility of the dividend strip as well as the Sharpe ratio are decreasing in the transitory shock z_t. Hence, income insurance endogenously leads to such countercyclical dynamics which is also a feature of the real data. The third row of Figure 13 shows the expected dividend growth and the equity yield as a function of both the maturity τ and the transitory shock z_t. The former is decreasing in z_t, whereas the latter is increasing. These relationships are consistent with the empirical results of Section II.D: in the data, the labor-share (which is a decreasing function of z_t in the model) predicts dividend growth and equity returns respectively with positive and negative sign. The same holds in the model.

VIII. Conclusion

This paper includes labor relations in an otherwise standard and parsimonious general equilibrium asset pricing model. Unlike standard Walrasian models, wages incorporate an income insurance that workers exploit within the firm. In turn, the labor-share has counter-cyclical dynamics and, hence, the riskiness of owning capital enhances. A term-structure effect takes place: equilibrium dividends and discount rates lead to short-term risk in equity returns. Under standard preferences, downward sloping term structures of both premia and volatility of equity obtain. Furthermore, the model simultaneously accounts for i) a good fit of the first two moments of both the market asset return and the risk-free rate; ii) endogenously counter-cyclical price of risk and equity premium as well as variation in return volatility; iii) cross-sectional value premium of equities; iv) upward-sloping real yields on bonds and downward-sloping premia on equity yields.

An empirical analysis supports the main model mechanism documenting a connection between income insurance and i) the upward-sloping term-structure of wage risk and downward-sloping term structure dividend risk; and ii) the counter-cyclical dynamics of the wage share and the pro-cyclical one of the dividend share. Consistently with the model, the time-series of the wage-to-dividend ratios strongly explain the distance between the variance ratios of wages and dividends over time.

The paper sheds lights on the implications of the equilibrium term-structures on traditional asset pricing facts, such as the equity premium. A timing decomposition of the premium shows that investors have a different perception of priced risk than in leading models. In the latter the equity premium comes out from variation in long-run discounted cash-flows, whereas accounting for the term-structures of macroeconomic variables implies that priced risk significantly shifts toward the short-run.

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Appendix A. Income Insurance

Proof of Lemma 1: Under Assumption 3, the transitory components of wages and dividends are given by Eq. (11). Therefore, the optimal degree of income insurance maximizes the geometric average of their unconditional expectations: $\left(\mathbb{E}_z[\tilde{W}(\phi)]\right)^{\vartheta}\left(\mathbb{E}_z[\tilde{D}(\phi)]\right)^{1-\vartheta}$ with bargaining parameter ϑ given in Eq. (14). We have

$$\begin{aligned}\mathbb{E}_z[\tilde{W}(\phi)] &= \alpha^{1-\phi}e^{\bar{z}(1-\phi)+\sigma_z^2(1-\phi)^2/(4\lambda_z)},\\ \mathbb{E}_z[\tilde{D}(\phi)] &= e^{\bar{z}+\sigma_z^2/(4\lambda_z)} - \alpha^{1-\phi}e^{\bar{z}(1-\phi)+\sigma_z^2(1-\phi)^2/(4\lambda_z)}.\end{aligned}$$

Taking the derivative with respect to ϕ,

$$\begin{aligned}0 = &\frac{\alpha^{-\phi}}{2\lambda_z}\left(\alpha^{1-\phi}e^{\frac{(-4\lambda_z\bar{z}+\sigma_z^2(\phi-1))(\phi-1)}{4\lambda_z}}\right)^{\vartheta}\left(e^{\bar{z}+\frac{\sigma_z^2}{4\lambda_z}} - \alpha^{1-\phi}e^{\frac{(-4\lambda_z\bar{z}+\sigma_z^2(\phi-1))(\phi-1)}{4\lambda_z}}\right)^{-\vartheta}\left(\alpha e^{\frac{(-4\lambda_z\bar{z}+\sigma_z^2(\phi-1))(\phi-1)}{4\lambda_z}} - \alpha^{\phi}e^{\bar{z}+\frac{\sigma_z^2}{4\lambda_z}}\vartheta\right)\\ &\times\left(2\lambda_z\bar{z} - \sigma_z^2(\phi-1) + 2\lambda_z\log\alpha\right),\end{aligned}$$

we get three solutions:

$$\begin{aligned}\tilde{\phi} &= 1 + \tfrac{2\lambda_z}{\sigma_z^2}(\bar{z}+\log\alpha),\\ \tilde{\phi} &= 1 + \tfrac{2\lambda_z}{\sigma_z^2}(\bar{z}+\log\alpha) - \sqrt{(-4\lambda_z\bar{z} - 2\sigma_z^2 - 4\lambda_z\log\alpha)^2 + 16\lambda_z\sigma_z^2\log(\vartheta/\alpha)}\,\sigma_z^{-2}/2,\\ \tilde{\phi} &= 1 + \tfrac{2\lambda_z}{\sigma_z^2}(\bar{z}+\log\alpha) + \sqrt{(-4\lambda_z\bar{z} - 2\sigma_z^2 - 4\lambda_z\log\alpha)^2 + 16\lambda_z\sigma_z^2\log(\vartheta/\alpha)}\,\sigma_z^{-2}/2.\end{aligned}$$

Since we are looking for a potential solution $\phi^* \in (0,1)$, we can exclude the first two solutions for $\tilde{\phi}$ given the choice $\bar{z} = 0$, which is used in the model calibration and which guarantees $D_t > 0\,\forall t$ under the log-linearized dynamics of Eq. (15) (under reasonable parameters the second term common to all solutions for $\tilde{\phi}$ is by far lower than minus one). Therefore, we consider the third solution and set $\phi^* = \max(0, \min(1, \tilde{\phi}))$ as in Eq. (13). *q.e.d.*

Appendix B. Equilibrium, Cash-Flows and Asset Prices

Proof of Proposition 1, 2 and 3: Under the infinite horizon, the utility process J satisfies the following Bellman equation: $\mathcal{D}J(X,\mu,z) + f(C_s,J) = 0$, where $\mathcal{D}$ denotes the differential operator. Then we have

$$0 = J_X\mu X + \tfrac{1}{2}J_{X,X}\sigma_x^2X^2 + J_\mu\lambda_\mu(\bar{\mu}-\mu) + \tfrac{1}{2}J_{\mu,\mu}\sigma_\mu^2 + J_z\lambda_z(\bar{z}-z) + \tfrac{1}{2}J_{z,z}\sigma_z^2 + f(C_s,J).$$

Guess a solution of the form $J(X,\mu,z) = \frac{1}{1-\gamma_s}X^{1-\gamma_s}g(\mu,z)$. The Bellman equation reduces to

$$0 = \mu - \tfrac{1}{2}\gamma_s\sigma_x^2 + \tfrac{g_\mu}{g}\tfrac{\lambda_\mu(\bar{\mu}-\mu)}{1-\gamma_s} + \tfrac{1}{2}\tfrac{g_{\mu,\mu}}{g}\tfrac{\sigma_\mu^2}{1-\gamma_s} + \tfrac{g_z}{g}\tfrac{\lambda_z(\bar{z}-z)}{1-\gamma_s} + \tfrac{1}{2}\tfrac{g_{z,z}}{g}\tfrac{\sigma_z^2}{1-\gamma_s} + \tfrac{\beta}{1-1/\psi}\left(g^{-1/\chi}e^{(1-1/\psi)(d_0+d_zz)} - 1\right). \tag{B1}$$

Under Assumption 2 and stochastic differential utility, the pricing kernel has dynamics given by

$$d\xi_{0,t} = \xi_{0,t}\tfrac{df_C}{f_C} + \xi_{0,t}f_Jdt = -r(t)\xi_{0,t} - \theta_x(t)\xi_{0,t}dB_{x,t} - \theta_\mu(t)\xi_{0,t}dB_{\mu,t} - \theta_z(t)\xi_{0,t}dB_{z,t}, \tag{B2}$$

where, by use of Itô's Lemma and Eq. (B1), we get

$$r(t) = -\tfrac{\partial_X f_C}{f_C}\mu X - \tfrac{1}{2}\tfrac{\partial_{X,X}f_C}{f_C}\sigma_x^2X^2 - \tfrac{\partial_\mu f_C}{f_C}\lambda_\mu(\bar{\mu}-\mu) - \tfrac{1}{2}\tfrac{\partial_{\mu,\mu}f_C}{f_C}\sigma_\mu^2 - \tfrac{\partial_z f_C}{f_C}\lambda_z(\bar{z}-z) - \tfrac{1}{2}\tfrac{\partial_{z,z}f_C}{f_C}\sigma_z^2 - f_J,$$

$$\theta_x(t) = -\frac{\partial_X fc}{fc}\sigma_x X, \qquad \theta_\mu(t) = -\frac{\partial_\mu fc}{fc}\sigma_\mu, \qquad \theta_z(t) = -\frac{\partial_z fc}{fc}\sigma_z,$$

An exact solution for $g(\mu, z)$ satisfying Eq. (B1) does not exist for $\psi \neq 1$. Therefore, I look for a solution of $g(\mu, z)$ around the unconditional mean of the consumption-wealth ratio. Aggregate wealth is given by

$$Q_{s,t} = \mathbb{E}_t\left[\int_t^\infty \xi_{t,u} C_{s,u} du\right],$$

and, applying Fubini's Theorem and taking standard limits, the consumption-wealth ratio satisfies

$$\frac{C_{s,t}}{Q_{s,t}} = r(t) - \frac{1}{dt}\mathbb{E}_t\left[\frac{dQ}{Q}\right] - \frac{1}{dt}\mathbb{E}_t\left[\frac{d\xi}{\xi}\frac{dQ}{Q}\right]. \tag{B3}$$

Guess

$$Q_{s,t} = C_{s,t}\beta^{-1}\left(g(\mu_t, z_t)e^{(\gamma_s-1)(d_0+d_z z_t)}\right)^{1/\chi}$$

and apply Itô's Lemma to get $\frac{dQ}{Q}$. Then, plug in the wealth dynamics, the risk-free rate and the pricing kernel into Eq. (B3): after tedious calculus you can recognize that the guess solution is correct. Notice that the consumption-wealth ratio approaches to β when $\psi \to 1$ as usual.

Denote $cq = \mathbb{E}[\log C_{s,t} - \log Q_{s,t}]$, hence, a first-order approximation of the consumption-wealth ratio around cq produces

$$\frac{C_{s,t}}{Q_{s,t}} = \beta g(\mu_t, z_t)^{-1/\chi} e^{(1-1/\psi)(d_0+d_z z_t)} \approx e^{cq}\left(1 - cq + \log\beta - \frac{1}{\chi}\left(\log g(\mu_t, z_t) + (\gamma_s - 1)(d_0 + d_z z_t)\right)\right).$$

Using such approximation in the Bellman equation (B1) leads to

$$\begin{aligned} 0 =& \mu - \frac{1}{2}\gamma_s\sigma_x^2 + \frac{g_\mu}{g}\frac{\lambda_\mu(\bar{\mu}-\mu)}{1-\gamma_s} + \frac{1}{2}\frac{g_{\mu,\mu}}{g}\frac{\sigma_\mu^2}{1-\gamma_s} + \frac{g_z}{g}\frac{\lambda_z(\bar{z}-z)}{1-\gamma_s} + \frac{1}{2}\frac{g_{z,z}}{g}\frac{\sigma_z^2}{1-\gamma_s} \\ &+ \frac{1}{1-1/\psi}\left(e^{cq}\left(1 - cq + \log\beta - \frac{1}{\chi}\log g(\mu, z) + (1-1/\psi)(d_0 + d_z z)\right) - \beta\right), \end{aligned}$$

which has exponentially affine solution $g(\mu, z) = e^{u_0 + (1-\gamma_s)d_0 + u_\mu\mu + (u_z + (1-\gamma_s)d_z)z}$, where u_0, u_μ and u_z have explicit solutions and the endogenous constant cq satisfies $cq = \log\beta - \chi^{-1}(u_0 + u_\mu\bar{\mu} + u_z\bar{z})$. The risk-free rate and the prices of risk take the form:

$$\begin{aligned} r_0 =& \frac{1}{2}\left(\frac{2\left(\beta\psi^2(\gamma_s-1)^2 + e^{cq}((1-\psi)u_0 + \psi(cq - 1 - \log(\beta))(\gamma_s-1))(\psi\gamma_s-1)\right)}{\psi(1-\psi)(\gamma_s-1)} + \frac{2\bar{z}(u_z - \psi(u_z - d_z(\gamma_s-1))\gamma_s)\lambda_z}{\psi(-1+\gamma_s)}\right. \\ &\left. - \frac{2\bar{\mu}u_\mu(\psi\gamma_s-1)\lambda_\mu}{\psi(\gamma_s-1)} - \gamma_s(1+\gamma_s)\sigma_x^2 - \frac{(u_z - \psi(u_z - d_z(\gamma_s-1))\gamma_s)^2\sigma_z^2}{\psi^2(\gamma_s-1)^2} - \frac{u_\mu^2(\psi\gamma_s-1)^2\sigma_\mu^2}{(\psi-\psi\gamma_s)^2}\right), \\ r_\mu =& \frac{\psi(\gamma_s-1)\gamma_s + u_\mu(\psi\gamma_s-1)\left(e^{cq}+\lambda_\mu\right)}{\psi(\gamma_s-1)}, \\ r_z =& \frac{-\psi d_z(\gamma_s-1)\gamma_s\lambda_z + u_z(\psi\gamma_s-1)(e^{cq}+\lambda_z)}{\psi(\gamma_s-1)}, \\ \theta_x(t) =& \gamma_s\sigma_x, \qquad \theta_\mu(t) = \frac{u_\mu\left(\gamma_s - \frac{1}{\psi}\right)}{1-\gamma_s}\sigma_\mu, \qquad \theta_z(t) = \left(d_z\gamma_s + \frac{u_z(1-\psi\gamma_s)}{\psi(\gamma_s-1)}\right)\sigma_z, \end{aligned}$$

and the results of Proposition 2 and 3 easily follow. *q.e.d.*

Proof of Lemma 2: All results simply obtain from the dynamics in Eq. (6)-(8)-(26) and standard calculus. *q.e.d.*

Proposition A *The following conditional expectation has exponential affine solution:*

$$\mathcal{M}_{t,\tau}(\bar{c}) = \mathbb{E}_t[e^{c_0+c_1\log\xi_{0,t+\tau}+c_2x_{t+\tau}+c_3\mu_{t+\tau}+c_4z_{t+\tau}}] = \xi_{0,t}^{c_1}X_t^{c_2}e^{\ell_0(\tau,\bar{c})+\ell_\mu(\tau,\bar{c})\mu_t+\ell_z(\tau,\bar{c})z_t}, \tag{B4}$$

where $\bar{c} = (c_0, c_1, c_2, c_3, c_4)$, model parameters are such that the expectation exists finite and ℓ_0, ℓ_μ and ℓ_z are deterministic functions of time derived in the Appendix B.

Proof of Proposition A: Consider the following conditional expectation:

$$\mathcal{M}_{t,\tau}(\bar{c}) = \mathbb{E}_t[e^{c_0+c_1\log\xi_{0,t+\tau}+c_2\log X_{t+\tau}+c_3\mu_{t+\tau}+c_4z_{t+\tau}}] \tag{B5}$$

where $\bar{c} = (c_0, c_1, c_2, c_3, c_4)$ is a coefficient vector such that the expectation exists. Guess an exponential affine solution of the kind:

$$\mathcal{M}_{t,\tau}(\bar{c}) = e^{c_1\log\xi_{0,t}+c_2\log X_t+\ell_0(\tau,\bar{c})+\ell_\mu(\tau,\bar{c})\mu_t+\ell_z(\tau,\bar{c})z_t}, \tag{B6}$$

where $\ell_0(\tau,\bar{c}), \ell_\mu(\tau,\bar{c})$ and $\ell_z(\tau,\bar{c})$ are deterministic functions of time. Feynman-Kac gives that $\mathcal{M}$ has to meet the following partial differential equation

$$\begin{aligned}
0 = &\mathcal{M}_t - \mathcal{M}_\xi(r_0 + r_\mu\mu + r_z z) + \tfrac{1}{2}\mathcal{M}_{\xi,\xi}(\theta_x(t)^2 + \theta_\mu(t)^2 + \theta_z(t)^2) + \mathcal{M}_X(\mu X) + \tfrac{1}{2}\mathcal{M}_{X,X}\sigma_x^2X^2 \\
&+ \mathcal{M}_\mu\lambda_\mu(\bar{\mu}-\mu) + \tfrac{1}{2}\mathcal{M}_{\mu,\mu}\sigma_\mu^2 + \mathcal{M}_z\lambda_z(\bar{z}-z) + \tfrac{1}{2}\mathcal{M}_{z,z}\sigma_z^2 - \mathcal{M}_{\xi,X}\theta_x(t)\sigma_x X \\
&- \mathcal{M}_{\xi,\mu}\theta_\mu(t)\sigma_\mu - \mathcal{M}_{\xi,z}\theta_z(t)\sigma_z,
\end{aligned}$$

where the arguments have been omitted for ease of notation. Plugging the resulting partial derivatives from the guess solution into the pde and simplifying gives a linear function of the states μ and z. Hence, we get three ordinary differential equations for $\ell_0(\tau,\bar{c}), \ell_\mu(\tau,\bar{c})$ and $\ell_z(\tau,\bar{c})$:

$$\begin{aligned}
0 = &\ell_0'(\tau,\bar{c}) - c_1r_0 + \tfrac{1}{2}c_1(c_1-1)(\theta_x(t)^2 + \theta_\mu(t)^2 + \theta_z(t)^2) + \tfrac{1}{2}c_2(c_2-1)\sigma_x^2 + \ell_\mu(\tau,\bar{c})\lambda_\mu\bar{\mu} \\
&+ \tfrac{1}{2}\ell_\mu(\tau,\bar{c})^2\sigma_\mu^2 + \ell_z(\tau,\bar{c})\lambda_z\bar{z} + \tfrac{1}{2}\ell_z(\tau,\bar{c})^2\sigma_z^2 - c_1c_2\theta_x(t)\sigma_x - c_1\ell_\mu(\tau,\bar{c})\theta_\mu(t)\sigma_\mu \\
&- c_1\ell_z(\tau,\bar{c})\theta_z(t)\sigma_z \\
0 = &\ell_\mu'(\tau,\bar{c}) - c_1r_\mu + c_2 - \ell_\mu(\tau,\bar{c})\lambda_\mu \\
0 = &\ell_z'(\tau,\bar{c}) - c_1r_z - \ell_z(\tau,\bar{c})\lambda_z
\end{aligned}$$

with initial conditions: $\ell_0(0,\bar{c}) = c_0, \ell_\mu(0,\bar{c}) = c_3$ and $\ell_z(0,\bar{c}) = c_4$. Explicit solutions are avaliable. *q.e.d.*

Proof of Corollary 1: The conditional moment generating function $\mathbb{D}_t(\tau,n)$ of Eq. (31) obtains as a special case of $\mathcal{M}_{t,\tau}(\bar{c})$ with $\bar{c} = (nd_0, 0, n, 0, nd_z)$. Therefore, it is given by $\mathbb{D}_t(\tau,n) = X_t^n e^{B_0(\tau,n)+B_\mu(\tau,n)\mu_t+B_z(\tau,n)z_t}$, with

$$\begin{aligned}
B_0(n,\tau) = \ell_0(\tau,\bar{c}) = &\tfrac{1}{4}n\Bigg(4d_0 + 4\left(1-e^{-\tau\lambda_z}\right)\bar{z}d_z + 4\bar{\mu}\left(\tau + \tfrac{-1+e^{-\tau\lambda_\mu}}{\lambda_\mu}\right) + 2(-1+n)\tau\sigma_x^2 \\
&+ \frac{e^{-2\tau(\lambda_z+\lambda_\mu)}n\left(e^{2\tau\lambda_\mu}\left(-1+e^{2\tau\lambda_z}\right)d_z^2\lambda_\mu^3\sigma_z^2 + e^{2\tau\lambda_z}\lambda_z\left(-1+4e^{\tau\lambda_\mu}+e^{2\tau\lambda_\mu}\left(-3+2\tau\lambda_\mu\right)\right)\sigma_\mu^2\right)}{\lambda_z\lambda_\mu^3}\Bigg), \\
B_\mu(n,\tau) = \ell_\mu(\tau,\bar{c}) = &\tfrac{n-e^{-\tau\lambda_\mu}n}{\lambda_\mu}, \\
B_z(n,\tau) = \ell_z(\tau,\bar{c}) = &e^{-\tau\lambda_z}nd_z,
\end{aligned}$$

and $B_0(0,n) = nd_0, B_\mu(0,n) = 0$ and $B_z(0,n) = nd_z$. Therefore, the coefficients in Eq. (32)-(33) are given by

$$g_{D,\tau,\mu} = \tfrac{1}{\lambda_\mu \tau}(1 - e^{-\lambda_\mu \tau}), \quad g_{D,\tau,z} = \tfrac{d_z}{\tau}(e^{-\lambda_z \tau} - 1),$$

and

$$v_{D,\tau,x} = 1, \quad v_{D,\tau,\mu} = \tfrac{4e^{-\lambda_\mu \tau} - e^{-2\lambda_\mu \tau} + 2\lambda_\mu \tau - 3}{2\lambda_\mu^3 \tau}, \quad v_{D,\tau,z} = \tfrac{e^{-\lambda_z \tau}\sinh(\lambda_z \tau) d_z^2}{\lambda_z \tau}.$$

Finally, the results about the limits automatically follow.
Using the true dynamics of dividends of Eq. (26), we have the following results:

$$\mathbb{E}_t\left[D_{t+\tau}\right] = \mathbb{E}_t\left[X_{t+\tau}\left(e^{z_{t+\tau}} - \alpha^{1-\phi^*} e^{(1-\phi^*)z_{t+\tau}}\right)\right] = \mathcal{M}_{t,\tau}(\bar{c}) - \mathcal{M}_{t,\tau}(\bar{c}'),$$

where $\bar{c} = (0,0,1,0,1)$ and $\bar{c}' = ((1-\phi^*)\log\alpha, 0, 1, 0, 1-\phi^*)$, and

$$\mathbb{E}_t\left[D_{t+\tau}^2\right] = \mathbb{E}_t\left[X_{t+\tau}^2\left(e^{z_{t+\tau}} - \alpha^{1-\phi^*} e^{(1-\phi^*)z_{t+\tau}}\right)^2\right] = \mathcal{M}_{t,\tau}(\bar{c}) - 2\mathcal{M}_{t,\tau}(\bar{c}') + \mathcal{M}_{t,\tau}(\bar{c}''),$$

where $\bar{c} = (0,0,2,0,2)$, $\bar{c}' = ((1-\phi^*)\log\alpha, 0, 2, 0, 2-\phi^*)$ and $\bar{c}'' = (2(1-\phi^*)\log\alpha, 0, 2, 0, 2(1-\phi^*))$. With this results in hand, it is possible to compute the term-structures of the expected growth and volatility. *q.e.d.*

Proof of Proposition 4: The equilibrium price of the market dividend strip with maturity τ of Eq. (36) obtains as a special case of $\mathcal{M}_{t,\tau}(\bar{c})$ with $\bar{c} = (d_0, 1, 1, 0, d_z)$. Therefore, it is given by $P_{t,\tau} = \xi_{0,t}^{-1}\mathcal{M}_{t,\tau}(\bar{c}) = X_t e^{A_0(\tau) + A_\mu(\tau)\mu_t + A_z(\tau) z_t}$ with

$$\begin{aligned}
A_0(\tau) = \ell_0(\tau,\bar{c}) = \tfrac{1}{4}\Big(&-\tfrac{4e^{-\tau\lambda_\mu}\bar{\mu}(-1+r_\mu)(1+e^{\tau\lambda_\mu}(-1+\tau\lambda_\mu))}{\lambda_\mu} + \tfrac{e^{-2\tau(\lambda_z+\lambda_\mu)}}{\lambda_z^3\lambda_\mu^3} \\
&\times\Big(-e^{2\tau\lambda_\mu}(r_z + d_z\lambda_z)^2\lambda_\mu^3\sigma_z^2 + 4e^{\tau(\lambda_z+2\lambda_\mu)}(r_z + d_z\lambda_z)\lambda_\mu^3\left(-\bar{z}\lambda_z^2 + \sigma_z(\theta_z\lambda_z + r_z\sigma_z)\right) \\
&-e^{2\tau\lambda_z}(-1+r_\mu)^2\lambda_z^3\sigma_\mu^2 + 4e^{\tau(2\lambda_z+\lambda_\mu)}(-1+r_\mu)\lambda_z^3\sigma_\mu(\theta_\mu\lambda_\mu + (-1+r_\mu)\sigma_\mu) \\
&+e^{2\tau(\lambda_z+\lambda_\mu)}\Big(\lambda_\mu^3\left(4\lambda_z^2(\bar{z}(r_z + (d_z - \tau r_z)\lambda_z) + \lambda_z(d_0 - \tau(r_0 + \theta_x\sigma_x))) - 4\theta_z\lambda_z(r_z + (d_z - \tau r_z)\lambda_z)\sigma_z\right. \\
&\left.+\left(d_z^2\lambda_z^2 - 2d_z r_z\lambda_z + r_z^2(2\tau\lambda_z - 3)\right)\sigma_z^2\right) + 4(r_\mu - 1)\theta_\mu\lambda_z^3\lambda_\mu(\tau\lambda_\mu - 1)\sigma_\mu + (r_\mu - 1)^2\lambda_z^3(2\tau\lambda_\mu - 3)\sigma_\mu^2\Big)\Big),
\end{aligned}$$

$$\begin{aligned}
A_\mu(\tau) &= \ell_\mu(\tau,\bar{c}) = \tfrac{1 + e^{-\tau\lambda_\mu}(-1+r_\mu) - r_\mu}{\lambda_\mu}, \\
A_z(\tau) &= \ell_z(\tau,\bar{c}) = \tfrac{-r_z + e^{-\tau\lambda_z}(r_z + d_z\lambda_z)}{\lambda_z},
\end{aligned}$$

and $A_0(0) = d_0, A_\mu(0) = 0$ and $A_z(0) = d_z$. Itô's Lemma gives the dynamics of the market dividend strip price:

$$dP_{t,\tau} = [\cdot]dt + \partial_x P_{t,\tau}\sigma_x dB_{x,t} + \partial_\mu P_{t,\tau}\sigma_\mu dB_{\mu,t} + \partial_z P_{t,\tau}\sigma_z dB_{z,t}.$$

Therefore the return volatility is given by

$$\begin{aligned}
\sigma_P(t,\tau) &= P_{t,\tau}^{-1}\sqrt{(\partial_x P_{t,\tau}\sigma_x)^2 + (\partial_\mu P_{t,\tau}\sigma_\mu)^2 + (\partial_z P_{t,\tau}\sigma_z)^2} = \sqrt{\sigma_x^2 + (A_\mu(\tau)\sigma_\mu)^2 + (A_z(\tau)\sigma_z)^2}, \\
(\mu_P - r)(t,\tau) &= -\tfrac{1}{dt}\left\langle \tfrac{d\xi_{0,t}}{\xi_{0,t}}, \tfrac{dP_{t,\tau}}{P_{t,\tau}}\right\rangle = \theta_x(t)\sigma_x + \theta_\mu(t)A_\mu(\tau)\sigma_\mu + \theta_z(t)A_z(\tau)\sigma_z.
\end{aligned}$$

q.e.d.

Proof of Corollary 2: Given the results of Proposition 4, the limits and the slopes of the return volatility and premium for the market dividend strip obtain by standard calculus:

$$\begin{aligned}
\lim_{\tau\to 0}\sigma_P^2(t,\tau) &= \lim_{\tau\to 0}\left(\sigma_x^2+(A_\mu(\tau)\sigma_\mu)^2+(A_z(\tau)\sigma_z)^2\right),\\
\lim_{\tau\to \infty}\sigma_P^2(t,\tau) &= \lim_{\tau\to \infty}\left(\sigma_x^2+(A_\mu(\tau)\sigma_\mu)^2+(A_z(\tau)\sigma_z)^2\right),\\
\partial_\tau\sigma_P^2(t,\tau) &= \partial_\tau\left(\sigma_x^2+(A_\mu(\tau)\sigma_\mu)^2+(A_z(\tau)\sigma_z)^2\right),\\
\lim_{\tau\to 0}(\mu_P-r)(t,\tau) &= \lim_{\tau\to 0}\left(\theta_x(t)\sigma_x+\theta_\mu(t)A_\mu(\tau)\sigma_\mu+\theta_x(t)A_z(\tau)\sigma_z\right),\\
\lim_{\tau\to \infty}(\mu_P-r)(t,\tau) &= \lim_{\tau\to \infty}\left(\theta_x(t)\sigma_x+\theta_\mu(t)A_\mu(\tau)\sigma_\mu+\theta_x(t)A_z(\tau)\sigma_z\right),\\
\partial_\tau(\mu_P-r)(t,\tau) &= \partial_\tau\left(\theta_x(t)\sigma_x+\theta_\mu(t)A_\mu(\tau)\sigma_\mu+\theta_x(t)A_z(\tau)\sigma_z\right).
\end{aligned}$$

q.e.d.

Proof of Proposition 5: Under Assumption 2, the shareholders act as a representative agent on the financial markets and, hence, the equilibrium price of the market asset is equal to the shareholders' wealth. Therefore, using the results of Proposition 1, the market asset price can be written as

$$P_t = Q_{s,t} = C_{s,t}e^{-cq_t} = X_t e^{-\log\beta+u_0\chi^{-1}+d_0+u_\mu\chi^{-1}\mu_t+(u_z\chi^{-1}+d_z)z_t}.$$

The dynamics of the market asset price obtains by applying Itô's Lemma to P_t:

$$dP_t = [\cdot]dt+\partial_x P_t\sigma_x dB_{x,t}+\partial_\mu P_t\sigma_\mu dB_{\mu,t}+\partial_z P_t\sigma_z dB_{z,t}.$$

Therefore the return volatility and premium are given by

$$\begin{aligned}
\sigma_P(t) &= P_t^{-1}\sqrt{(\partial_x P_t\sigma_x)^2+(\partial_\mu P_t\sigma_\mu)^2+(\partial_z P_t\sigma_z)^2} = \sqrt{\sigma_x^2+(u_\mu\chi^{-1}\sigma_\mu)^2+((u_z\chi^{-1}+d_z)\sigma_z)^2},\\
(\mu_P-r)(t) &= -\tfrac{1}{dt}\left\langle \tfrac{d\xi_{0,t}}{\xi_{0,t}},\tfrac{dP_t}{P_t}\right\rangle = \theta_x(t)\sigma_x+\theta_\mu(t)u_\mu\chi^{-1}\sigma_\mu+\theta_z(t)\left(u_z\chi^{-1}+d_z\right)\sigma_z.
\end{aligned}$$

q.e.d.

Proof of Proposition 6: Using the definition of the dividend strip price $P_{t,\tau}=\mathbb{E}_t[\xi_{t,t+\tau}D_{t+\tau}]$ and market asset price $P_t=\int_0^\infty P_{t,\tau}d\tau$, we can write the instantaneous premium of the market return as follows:

$$\begin{aligned}
(\mu_P-r)(t) &= \theta_x(t)\tfrac{\partial_x P_t}{P_t}\sigma_x+\theta_\mu(t)\tfrac{\partial_\mu P_t}{P_t}\sigma_\mu+\theta_z(t)\tfrac{\partial_z P_t}{P_t}\sigma_z\\
&= \theta_x(t)\tfrac{\int_0^\infty \partial_x P_{t,\tau}d\tau}{P_t}\sigma_x+\theta_\mu(t)\tfrac{\int_0^\infty \partial_\mu P_{t,\tau}d\tau}{P_t}\sigma_\mu+\theta_z(t)\tfrac{\int_0^\infty \partial_z P_{t,\tau}d\tau}{P_t}\sigma_z\\
&= \int_0^\infty P_t^{-1}P_{t,\tau}\left(\theta_x(t)\sigma_x+\theta_\mu(t)A_\mu(\tau)\sigma_\mu+\theta_z(t)A_z(\tau)\sigma_z\right)d\tau\\
&= \int_0^\infty \Pi(t,\tau)d\tau,
\end{aligned}$$

and $\Pi(t,\tau)$ easily follows using the results of Proposition 4 and 5. *q.e.d.*

Proof of Proposition 7: The equilibrium price of the zero-coupon bond with maturity τ of Eq. (49) obtains as a

special case of $\mathcal{M}_{t,\tau}(\bar{c})$ with $\bar{c}=(0,1,0,0,0)$. Then, it is given by $B_{t,\tau}=\xi_{0,t}^{-1}\mathcal{M}_{t,\tau}(\bar{c})=e^{K_0(\tau)+K_\mu(\tau)\mu_t+K_z(\tau)z_t}$ with

$$\begin{aligned}
K_0(\tau)=\ell_0(\tau,\bar{c})=\tfrac{e^{-2\tau(\lambda_z+\lambda_\mu)}}{4\lambda_z^3\lambda_\mu^3}\Big(&-e^{2\tau\lambda_\mu}r_z^2\lambda_\mu^3\sigma_z^2+4e^{\tau(\lambda_z+2\lambda_\mu)}r_z\lambda_\mu^3\left(-\bar{z}\lambda_z^2+\sigma_z\left(\theta_z\lambda_z+r_z\sigma_z\right)\right)-e^{2\tau\lambda_z}r_\mu^2\lambda_z^3\sigma_\mu^2\\
&-e^{2\tau(\lambda_z+\lambda_\mu)}\left(\lambda_\mu^2\left(4\bar{\mu}r_\mu\lambda_z^3\left(-1+\tau\lambda_\mu\right)+\lambda_\mu\left(4\tau r_0\lambda_z^3+4\bar{z}r_z\lambda_z^2\left(-1+\tau\lambda_z\right)\right.\right.\right.\\
&\left.\left.+r_z\sigma_z\left(-4\theta_z\lambda_z\left(-1+\tau\lambda_z\right)+r_z\left(3-2\tau\lambda_z\right)\sigma_z\right)\right)\right)-4r_\mu\theta_\mu\lambda_z^3\lambda_\mu\left(-1+\tau\lambda_\mu\right)\sigma_\mu\\
&\left.+r_\mu^2\lambda_z^3\left(3-2\tau\lambda_\mu\right)\sigma_\mu^2\right)+4e^{\tau(2\lambda_z+\lambda_\mu)}r_\mu\lambda_z^3\left(-\bar{\mu}\lambda_\mu^2+\sigma_\mu\left(\theta_\mu\lambda_\mu+r_\mu\sigma_\mu\right)\right)\Big),\\
K_\mu(\tau)=\ell_\mu(\tau,\bar{c})=&\tfrac{\left(e^{-\tau\lambda_\mu}-1\right)r_\mu}{\lambda_\mu},\\
K_z(\tau)=\ell_z(\tau,\bar{c})=&\tfrac{\left(e^{-\tau\lambda_z}-1\right)r_z}{\lambda_z},
\end{aligned}$$

and $K_0(0)=0, K_\mu(0)=0$ and $K_z(0)=0$. And the real yield in Eq. (50) easily follows. Given $p(t,\tau)$ and $\rho(t,\tau)$ in Eq. (52) and (53), it is easy to verify that the premium on the equity yield is state-independent:

$$\begin{aligned}
\rho(t,\tau)&=\tfrac{1}{\tau}\left(K_0(\tau)+B_0(1,\tau)-A_0(\tau)\right)+\partial_\mu\left(p(t,\tau)-\varepsilon(t,\tau)+g_D(t,\tau)\right)\mu_t+\partial_z\left(p(t,\tau)-\varepsilon(t,\tau)+g_D(t,\tau)\right)z_t\\
&=\tfrac{1}{\tau}\left(K_0(\tau)+B_0(1,\tau)-A_0(\tau)\right),
\end{aligned}$$

q.e.d.

Proof of Proposition 8: Using the definition of the share process in Eq. (54) and of the price of a dividend claim on a firm with residual life T in Eq. (55), we get

$$P_t^T=\int_0^T s(T-\tau)\mathbb{E}_t[\xi_{t,t+\tau}D_{t+\tau}]d\tau,$$

since $D_{t+\tau}^{T-\tau}=s(T-\tau)D_{t+\tau}$ and $s(T)$ is deterministic. Therefore, we can use the results of Proposition 4 to obtain:

$$P_t^T=\int_0^T s(T-\tau)P_{t,\tau}d\tau\approx X_t\int_0^T s(T-\tau)e^{A_0(\tau)+A_\mu(\tau)\mu_t+A_z(\tau)z_t}d\tau.$$

By setting $H_0(\tau,T)=\log s(T-\tau)+A_0(\tau)$, $H_\mu(\tau)=A_\mu(\tau)$ and $H_z(\tau)=A_z(\tau)$, the final result automatically follows. The dynamics of the single firm price obtains by applying Itô's Lemma to P_t^T:

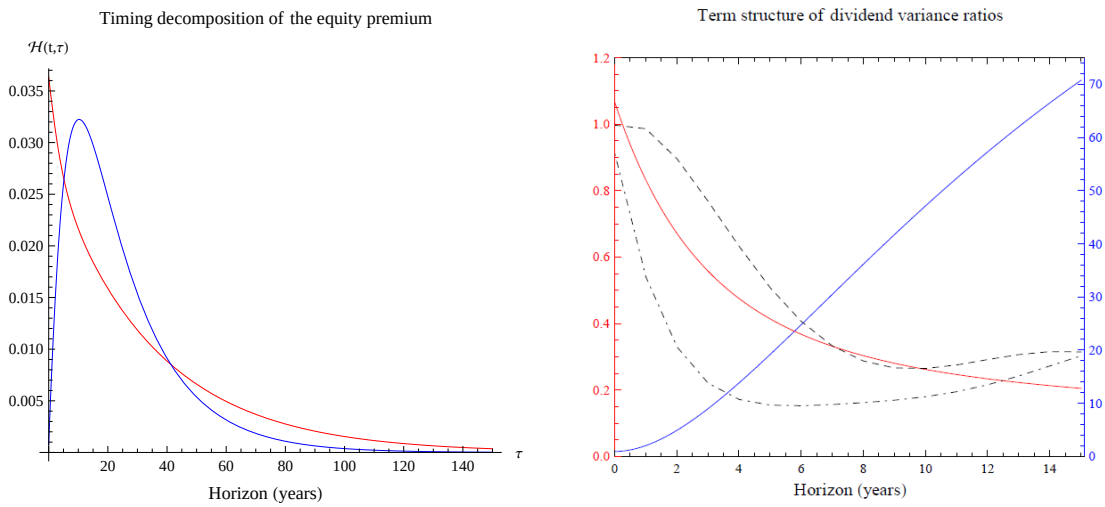
$$dP_t^T=[\cdot]dt+\partial_x P_t^T\sigma_x dB_{x,t}+\partial_\mu P_t^T\sigma_\mu dB_{\mu,t}+\partial_z P_t^T\sigma_z dB_{z,t}.$$

Therefore the return volatility and premium are given by

$$\begin{aligned}
\sigma_P^T(t)&=\left(P_t^T\right)^{-1}\sqrt{(\partial_x P_t^T\sigma_x)^2+(\partial_\mu P_t^T\sigma_\mu)^2+(\partial_z P_t^T\sigma_z)^2},\\
(\mu_P^T-r)(t)&=-\tfrac{1}{dt}\left\langle\tfrac{d\xi_{0,t}}{\xi_{0,t}},\tfrac{dP_t^T}{P_t^T}\right\rangle=\theta_x(t)\sigma_x+\theta_\mu(t)\tfrac{\partial_\mu P_t^T}{P_t^T}\sigma_\mu+\theta_z(t)\tfrac{\partial_z P_t^T}{P_t^T}\sigma_z.
\end{aligned}$$

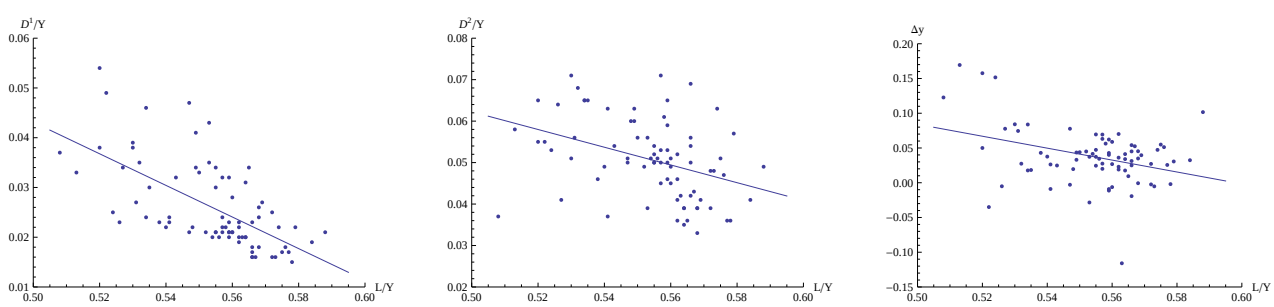
q.e.d.

Figure 1. Equity premium and the timing of macroeconomic risk



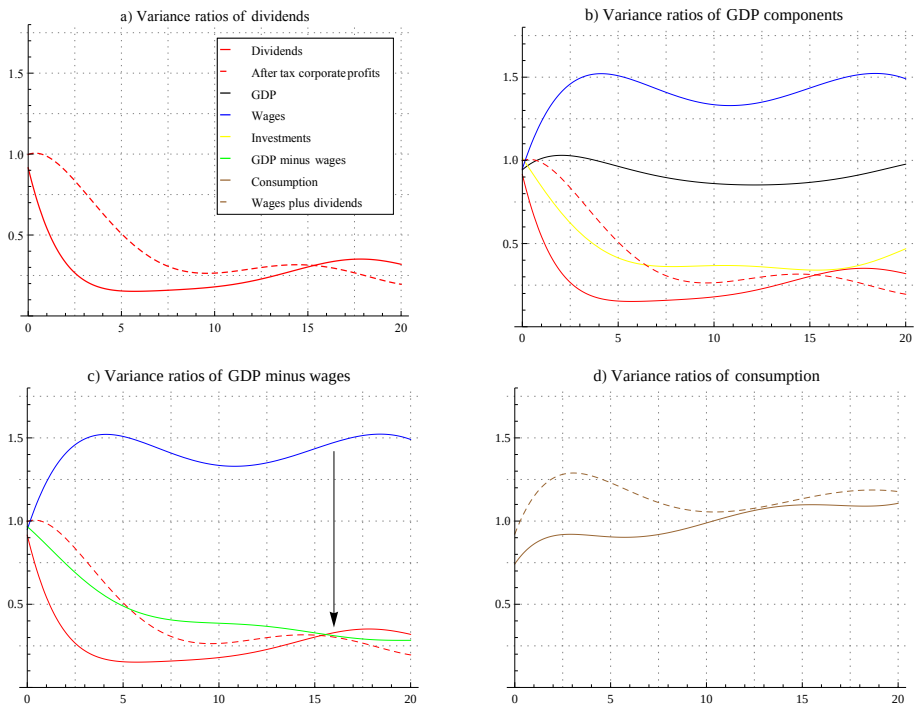
Left panel: Equilibrium density $\mathcal{H}(t,\tau)$ as a function of the horizon τ. Right panel: Variance ratios of the growth rates of dividends $VR_D(t,\tau)$ as a function of the horizon τ, standardized with one-quarter variance. The red line denotes the baseline model calibration of Table V. The blue line denotes the sub-case $Y_t = X_t$ (with $\bar{\mu} = 2\%, \sigma_x = 1\%, \lambda_\mu = .15, \sigma_\mu = 2\%, \beta = 2.5\%, \gamma_s = 10, \psi = 2$), which is similar to Hore (2015) and reproduces in continuous-time the main features of the standard long-run risk model of Bansal and Yaron (2004). In the right panel, the black lines (left axis) denote the polynomial fits of the variance ratios of the growth rates of net dividends (dot-dashed), after tax corporate profits (dashed). Data are quarterly on 1947-2012 from the NIPA tables. The variance ratio procedure uses the theoretical exposition of Campbell, Lo and MacKinlay (1997, pp. 48-55), which accounts for heteroscedasticity and overlapping observations. → Back to the text.

Figure 2. GDP shares of wages and dividends



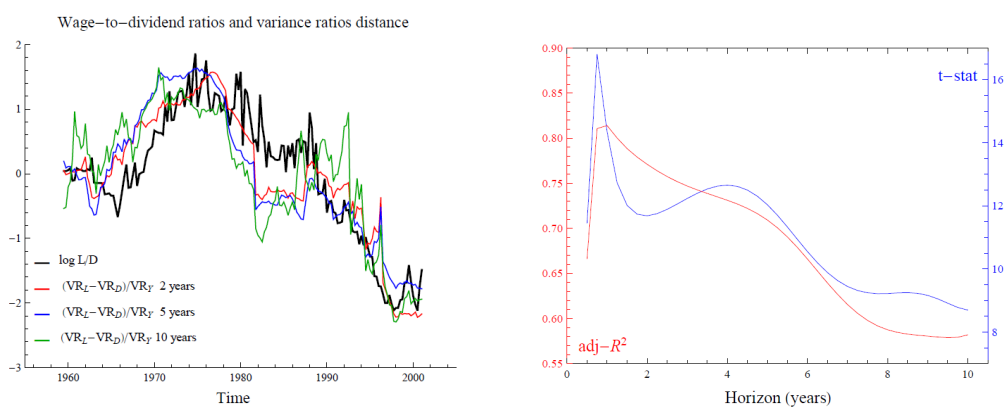
Scatter plots of the labor-share (L/Y) respectively with the dividend-share (D^1/Y), the profit-share (D^2/Y) and the GDP growth rates (Δy). Data are yearly on 1935-2012. → Back to the text.

Figure 3. The term-structure of variance ratios of dividends and other GDP components



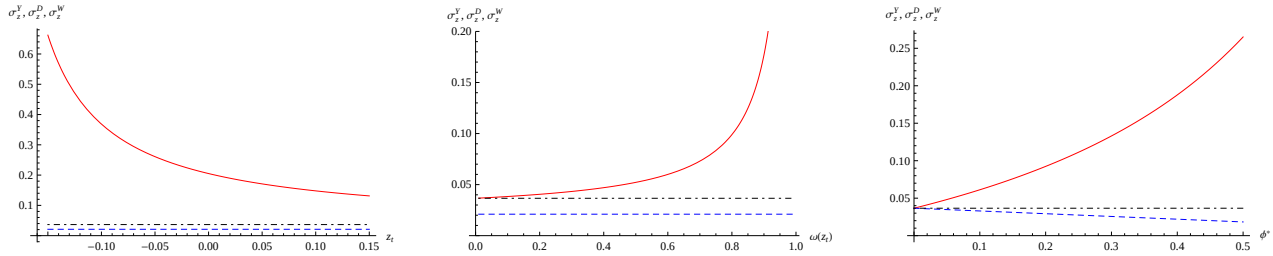
Upper panels a) and b): Polynomial fits of the variance ratios of the growth rates of net dividends (red), after tax corporate profits (dashed red), GDP (black), wages (blue) and investments (yellow) as a function of the horizon (in years). Lower panels c) and d): Polynomial fits of the variance ratios of the growth rates of net dividends (red), after tax corporate profits (dashed red), wages (blue), GDP minus wages (green), consumption (brown) and wages plus net dividends (dashed brown) as a function of the horizon (in years). Data are quarterly on 1947-2012 from the NIPA tables. The variance ratio procedure uses the theoretical exposition of Campbell, Lo and MacKinlay (1997, pp. 48-55), which accounts for heteroscedasticity and overlapping observations. → Back to the text.

Figure 4. The term-structure effect of income isurance



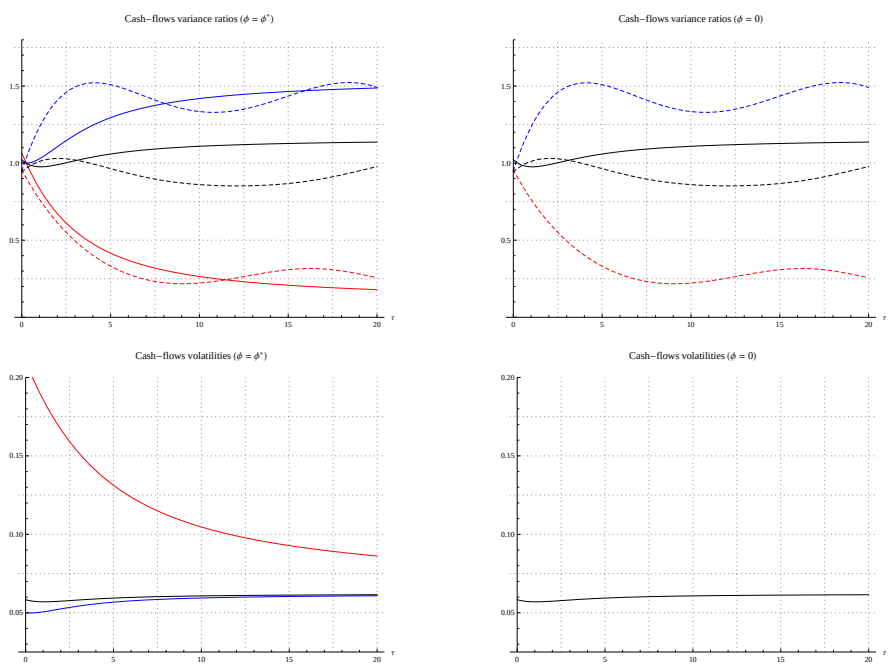
Left panel: Standardized time-series of log wage-to-dividend ratios (black) and variance ratios distance $\Delta_{t,\tau} = (VR_L - VR_D)/VR_Y$ with 2 (red), 5 (blue) and 10 (green) years horizon τ. Right panel: adjusted-R^2 (left axis, red) and Newey-West corrected t-statistics (right axis, blue) from the regressions of $\Delta_{t,\tau}$ on the log wage-to-dividend ratio as a function of the horizon τ (Table III, panel A). → Back to the text.

Figure 5. Instantaneous volatilities of cash flows



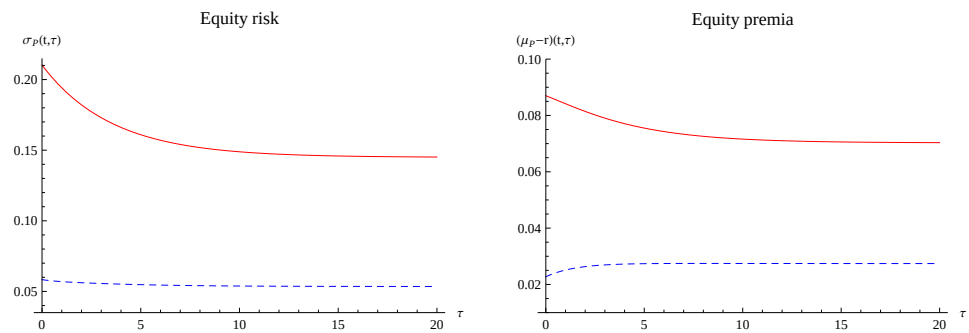
Instantaneous volatilities of cash-flows as a function of the transitory shock (z_t), the labor-share ($\omega(z_t)$) and the degree of income insurance are shown respectively in the upper, middle and lower panel. Black dot-dashed, red solid and blue dashed lines denote respectively total consumption, dividends and wages volatilities. All unreported parameters are from Table V. → Back to the text.

Figure 6. Term structures of cash flows



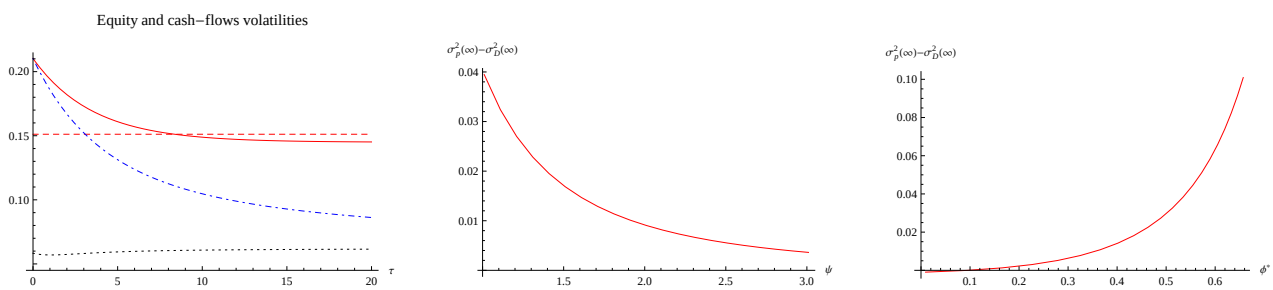
The upper panels plot the model variance ratios of dividends (red), wages (blue) and total resources (black) as a function of the horizon in the cases of $\phi = \phi^*$ (left) and $\phi = 0$ (right) at the steady state ($\mu_t = \bar{\mu}, z_t = \bar{z}$). Dashed lines denote empirical data as in Figure 3 (dividends moments are computed as the average of those of net dividends and after tax corporate profits). The lower panels plot the growth rates volatilities of dividends (red), wages (blue) and total resources (black) as a function of the horizon in the cases of $\phi = \phi^*$ (left) and $\phi = 0$ (right) at the steady state ($\mu_t = \bar{\mu}, z_t = \bar{z}$). All unreported parameters are from Table V. → Back to the text.

Figure 7. Term structures of premia and volatility on dividend strips



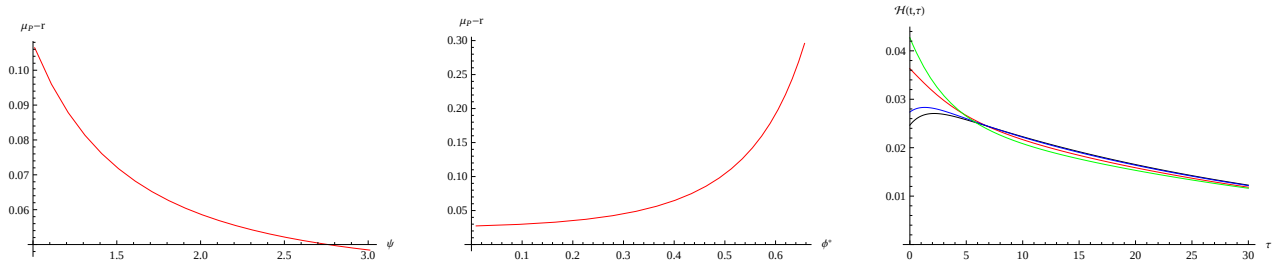
The left and right panels plot the term structures of respectively the volatility and the premia of the dividend strips returns at the steady state ($\mu_t = \bar{\mu}, z_t = \bar{z}$). The cases $\phi = \phi^*$ and $\phi = 0$ are denoted respectively by red solid and blue dashed lines. All unreported parameters are from Table V. → Back to the text.

Figure 8. Term structures of cash flows and equity risk and excess volatility



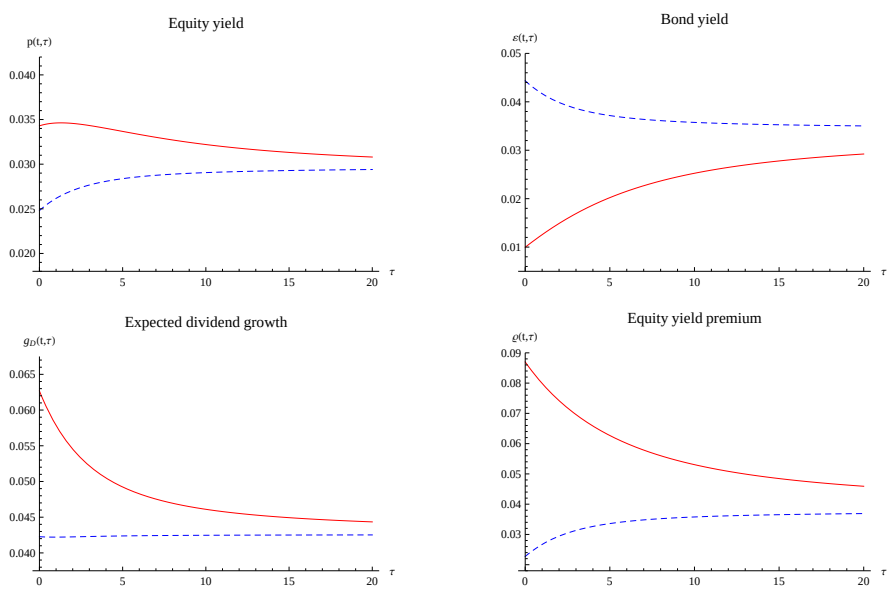
The left panel plots the term structures of volatility of cash flows and dividend strips returns at the steady state ($\mu_t = \bar{\mu}, z_t = \bar{z}$). The volatility of dividend strips' returns, dividends and total resources are denoted with red solid, blue dot-dashed and black dotted lines. The red dashed line denotes the market asset volatility. The middle and right panels plot the long-run excess volatility of dividend strips returns over dividends, $\sigma_P^2(t,\infty) - \sigma_D^2(t,\infty)$, as a function of ψ and ϕ^*. All parameters are from Table V. → Back to the text.

Figure 9. Equity premium, preferences and income insurance



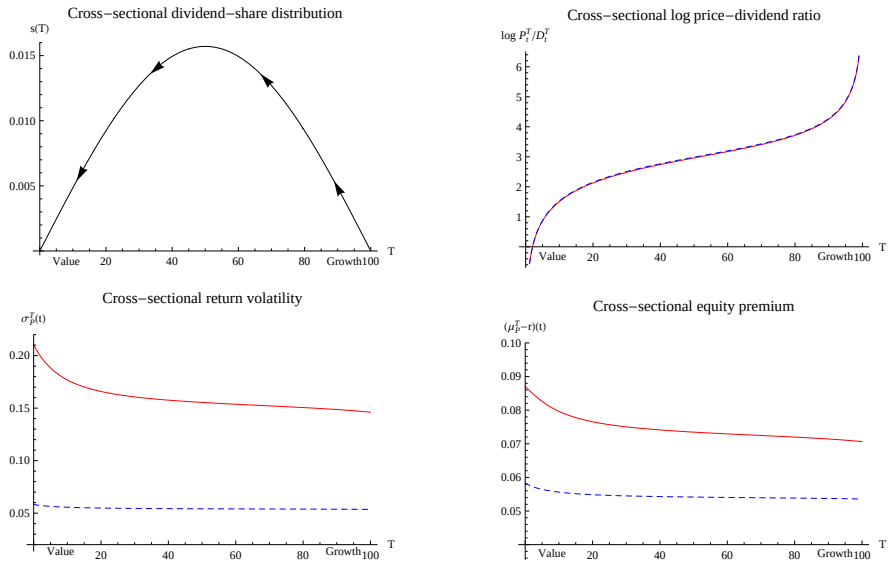
The premium on the market asset, $(\mu_P - r)(t)$, is plotted as a function of ψ (left panel) and ϕ^* (middle panel). The right panel plots the equilibrium density $\mathcal{H}(t,\tau)$ as a function of the horizon τ (i.e. the relative contribution of any horizon τ to the whole equity premium $(\mu_P - r)(t)$). The red line denotes the baseline calibration of Table V with $\phi = \phi^*$. The black, blue and green lines denote respectively the cases $\phi = 0$, $\phi = .15$ and $\phi = .60$. All unreported parameters are from Table V.
→ Back to the text.

Figure 10. Term structures of bond and equity yields



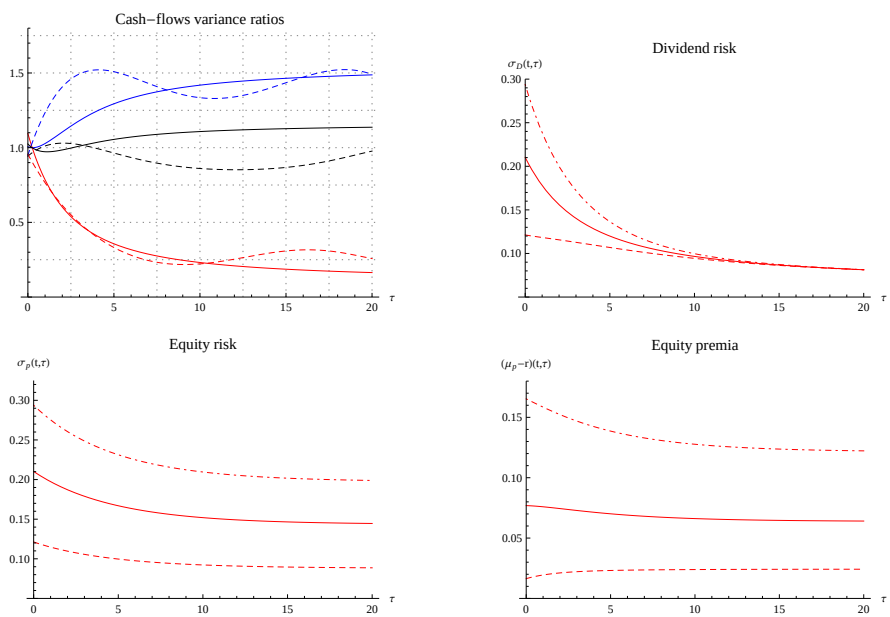
The term structures of equity yields (upper left panel), real bond yields (upper right panel), dividend growth rates (lower left panel) and premia on equity yields (lower right panel) are plotted as a function of the horizon at the steady state $(\mu_t = \bar{\mu}, z_t = \bar{z})$. The cases $\phi = \phi^*$ and $\phi = 0$ are denoted with red solid and blue dashed lines. All unreported parameters are from Table V.
→ Back to the text.

Figure 11. Cross-sectional returns, value premium and income insurance



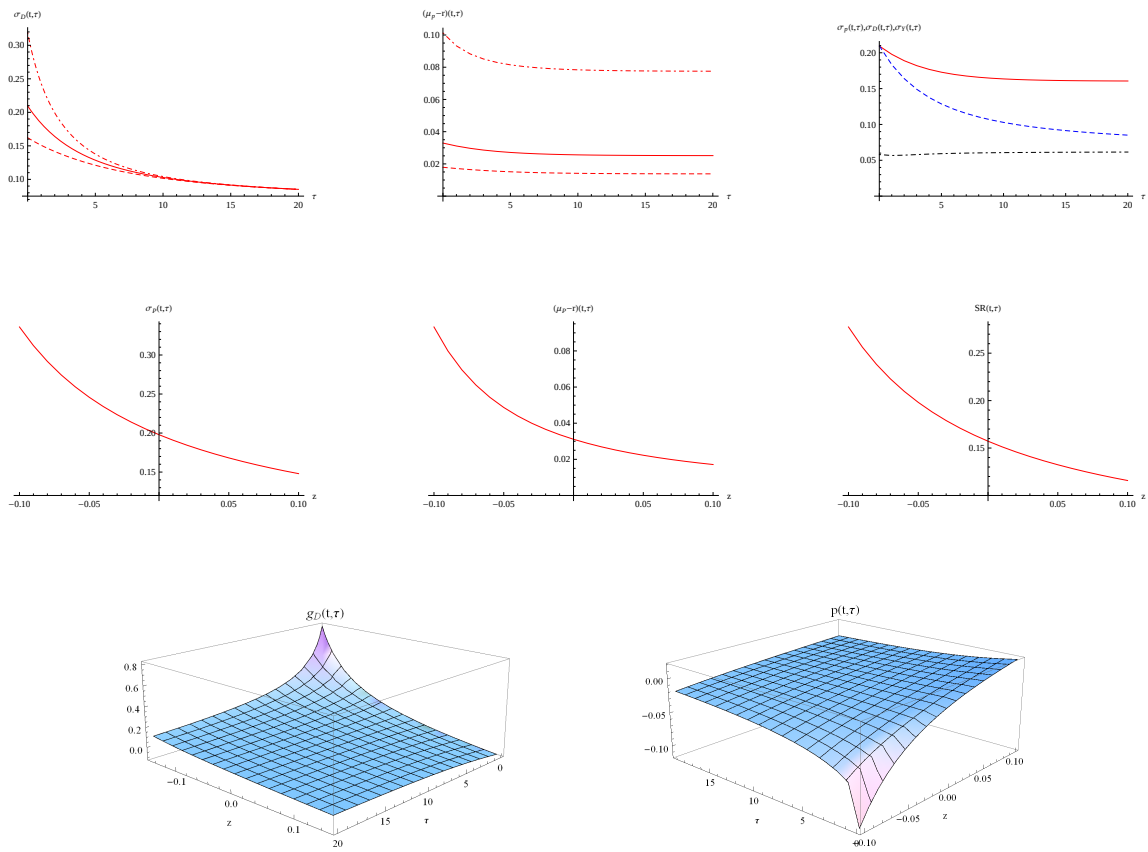
Firm share process (upper left panel), log price-dividend ratios (upper right panel), return volatility (lower left panel) and instantaneous premium (lower right panel) of single firm claims are plotted as a function of the firm residual life (T) at the steady state ($\mu_t = \bar{\mu}, z_t = \bar{z}$). The cases $\phi = \phi^*$ and $\phi = 0$ are denoted with red solid and blue dashed lines. $T^{\max}$ is 100 years. All unreported parameters are from Table V. → Back to the text.

Figure 12. Cash-flows and equity term-structures with countercyclical heteroscedasticity



The upper left panel shows the model variance ratios of dividends (red), wages (blue) and total resources (black) as a function of the horizon τ at the steady state ($\mu_t = \bar{\mu}, z_t = 0$). Dashed lines denote empirical data as in Figure 3 (dividends moments are computed as the average of those of net dividends and after tax corporate profits). The volatility of dividends (upper right panel), the volatility of the dividend strip returns (lower left panel), and the premium of the dividend strip return (lower right panel) are plotted as a function of the horizon τ for z at the steady-state (solid line) and its 5-95% probability interval (dot-dashed and dashed lines). All parameters are from Table V apart $\bar{z} = 0.1$. → Back to the text.

Figure 13. Conditional return moments under power utility



Upper panels: the left plot shows the dividend growth rates volatility as a function of the horizon τ for z at the steady-state (solid line) and its 5-95% probability interval (dot-dashed and dashed lines); the central plot shows the premium of the dividend strip as a function of the horizon τ for z at the steady-state (solid line) and its 5-95% probability interval (dot-dashed and dashed lines); the right plot shows the volatility of the dividend strip, the dividend growth rates volatility and the total consumtpion growth rates volatility respectively with red solid, blue dashed and black dot-dashed lines. Middle panels: the left, central and right plots show respectively the volatility, the premium and the Sharpe ratio on the dividend strip as a function of z, given a maturity of $\tau = 1$ year. Lower panels: the left plot and the right plot show respectively the dividend expected growth rate and the equity yield as a function of the maturity τ and of the transitory shock z. All parameters are from Table V apart $\gamma_s = 0.75$.

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Table I
GDP Shares and Growth Rates

Panel A reports yearly mean, standard deviation, first order autocorrelation and cross-correlations (significance levels in parenthesis) about the labor-share (L/Y), the investment-share (I/Y), the dividend-share (D^1/Y) and the profit-share (D^2/Y) as well as their log differences. Panel B reports the Johansen's trace statistics and the associated 5% and 1% critical values for the number of cointegrating equations in a VECM model for cointegration among the logarithm of GDP, labor, investment, dividends and profits. Panel C reports the estimates with Newey-West corrected t-statistics from univariate regressions of first differences in labor-share, investment-share, dividend-share and profit-share on their lagged levels. Data are yearly on the sample 1932:2012 from NIPA tables.

Panel A – Summary Statistics

	Shares					Growth Rates				
	L/Y	I/Y	D^1/Y	D^2/Y		Δl	Δi	Δd^1	Δd^2	Δy
mean	0.554	0.157	0.026	0.051		0.038	0.051	0.038	0.052	0.038
std	0.017	0.036	0.009	0.010		0.047	0.210	0.145	0.177	0.042
ac(1)	0.907	0.830	0.844	0.686		0.492	0.187	-0.258	0.346	0.477
correlations										
I/Y	0.377				Δi	-0.188				
	(0.001)					(0.099)				
D^1/Y	-0.632	-0.124			Δd^1	0.172	0.315			
	(0.000)	(0.277)				(0.131)	(0.005)			
D^2/Y	-0.383	-0.022	0.238		Δd^2	0.233	0.224	0.287		
	(0.001)	(0.845)	(0.036)			(0.040)	(0.049)	(0.011)		
Δy	-0.256	-0.215	0.103	0.299	Δy	0.960	-0.040	0.213	0.396	
	(0.023)	(0.058)	(0.368)	(0.008)		(0.000)	(0.727)	(0.061)	(0.000)	

Panel B – Cointegration

Johansen Test for Cointegration for l, i, d^1, d^2 and y

Rank	0	1	2	3	4	5
Trace	115.83	73.19	37.72	17.58	0.35	–
5% c.v.	59.46	39.89	24.31	12.53	3.84	–
1% c.v.	66.52	45.58	29.75	16.31	6.51	–

Panel C – Stationarity of GDP Shares

$\Delta L/Y$		$\Delta I/Y$		$\Delta D^1/Y$		$\Delta D^2/Y$	
const	0.056	const	0.033	const	0.004	const	0.017
t	(2.04)	t	(2.22)	t	(2.67)	t	(4.10)
L/Y_{t-1}	-0.101	I/Y_{t-1}	-0.209	D^1/Y_{t-1}	-0.156	D^2/Y_{t-1}	-0.327
t	(-2.05)	t	(-2.39)	t	(-2.74)	t	(-3.98)
rmse	0.007	rmse	0.020	rmse	0.005	rmse	0.007
R^2	0.055	R^2	0.134	R^2	0.078	R^2	0.173

→ Back to the text.

Table II
Determinants of the Dividend-Share: Granger Causality

The table reports the estimates of the model

$$D^i/Y_t = b_0 + b_{01}D^i/Y_{t-1} + b_{01}D^i/Y_{t-2} + b_{11}Lev_{t-1} + b_{12}Lev_{t-2} + b_{21}I/Y_{t-1} + b_{22}I/Y_{t-2} + b_{31}L/Y_{t-1} + b_{32}L/Y_{t-2} + \varepsilon_t, \qquad i = \{1,2\},$$

where the dependent variable is either the dividend-share (D^1/Y) or the profit-share (D^2/Y) at time t; the independent variables are the first and second lag of the aggregate leverage ratio (Lev), the investment-share (I/Y) and the labor-share (L/Y). The t-statistics are reported in parenthesis. The symbols *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels. A set of Granger causality tests reports the F statistics and p-values of the hypothesis that all coefficients on the lags of each independent variable are jointly zero. The number of lags (2) is selected according to the final prediction error (FPE), Akaike's information criterion (AIC) and the Hannan and Quinn information criterion (HQIC). Data are yearly on the sample 1932:2012 from NIPA tables and 1945:2012 from Flow of Funds.

Panel A

D^1/Y_t		coeff		Granger test F	prob
		(t)			
D^1/Y	$t-1$	0.525***		43.77	0.000
		(4.20)			
	$t-2$	0.288**			
		(2.32)			
Lev	$t-1$	-0.019		1.16	0.322
		(−1.49)			
	$t-2$	0.014			
		(1.11)			
I/Y	$t-1$	0.003		1.26	0.293
		(0.08)			
	$t-2$	0.044			
		(1.20)			
L/Y	$t-1$	-0.239**		3.43	0.039
		(−2.47)			
	$t-2$	0.161*			
		(1.77)			
N		66			
R^2		0.780	All:	25.28	0.000
adj-R^2		0.749			

Panel B

D^2/Y_t		coeff		Granger test F	prob
		(t)			
D^2/Y	$t-1$	0.758***		26.44	0.000
		(5.61)			
	$t-2$	-0.132			
		(−0.85)			
Lev	$t-1$	-0.132		0.95	0.392
		(1.11)			
	$t-2$	-0.025			
		(−1.38)			
I/Y	$t-1$	-0.221***		8.90	0.000
		(−3.93)			
	$t-2$	0.068			
		(1.37)			
L/Y	$t-1$	-0.411**		2.96	0.060
		(−2.37)			
	$t-2$	0.351**			
		(2.02)			
N		66			
R^2		0.731	All:	19.38	0.000
adj-R^2		0.693			

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Table III
The Term-Structure Effect of Income Insurance

The table reports the estimates of the model

$$\Delta_{t,\tau} = \frac{VR_L - VR_D}{VR_Y}\Big|_{t,\tau} = b_0 + b_1' x_t + \varepsilon_t, \qquad \forall\, \tau \in \{0.5, 1, 2, 3, 5, 7, 10\},$$

where the dependent variable is the distance of wage and dividends variance ratio relative to the GDP variance ratio at time t and with horizon τ; the independent variables are the logarithm of the wage to dividend ratio (L/D) in panel A, the labor-share (L/Y) in panel B and the labor-share (L/Y), the investment-share (I/Y) and the aggregate financial leverage ratio (Lev) in panel C. Data are quarterly on 1947-2012 from the NIPA tables. Variance ratios at time t are computed with a rolling window of 100 quarterly growth rates centered at t. The variance ratio procedure uses the theoretical exposition of Campbell, Lo and MacKinlay (1997, pp. 48-55), which accounts for heteroscedasticity and overlapping observations. The Newey-West corrected t-statistics are reported in parenthesis. Economic significance denotes standardized coefficients. The symbols *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Panel A	Horizon τ (years)						
	0.5	1	2	3	5	7	10
L/D	0.53***	0.74***	1.05***	1.27***	1.47***	1.39***	1.39***
t-stat	(11.45)	(14.35)	(11.68)	(12.23)	(12.04)	(9.44)	(8.70)
economic significance	0.82	0.90	0.88	0.87	0.84	0.79	0.76
adj-R^2	0.67	0.81	0.77	0.75	0.71	0.62	0.58

Panel B	Horizon τ (years)						
	0.5	1	2	3	5	7	10
L/Y	12.04***	14.36***	20.43***	26.13***	32.20***	32.13***	32.61***
t-stat	(5.65)	(6.05)	(6.99)	(7.92)	(8.07)	(6.86)	(6.50)
economic significance	0.64	0.60	0.59	0.61	0.63	0.62	0.62
adj-R^2	0.41	0.36	0.34	0.37	0.40	0.38	0.37

Panel C	Horizon τ (years)						
	0.5	1	2	3	5	7	10
Lev	-0.03	0.65**	0.78*	0.61	0.29	-0.17	0.55
t-stat	(-0.16)	(2.31)	(1.81)	(1.14)	(0.46)	(-0.26)	(0.88)
economic significance	-0.02	0.35	0.29	0.18	0.07	-0.04	0.13
I/Y	-0.83	-1.92	-3.06	-2.89	-1.57	-2.48	-7.13*
t-stat	(-0.63)	(-1.01)	(-1.02)	(-0.77)	(-0.36)	(-0.58)	(-1.74)
economic significance	-0.09	-0.16	-0.18	-0.14	-0.06	-0.10	-0.27
L/Y	11.76***	13.61***	19.28***	25.05***	31.62***	31.29***	30.07***
t-stat	(5.44)	(6.85)	(7.53)	(8.03)	(7.86)	(6.62)	(6.53)
economic significance	0.63	0.57	0.56	0.59	0.62	0.61	0.57
adj-R^2	0.41	0.45	0.40	0.39	0.40	0.39	0.43

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Table IV
Premia on dividend strips

Panel A – Sign of compensations

	transient risk	non-transient long-run risk	non-transient short-run risk
a) $Y_t = X_t, \mu_t = \bar{\mu}$	> 0	–	–
b) $Y_t = X_t$	> 0	> 0 if $\begin{cases} \text{ERU \& ISE} \\ \text{LRU \& WE} \end{cases}$	–
c) $Y_t = Z_t$	> 0	–	< 0 if ERU
d) $Y_t = X_t Z_t, \mu_t = \bar{\mu}$	> 0	–	< 0 if ERU
e) $Y_t = X_t Z_t$	> 0	> 0 if $\begin{cases} \text{ERU \& ISE} \\ \text{LRU \& WE} \end{cases}$	< 0 if ERU

Panel B – Sign of compensations' slopes

	transient risk	non-transient long-run risk	non-transient short-run risk
a) $Y_t = X_t, \mu_t = \bar{\mu}$	–	–	–
b) $Y_t = X_t$	–	> 0 if $\begin{cases} \text{ERU \& ISE} \\ \text{LRU \& WE} \end{cases}$	–
c) $Y_t = Z_t$	< 0 if ISE	–	< 0 if ISE
d) $Y_t = X_t Z_t, \mu_t = \bar{\mu}$	< 0 if ISE	–	< 0 if ISE
e) $Y_t = X_t Z_t$	< 0 if ISE	> 0 if $\begin{cases} \text{ERU \& ISE} \\ \text{LRU \& WE} \end{cases}$	< 0 if ISE

Panel A reports the sign of the components of the premia on dividend strip's returns associated respectively to transient risk, non-transient long-run risk and non-transient short-run risk as defined in Section V.A. Panel B reports the sign of the components of the slope of term-structure of the premia on dividend strip's returns associated respectively to transient risk, non-transient long-run risk and non-transient short-run risk as defined in Section V.A. ISE and WE stay respectively for intertemporal substitution effect and wealth effect. ERU and LRU stay respectively for preference for the early and the late resolution of uncertainty.

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Table V
Model parameters – Baseline calibration

Cash-flows		
volatility of log permanent shock	σ_x	.045
steady-state of long-run growth	$\bar{\mu}$	.042
speed of reversion of long-run growth	λ_μ	.644
volatility of long-run growth	σ_μ	.027
steady-state of log transitory shock	$\bar{z}$	.000
speed of reversion of log transitory shock	λ_z	.279
volatility of log transitory shock	σ_z	.037

Labor relations		
labor-share Walrasian benchmark	α	.857
preference heterogeneity	h	1.784

Preferences		
time-discount rate	β	.037
shareholders' relative risk aversion	γ_s	10
elasticity of intertemporal substitution	ψ	1.5

Implied parameters		
workers' relative risk aversion	γ_w	17.84
bargaining power	ϑ	.914
optimal degree of income insurance	ϕ^*	.425
steady-state labor share	$\omega(\bar{z})$	.915
dividends' leverage coefficient	d_z	5.61

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Table VI
Cash-flows moments

		Data	Model	
			$\phi = 0$	$\phi = \phi^*$
consumption growth†	$g_Y(t,1)$	.038	.042	.042
consumption volatility†	$\sigma_Y(t,1)$	.042	.057	.057
2 years variance ratio of consumption	$VR_Y(t,2)$	.985	.989	.989
15 years variance ratio of consumption	$VR_Y(t,15)$	1.00	1.13	1.13
dividends growth	$g_D(t,1)$	.045	.042	.057
dividends volatility†	$\sigma_D(t,1)$	.161	.057	.185
2 years variance ratio of dividends†	$VR_D(t,2)$	.854	.989	.672
15 years variance ratio of dividends†	$VR_D(t,15)$	.423	1.13	.208
wages growth	$g_W(t,1)$	.038	.042	.042
wages volatility†	$\sigma_W(t,1)$	.047	.057	.050
2 years variance ratio of wages†	$VR_W(t,2)$	1.11	.989	1.11
15 years variance ratio of wages†	$VR_W(t,15)$	1.52	1.13	1.46
unconditional dividend-share	D/Y	.065	.143	.085
dividend-share volatility	$\sigma_{D/Y}(t,1)$	.012	.000	.019
dividend-share autocorrelation	$AC_{D/Y}(t,1)$	.775	.000	.721
log consumption and dividend-share correlation	$\rho_{d\log Y,D/Y}$	.280	.000	.194
log consumption and labor-share correlation	$\rho_{d\log Y,W/Y}$	-.280	.000	-.194
2 years variance ratios distance	$\Delta_{t,2}$	.260	.000	.443
15 years variance ratios distance	$\Delta_{t,15}$	1.097	.000	1.108

	$\phi = .00$	$\phi = .15$	$\phi = \phi^*$	$\phi = .60$
$\sigma_D(t,1)$	.057	.081	.185	.337
$\sigma_W(t,1)$	.057	.054	.050	.049
$VR_D(t,2)$	.989	.799	.672	.653
$VR_W(t,2)$	.989	1.03	1.11	1.15
$\sigma_{D/Y}(t,1)$	.000	.006	.019	.028
$\rho_{d\log Y,D/Y}$	.000	.218	.194	.156

Steady-state values of model cash-flows moments are compared with the data (see Section II and Table I). The variance ratios distance is computed as $\Delta_{t,\tau} = (VR_W(t,\tau) - VR_D(t,\tau))/VR_Y(t,\tau)$. The symbol † denotes the eight moment conditions used in the calibration of model parameters in Table V. The cases of ϕ equal to 0, .15 and .60 obtain by setting h equal to respectively 1, 1.19 and 2.58. All unreported parameters are from Table V.

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Table VII
Term-structures of cash-flows and equity returns

Panel A – Volatilities

	$\phi = \phi^*$				$\phi = 0$			
τ	1	10	50	∞	1	10	50	∞
$\sigma_Y(t,\tau)$	.057	.061	.062	.062	.057	.061	.062	.062
$\sigma_D(t,\tau)$	.185	.105	.073	.062	.057	.061	.062	.062
$\sigma_W(t,\tau)$	.050	.059	.061	.062	.057	.061	.062	.062

Panel B – Spreads

	$\phi = \phi^*$				$\phi = 0$			
τ	1	10	50	∞	1	10	50	∞
$\sigma_Y(t,\tau) - \sigma_Y(t,0)$	-.001	.002	.003	.004	-.001	.002	.003	.004
$\sigma_D(t,\tau) - \sigma_D(t,0)$	-.024	-.106	-.137	-.148	-.001	.002	.003	.004
$\sigma_W(t,\tau) - \sigma_W(t,0)$	.001	.009	.011	.012	-.001	.002	.003	.004

Panel C – Equity risk, premia and excess-volatility

	$\phi = \phi^*$				$\phi = 0$			
τ	1	10	50	∞	1	10	50	∞
$\sigma_P(t,\tau)$	.194	.149	.145	.145	.057	.054	.054	.054
$(\mu_P - r)(t,\tau)$	.084	.072	.070	.070	.025	.027	.027	.027
$\sigma_P(t,\tau) - \sigma_D(t,\tau)$	.008	.044	.072	.082	-.000	-.007	-.008	-.009
$\sigma_P(t,\tau) - \sigma_Y(t,\tau)$	.137	.088	.083	.083	-.000	-.007	-.008	-.009

Panel D – Spreads on bond and equity yields

	$\phi = \phi^*$				$\phi = 0$			
τ	1	10	50	∞	1	10	50	∞
$\varepsilon(t,\tau) - \varepsilon(t,0)$	.003	.015	.022	.024	-.003	-.009	-.010	-.010
$p(t,\tau) - p(t,0)$	.000	.002	-.004	-.005	.001	.004	.005	.005
$\rho(t,\tau) - \rho(t,0)$	-.007	-.034	-.046	-.049	.004	.013	.015	.015

Model statistics are computed at the steady-state ($\mu_t = \bar{\mu}$, $z_t = \bar{z}$). The case $\phi = 0$ obtains by setting $h = 1$. All unreported parameters are from Table V.

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Table VIII
Steady-state moments of the market asset returns

Data

Sample	r	σ_r	$\mu_P - r$	σ_P	SR	$\log P/D$	$\sigma_{\log P/D}$
1931-2009	.006	.030	.062	.198	.313	3.38	.450
1947-2009	.010	.027	.063	.176	.358	3.47	.429

Model – Baseline calibration

γ_s	ψ	β	ϕ	r	σ_r	$\mu_P - r$	σ_P	SR	$\log P/D$	$\sigma_{\log P/D}$	(a)	(b)
10	1.50	.037	ϕ^*	.010	.042	.072	.151	.477	3.50	.347	+	+

Model – Alternative preferences settings

γ_s	ψ	β	ϕ	r	σ_r	$\mu_P - r$	σ_P	SR	$\log P/D$	$\sigma_{\log P/D}$	(a)	(b)
10	1.05	.120	ϕ^*	.010	.061	.170	.204	.837	2.12	.038	+	+
			0	.135	.028	.026	.058	.450	2.13	.002	−	−
	1.25	.075	ϕ^*	.010	.051	.118	.179	.661	2.64	.183	+	+
			0	.086	.023	.026	.055	.475	2.66	.010	−	−
	1.50	.037	ϕ^*	.010	.042	.072	.151	.477	3.50	.347	+	+
			0	.044	.020	.027	.054	.505	3.52	.018	−	−
	1.75	.025	ϕ^*	.010	.036	.054	.131	.411	4.19	.468	+	+
			0	.028	.017	.028	.053	.533	4.21	.023	−	−
7.5	1.05	.025	ϕ^*	.010	.061	.075	.201	.372	3.74	.051	+	+
			0	.048	.028	.018	.057	.311	3.74	.003	−	−
	1.25	.006	ϕ^*	.010	.051	.045	.171	.265	7.85	.230	+	+
			0	.024	.023	.019	.055	.343	7.85	.011	−	−
	1.50	.010	ϕ^*	.013	.042	.039	.145	.267	7.34	.383	+	+
			0	.022	.020	.020	.054	.377	7.34	.019	−	−
	1.75	.012	ϕ^*	.014	.036	.035	.127	.273	9.41	.493	+	+
			0	.021	.017	.021	.053	.400	9.41	.024	−	−
5	1.05	.002	ϕ^*	.012	.061	.049	.201	.245	7.74	.055	+	+
			0	.031	.028	.012	.057	.207	7.74	.003	−	−
	1.25	.007	ϕ^*	.016	.051	.039	.171	.228	8.66	.230	+	+
			0	.030	.023	.013	.055	.232	8.66	.011	−	−
	1.50	.011	ϕ^*	.019	.042	.032	.145	.219	10.16	.384	+	+
			0	.029	.020	.014	.054	.253	10.16	.019	−	−
	1.75	.014	ϕ^*	.021	.036	.028	.127	.218	7.81	.493	+	+
			0	.028	.017	.014	.053	.266	7.81	.024	−	−

Steady-state values for the risk-free rate (r) and its volatility (σ_r), the excess market return ($\mu_P - r$) and volatility (σ_P), the market Sharpe ratio (SR), the log price-dividend ratio ($\log P/D$) and its volatility ($\sigma_{\log P/D}$) are compared with empirical moments from Constantinides and Ghosh (2011). (a) and (b) denote the sign of respectively the market asset excess-volatility over dividend and consumption at 10 year horizon: $\sigma_P - \sigma_D(t,10)$ and $\sigma_P - \sigma_Y(t,10)$. For each pair (γ_s, ψ), β is set to match (if possible) a 1% risk-free rate. The case $\phi = 0$ obtains by setting $h = 1$. All unreported parameters are from Table V.

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Online Appendix of
"Income Insurance and the Equilibrium Term-Structure of Equity"

Roberto Marfè

Online Appendix A. Empirical Support: Aggregate Economy

Table OA.I
Determinants of the Dividend-Share

The table reports the estimates of the model

$$D^i/Y_{t+j} = b_0 + b_1 Lev_t + b_2 I/Y_t + b_3 L/Y_t + \varepsilon_t, \qquad i = \{1,2\},\ j = \{0,1\},$$

where the dependent variable is either the dividend-share (D^1/Y) or the profit-share (D^2/Y) at time t or $t+1$; the independent variables are the aggregate leverage ratio (Lev), the investment-share (I/Y) and the labor-share (L/Y). The t-statistics are reported in parenthesis. The symbols *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels. Data are yearly on the sample 1932:2012 from NIPA tables and 1945:2012 from Flow of Funds.

Panel A – Net Dividends

	D^1/Y_t	D^1/Y_{t+1}
	coeff	coeff
	(t)	(t)
Lev_t	-0.018*	-0.015
	(-1.79)	(-1.54)
R^2	0.046	0.035
I/Y_t	-0.030	-0.031
	(-1.09)	(-1.16)
R^2	0.015	0.018
L/Y_t	-0.318***	-0.307***
	(-7.11)	(-6.67)
R^2	0.400	0.372
Lev_t	-0.022**	-0.018*
	(-2.39)	(-2.00)
I/Y_t	0.074**	0.063*
	(2.13)	(1.79)
L/Y_t	-0.283***	-0.297***
	(-5.05)	(-5.09)
R^2	0.342	0.335

Panel B – Corporate Profits

	D^2/Y_t	D^2/Y_{t+1}
	coeff	coeff
	(t)	(t)
Lev_t	-0.048***	-0.0397***
	(-4.09)	(-3.18)
R^2	0.202	0.135
I/Y_t	-0.006	-0.041
	(-0.20)	(-1.38)
R^2	0.001	0.025
L/Y_t	-0.214***	-0.222***
	(-3.61)	(-3.74)
R^2	0.147	0.158
Lev_t	-0.052***	-0.032***
	(-4.77)	(-2.66)
I/Y_t	0.078*	-0.033
	(1.85)	(-0.71)
L/Y_t	-0.353***	-0.333***
	(-5.23)	(-4.33)
R^2	0.455	0.339

Table OA.II
The Term-Structure Effect of Income Insurance – Robustness Checks: Sub-Samples

The table reports the estimates of the model

$$\Delta_{t,\tau} = \frac{VR_L - VR_D}{VR_Y}\Big|_{t,\tau} = b_0 + b_1 x_t + \varepsilon_t, \qquad \forall\, \tau \in \{0.5, 1, 2, 3, 5, 7, 10\},$$

where the dependent variable is the distance of wage and dividends variance ratio relative to the GDP variance ratio at time t and with horizon τ; the independent variables are the logarithm of the wage to dividend ratio (L/D) in panel A, and the labor-share (L/Y) in panel B. Data are quarterly on the sub-samples 1947-1980 and 1980-2012 from the NIPA tables. Variance ratios at time t are computed with a rolling window of 100 quarterly growth rates centered at t. The variance ratio procedure uses the theoretical exposition of Campbell, Lo and MacKinlay (1997, pp. 48-55), which accounts for heteroscedasticity and overlapping observations. The Newey-West corrected t-statistics are reported in parenthesis. Economic significance denotes standardized coefficients. The symbols *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Panel A1 – 1947-1980	Horizon τ (years)						
	0.5	1	2	3	5	7	10
L/D	0.23**	0.60***	0.90***	1.12***	1.45***	1.22***	0.75***
t-stat	(2.48)	(7.76)	(7.94)	(6.97)	(5.88)	(4.26)	(3.69)
economic significance	0.44	0.84	0.85	0.82	0.78	0.67	0.56
adj-R^2	0.18	0.70	0.72	0.67	0.60	0.44	0.30

Panel A2 – 1980-2012	Horizon τ (years)						
	0.5	1	2	3	5	7	10
L/D	0.43***	0.68***	0.84***	0.90***	0.88***	0.71***	1.09***
t-stat	(10.41)	(9.54)	(6.72)	(6.59)	(8.06)	(7.28)	(5.26)
economic significance	0.86	0.86	0.81	0.81	0.84	0.78	0.66
adj-R^2	0.74	0.74	0.65	0.65	0.70	0.61	0.43

Panel B1 – 1947-1980	Horizon τ (years)						
	0.5	1	2	3	5	7	10
L/Y	4.80**	6.91***	10.16***	14.34***	20.06***	17.67***	12.15***
t-stat	(2.51)	(3.33)	(3.28)	(3.58)	(3.61)	(3.15)	(2.87)
economic significance	0.48	0.51	0.51	0.56	0.57	0.51	0.48
adj-R^2	0.22	0.25	0.25	0.30	0.32	0.25	0.22

Panel B2 – 1980-2012	Horizon τ (years)						
	0.5	1	2	3	5	7	10
L/Y	11.04***	13.99***	15.37***	15.55***	15.12***	14.93***	32.87***
t-stat	(3.93)	(3.28)	(2.79)	(2.64)	(2.75)	(3.43)	(4.20)
economic significance	0.59	0.47	0.40	0.38	0.39	0.44	0.53
adj-R^2	0.34	0.22	0.15	0.13	0.14	0.18	0.28

Table OA.III
The Term-Structure Effect of Income Insurance – Robustness Checks: Time Trend

The table reports the estimates of the model

$$\text{step 1:} \qquad \Delta_{t,\tau} = \frac{VR_L - VR_D}{VR_Y}\Big|_{t,\tau} = \quad b_0 + b_1 t + \widetilde{\Delta}_{t,\tau}, \qquad \forall\, \tau \in \{0.5, 1, 2, 3, 5, 7, 10\},$$
$$x_t = \quad b_0 + b_1 t + \widetilde{x}_t,$$

$$\text{step 2:} \qquad \widetilde{\Delta}_{t,\tau} = \quad b_0 + b_1 \widetilde{x}_t + \varepsilon_t, \qquad \forall\, \tau \in \{0.5, 1, 2, 3, 5, 7, 10\},$$

where, in the second step, the dependent variable is the de-trended distance of wage and dividends variance ratio relative to the GDP variance ratio at time t and with horizon τ; the independent variables are the de-trended logarithm of the wage to dividend ratio (L/D) in panel A, and the de-trended labor-share (L/Y) in panel B. Data are quarterly on 1947-2012 from the NIPA tables. Variance ratios at time t are computed with a rolling window of 100 quarterly growth rates centered at t. The variance ratio procedure uses the theoretical exposition of Campbell, Lo and MacKinlay (1997, pp. 48-55), which accounts for heteroscedasticity and overlapping observations. The Newey-West corrected t-statistics are reported in parenthesis. Economic significance denotes standardized coefficients. The symbols *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Panel A	Horizon τ (years)						
	0.5	1	2	3	5	7	10
L/D	0.37***	0.67***	0.90***	1.04***	1.19***	1.00***	1.00***
t-stat	(6.92)	(11.99)	(9.59)	(8.40)	(7.00)	(5.22)	(6.55)
economic significance	0.71	0.85	0.81	0.79	0.75	0.65	0.62
adj-R^2	0.50	0.72	0.65	0.62	0.56	0.42	0.38

Panel B	Horizon τ (years)						
	0.5	1	2	3	5	7	10
L/Y	8.25***	10.64***	14.56***	18.52***	23.49***	22.56***	23.07***
t-stat	(5.22)	(5.15)	(5.46)	(5.84)	(5.80)	(5.51)	(6.16)
economic significance	0.64	0.54	0.52	0.56	0.59	0.58	0.57
adj-R^2	0.40	0.28	0.27	0.31	0.34	0.34	0.32

Table OA.IV
Long-Horizon Predictability of Dividends

The table reports the estimates of the model

$$g_D^i(\tau) = \frac{1}{\tau}\left(d_{t+\tau}^i - d_t^i\right) = b_0 + b_1 Lev_t + b_2 I/Y_t + b_3 L/Y_t + \varepsilon_{t+\tau}, \qquad i = \{1,2\}, \quad \tau = \{1,5,10,15,20\},$$

where the dependent variable is the yearly growth rate of either net dividends ($d^1 = \log D^1$) or profits ($d^2 = \log D^2$) computed over the future horizon of $\tau = \{1,5,10,15,20\}$ years; the independent variables are the current level of the aggregate leverage ratio (Lev), the investment-share (I/Y) and the labor-share (L/Y). The Newey-West corrected t-statistics are reported in parenthesis. The symbols *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels. Data are yearly on the sample 1932:2012 from NIPA tables and 1945:2012 from Flow of Funds.

		Panel A - Net Dividends					Panel B - Corporate Profits				
		Horizon					Horizon				
		1	5	10	15	20	1	5	10	15	20
Lev	Coeff	0.079	0.164***	0.167***	0.160***	0.120***	0.141	-0.064	0.012	0.046	0.054**
	t	(0.88)	(3.13)	(5.81)	(16.65)	(8.17)	(0.63)	(-0.65)	(0.22)	(1.15)	(2.25)
	R^2	0.003	0.186	0.526	0.736	0.604	0.007	0.014	0.002	0.059	0.156
	adj-R^2	-0.012	0.172	0.518	0.730	0.595	-0.008	-0.003	-0.016	0.040	0.138
I/Y	Coeff	-0.183	0.179*	0.281***	0.284***	0.251**	-0.906	-0.371	-0.121	-0.057	-0.107*
	t	(-0.49)	(1.67)	(2.89)	(3.33)	(2.63)	(-0.94)	(-1.29)	(-0.80)	(-0.55)	(-1.95)
	$R^2$2	0.002	0.032	0.183	0.278	0.304	0.038	0.040	0.030	0.013	0.083
	adj-R^2	-0.011	0.018	0.171	0.267	0.291	0.025	0.026	0.015	-0.003	0.067
L/Y	Coeff	-0.456	0.743*	0.600**	0.578***	0.574***	-1.384	-0.167	-0.241	-0.317**	-0.267**
	t	(-0.48)	(1.87)	(2.51)	(3.39)	(4.41)	(-1.37)	(-0.28)	(-1.17)	(-2.19)	(-2.06)
	R^2	0.003	0.115	0.179	0.261	0.365	0.020	0.003	0.025	0.091	0.119
	adj-R^2	-0.010	0.103	0.166	0.249	0.353	0.006	-0.011	0.011	0.076	0.103
Lev	Coeff	0.122	0.202***	0.175***	0.163***	0.113***	0.333	-0.036	0.055	0.090**	0.108***
	t	(1.01)	(3.38)	(5.45)	(12.60)	(6.40)	(1.52)	(-0.30)	(1.07)	(2.12)	(6.48)
I/Y	Coeff	-0.312	-0.391**	-0.082	-0.043	0.035	-1.849	-0.359	-0.380**	-0.314*	-0.375***
	t	(-0.58)	(-2.34)	(-0.96)	(-0.87)	(0.50)	(-1.10)	(-1.07)	(-2.47)	(-1.98)	(-5.58)
L/Y	Coeff	-0.853	0.282	0.144	0.331***	0.407***	0.053	0.762	-0.012	-0.277	-0.212
	t	(-0.98)	(0.58)	(0.52)	(4.50)	(4.65)	(0.04)	(1.29)	(-0.05)	(-1.61)	(-1.63)
	R^2	0.013	0.253	0.538	0.789	0.720	0.075	0.075	0.111	0.213	0.501
	adj-R^2	-0.034	0.215	0.512	0.776	0.700	0.030	0.027	0.061	0.165	0.467

Table OA.V
Long-Horizon Predictability of Excess Returns

The table reports the estimates of the model

$$\sum_{i=1}^{\tau} r_{t+\tau}^{ex} = b_0 + b_1 Lev_t + b_2 I/Y_t + b_3 L/Y_t + \varepsilon_{t+\tau}, \qquad \tau = \{1,5,10,15,20\},$$

where the dependent variable is the cumulative excess returns (r^{ex}) computed over the future horizon of $\tau = \{1,5,10,15,20\}$ years; the independent variables are the current level of the aggregate leverage ratio (Lev), the investment-share (I/Y) and the labor-share (L/Y). The Newey-West corrected t-statistics are reported in parenthesis. The symbols *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels. Data are yearly on the sample 1932:2012 from NIPA tables and 1945:2012 from Flow of Funds.

Cumulative Excess Returns

		Horizon				
		1	5	10	15	20
Lev	Coeff	0.378**	0.243**	0.189***	0.095**	0.015
	t	(2.33)	(2.29)	(3.69)	(2.31)	(0.40)
	R^2	0.048	0.107	0.267	0.208	0.010
	adj-R^2	0.034	0.093	0.254	0.193	-0.012
I/Y	Coeff	-0.649	-0.372*	-0.401***	-0.193**	-0.111
	t	(-1.37)	(-1.74)	(-3.70)	(-2.03)	(-1.33)
	R^2	0.021	0.040	0.166	0.093	0.065
	adj-R^2	0.008	0.026	0.154	0.078	0.049
L/Y	Coeff	-1.405	-1.146	-1.070***	-0.573***	-0.157
	t	(-1.43)	(-1.31)	(-6.13)	(-3.20)	(-0.82)
	R^2	0.021	0.080	0.254	0.186	0.030
	adj-R^2	0.008	0.067	0.243	0.172	0.013
Lev	Coeff	0.549***	0.447***	0.273***	0.102**	0.003
	t	(2.98)	(5.99)	(5.89)	(2.10)	(0.09)
I/Y	Coeff	-1.292	-1.599***	-0.657***	-0.043	0.065
	t	(-1.01)	(-3.14)	(-4.98)	(-0.29)	(0.42)
L/Y	Coeff	-2.971***	-2.601***	-1.211***	-0.136	0.442**
	t	(-2.69)	(-4.34)	(-4.02)	(-0.56)	(2.48)
	R^2	0.146	0.539	0.608	0.217	0.161
	adj-R^2	0.105	0.515	0.587	0.169	0.104

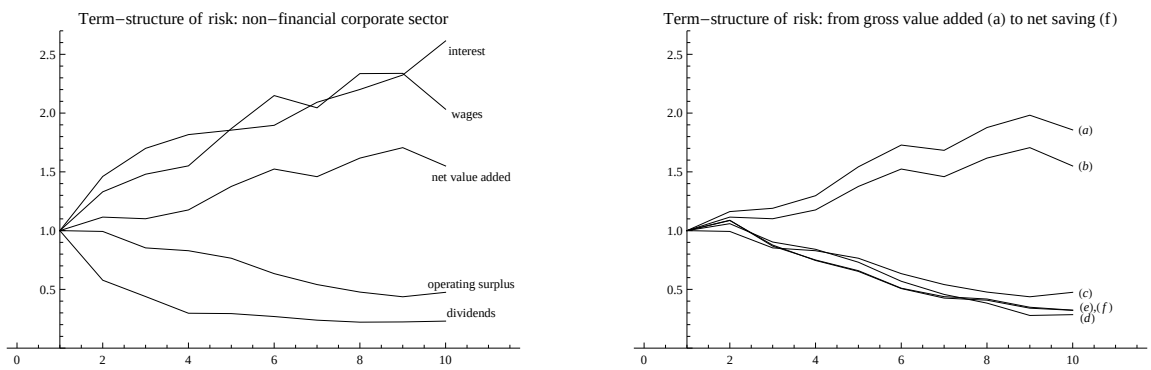
Online Appendix B. Empirical Support: Non-Financial Corporate Sector

This Appendix provides further support to the empirical Section II and focuses on the non-financial corporate sector. Data are from the Flow of Funds, Integrated macroeconomic accounts, table S.5.a from 1945 to 2013 at yearly frequency. The main quantities of interest are reported in the following table.

			% share
(a)		Gross value added	100
	(a.1)	- Capital depreciation	(11.6)
(b)		= Net value added	88.4
	(b.1)	- Compensations to employees	(62.9)
	(b.2)	- Taxes on production and imports less subsidies	(8.7)
(c)		= Net operating surplus	16.8
	(c.1)	- net interest paid	(2.5)
	(c.2)	- net dividends paid	(4.0)
	(c.3)	- net reinvestment of earnings	(-0.8)
(d)		= Net national income	11.1
	(d.1)	- Current taxes on income, wealth, and other transfers	(5.8)
(e)		= Net disposable income	5.3
	(e.1)	- Capital transfers	(0.0)
(f)		= Net saving	5.3

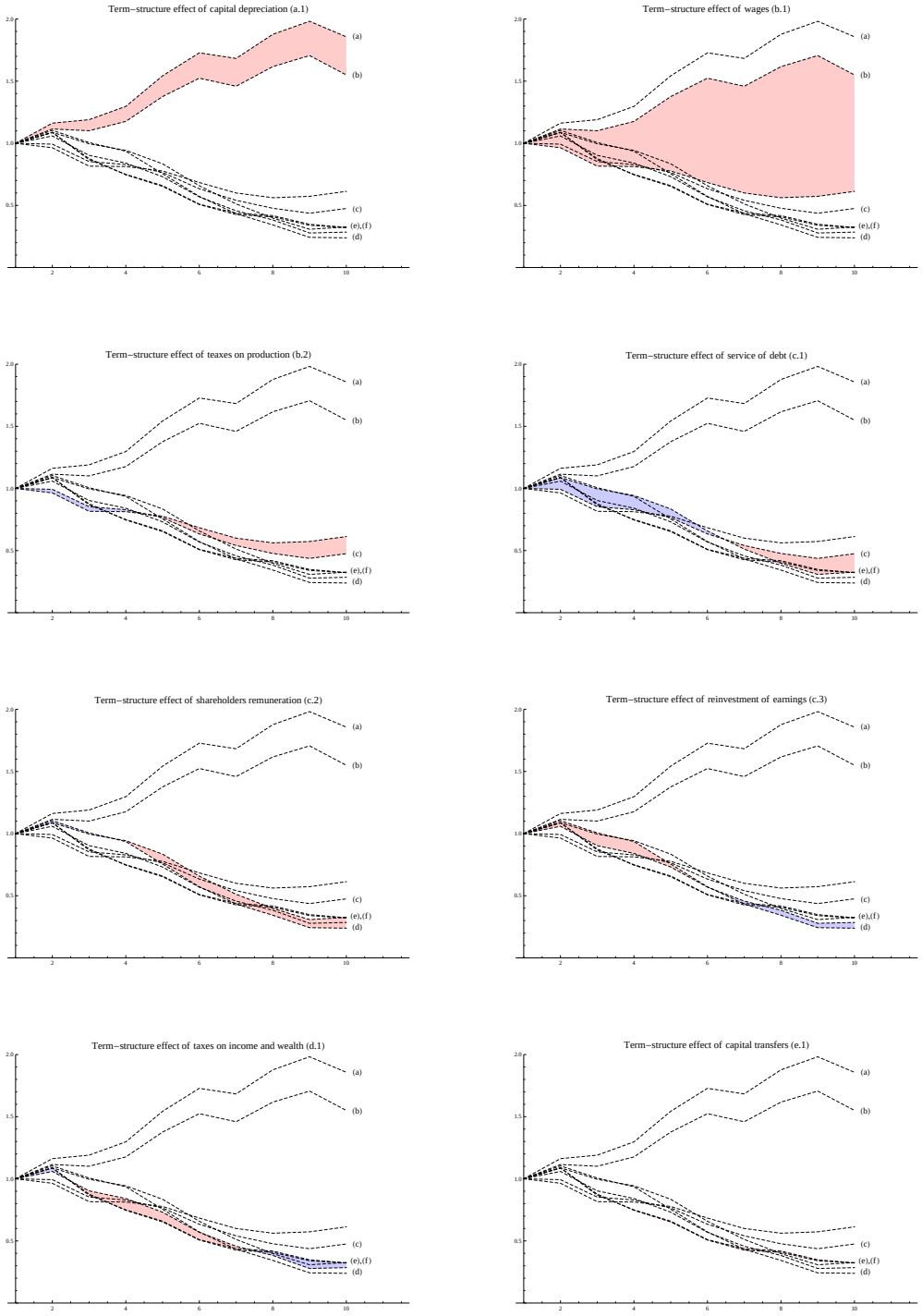
Summary of Integrated macroeconomic accounts, table S.5.a

The term structures of variance ratios (VR) of growth rates of net value added, wages, operating surplus, interest rates and dividends are reported in the left panel of the next figure.



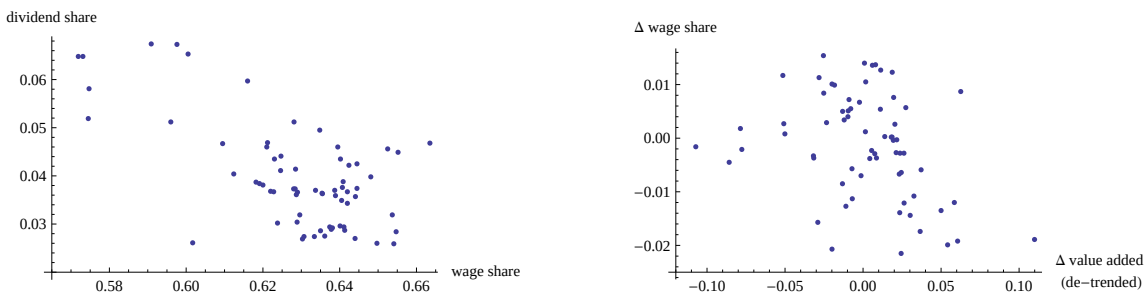
Similarly to the aggregate data (Figure 3), wages and dividends feature VR's which are respectively increasing and decreasing. Interest rates have VR's similar to those of wages. The right panel of the above figure reports the term structures of VR's of the various quantities in the table. We observe that both gross and net value added feature VR's increasing and larger than unity, whereas the operating surplus, as well as the disposable income and the net saving, have VR's strongly decreasing below unity. In particular, the shape of the VR's of dividends and operating surplus is quite similar.

In order to understand whence the change in the slope of the term-structure of risk comes from, the eight panels of the next figure show the term-structure effects due to each variable from the table.



Namely, each shadow area represents the increase (blue) or decrease (red) in the slope at a given horizon. Capital depreciation explains a limited decrease between the slopes of term-structures of gross and net value added. Then, wages do explain the large gap between the slope of net value added and essentially the slope of all other variables from the table. For instance, the term-structure effect due to interest payments is quite negligible and is positive up to about 6 years and then negative. Therefore, even if wages and interest payments have similar upward-sloping VR's, the former has a big impact whereas the latter does not affect the timing of risk.

Given co-integration, we can understand the heterogeneity in the slopes of the VR's as heterogeneity in the exposition of a given variable with the transitory component of value added. Wages and dividends load respectively more and less than value added on such a transitory shock and, hence, feature increasing and decreasing term-structures. This interpretation is consistent with the idea that income insurance within the firm from shareholders to workers is the main determinant of the timing of dividend risk. Such an hypothesis is confirmed by i) the strong negative correlation between the wage and dividend shares of value added, which is about -68.5% (p-value: .000); and ii) the counter-cyclical dynamics of the wage share (its correlation with the de-trended component of value added is about -33.1%, with p-value of .006). Instead, the fraction of value added imputed to interest payments –that is the bondholders' remuneration– is essentially uncorrelated with the dividend-share (about 2.4%, with p-value of .842). The left panel of the next figure shows the scatter plot of the wage- and the dividend-shares; the right panel shows the scatter plot of the (de-trended) increments of value added and the increments of the wage-share.



To provide further support to the interpretation of the data as a result of income insurance, I regress the dividend-share (D/V) on its lag and the lags of the interest-share (I/V) and the wage-share (W/V) (t-statistics are reported in parentheses):

$$D/V_t = \underset{(3.30)}{.12} + \underset{(6.34)}{.61}\ D/V_{t-1} + \underset{(.62)}{.04}\ I/V_{t-1} \underset{(-3.12)}{-.16}\ W/V_{t-1} + \varepsilon_t,$$

$$D/V_t = \underset{(2.33)}{.09} + \underset{(3.32)}{.39}\ D/V_{t-1} + \underset{(3.02)}{.35}\ D/V_{t-2} \underset{(-.21)}{-.05}\ I/V_{t-1} + \underset{(.45)}{.11}\ I/V_{t-2} \underset{(-1.99)}{-.18}\ W/V_{t-1} + \underset{(.50)}{.04}\ W/V_{t-2} + \varepsilon_t.$$

Adjusted-R^2 are respectively 64.1% and 68.8%. By testing for Granger causality, the wage-share Granger causes the dividend-share (p-value: .002 or .031 with 1 or 2 lags) with negative sign, whereas the interest-share does not (p-value: .538 or .544 with 1 or 2 lags).

Similarly to the analysis of Section II, also the non-financial corporate sector data suggest that income insurance is the main determinant of the timing of risk.

Online Appendix C. Results Under Countercyclical Heteroscedasticity

Recall that total resources are characterized by $Y_t = X_t Z_t$ with $\log X_t = x_t$, $\log Z_t = z_t = \bar{z} - \hat{z}_t$ and

$$dx_t = (\mu_t - \sigma_x^2/2)dt + \sigma_x dB_{x,t}, \tag{OA.1}$$

$$d\mu_t = \lambda_\mu(\bar{\mu} - \mu_t)dt + \sigma_\mu dB_{\mu,t}, \tag{OA.2}$$

$$d\hat{z}_t = \lambda_z(\bar{z} - \hat{z}_t)dt + \hat{\sigma}_z\sqrt{\hat{z}_t}dB_{z,t}. \tag{OA.3}$$

Wages and dividends satisfy

$$W_t = X_t \alpha^{1-\phi^*} e^{(1-\phi^*)(\bar{z}-\hat{z}_t)}, \tag{OA.4}$$

$$D_t = X_t \left(e^{\bar{z}-\hat{z}_t} - \alpha^{1-\phi^*} e^{(1-\phi^*)(\bar{z}-\hat{z}_t)} \right). \tag{OA.5}$$

The linearization of the logarithm of the dividend process implies:

$$\log D_t \approx \log X_t + \log \bar{D} + \partial_z \log D\big|_{z=0} z_t = x_t + d_0 + d_z \hat{z}_t, \tag{OA.6}$$

where $\log \bar{D} = d_0 = \log\left(1 - \alpha^{1-\phi^*}\right) - d_z \bar{z}$ with

$$d_z = -\left(1 + \frac{\phi^* \alpha}{\alpha^{\phi^*} - \alpha}\right). \tag{OA.7}$$

Proposition A *The shareholders' utility process under recursive preferences and dynamics in Eq. (OA.1)-(OA.2)-(OA.3)-(OA.6) is given by*

$$J(X_t, \mu_t, \hat{z}_t) = \tfrac{1}{1-\gamma_s} X_t^{1-\gamma_s} \exp\left(u_0 + (1-\gamma_s) d_0 + u_\mu \mu_t + (u_z + (1-\gamma_s) d_z) \hat{z}_t\right), \tag{OA.8}$$

where u_0,

$$u_\mu = \frac{1-\gamma_s}{e^{cq} + \lambda_\mu}, \qquad u_z = \frac{\lambda_z(\gamma_s - 1)}{e^{cq} + \lambda_z} \left(1 + \frac{\phi^* \alpha}{\alpha^{\phi^*} e^{\phi^* \bar{z}} - \alpha}\right),$$

and $cq = \mathbb{E}[cq_t]$ are endogenous constants depending on the primitive parameters. The shareholders' consumption-wealth ratio is equal to

$$cq_t = \log C_{s,t}/Q_{s,t} = \log \beta - \chi^{-1}(u_0 + u_\mu \mu_t + u_z \hat{z}_t). \tag{OA.9}$$

Proposition B *The equilibrium state price density has dynamics given by*

$$d\xi_{0,t} = \xi_{0,t} \tfrac{df_C}{f_C} + \xi_{0,t} f_J dt = -r(t)\xi_{0,t} dt - \theta_x(t)\xi_{0,t} dB_{x,t} - \theta_\mu(t)\xi_{0,t} dB_{\mu,t} - \theta_z(t)\xi_{0,t} dB_{z,t}, \tag{OA.10}$$

where the instantaneous risk-free rate satisfies

$$r(t) = r_0 + r_\mu \mu_t + r_z \hat{z}_t, \tag{OA.11}$$

and the instantaneous prices of risk are given by

$$\theta_x(t) = \gamma_s \sigma_x, \tag{OA.12}$$

$$\theta_\mu(t) = \frac{\gamma_s - 1/\psi}{e^{cq} + \lambda_\mu} \sigma_\mu, \tag{OA.13}$$

$$\theta_z(t) = \sqrt{\hat{z}_t} \left(d_z \gamma_s \hat{\sigma}_z - \tfrac{(\psi\gamma_s - 1)\left(e^{cq} - \Theta + \lambda_z + d_z(\gamma_s - 1)\hat{\sigma}_z^2\right)}{\hat{\sigma}_z \psi(\gamma_s - 1)} \right). \tag{OA.14}$$

Proof of Proposition A and B: Under the infinite horizon, the utility process J satisfies the following Bellman equation:

$$\mathcal{D}J(X, \mu, \hat{z}) + f(C_s, J) = 0, \tag{OA.15}$$

where $\mathcal{D}$ denotes the differential operator. Eq. (OA.15) can be written as

$$0 = J_X \mu X + \tfrac{1}{2} J_{X,X} \sigma_x^2 X^2 + J_\mu \lambda_\mu (\bar{\mu} - \mu) + \tfrac{1}{2} J_{\mu,\mu} \sigma_\mu^2 + J_z \lambda_z (\bar{z} - \hat{z}) + \tfrac{1}{2} J_{z,z} \sigma_z^2 \hat{z} + f(C_s, J).$$

Guess a solution of the form $J(X, \mu, \hat{z}) = \frac{1}{1-\gamma_s} X^{1-\gamma_s} g(\mu, \hat{z})$. The Bellman equation reduces to

$$0 = \mu - \tfrac{1}{2}\gamma_s \sigma_x^2 + \tfrac{g_\mu}{g} \tfrac{\lambda_\mu(\bar{\mu}-\mu)}{1-\gamma_s} + \tfrac{1}{2} \tfrac{g_{\mu,\mu}}{g} \tfrac{\sigma_\mu^2}{1-\gamma_s} + \tfrac{g_z}{g} \tfrac{\lambda_z(\bar{z}-\hat{z})}{1-\gamma_s} + \tfrac{1}{2} \tfrac{g_{z,z}}{g} \tfrac{\hat{\sigma}_z^2 \hat{z}}{1-\gamma_s} + \tfrac{\beta}{1-1/\psi} \left(g^{-1/\chi} e^{(1-1/\psi)(d_0 + d_z \hat{z})} - 1 \right). \tag{OA.16}$$

Under Assumption 2 (limited market participation) and stochastic differential utility, the pricing kernel has dynamics given by

$$d\xi_{0,t} = \xi_{0,t} \tfrac{df_C}{f_C} + \xi_{0,t} f_J dt = -r(t)\xi_{0,t} - \theta_x(t)\xi_{0,t} dB_{x,t} - \theta_\mu(t)\xi_{0,t} dB_{\mu,t} - \theta_z(t)\xi_{0,t} dB_{z,t}, \tag{OA.17}$$

where, by use of Itô's Lemma and Eq. (OA.16), we get

$$\begin{aligned}
r(t) &= -\tfrac{\partial_X f_C}{f_C} \mu X - \tfrac{1}{2} \tfrac{\partial_{X,X} f_C}{f_C} \sigma_x^2 X^2 - \tfrac{\partial_\mu f_C}{f_C} \lambda_\mu (\bar{\mu} - \mu) - \tfrac{1}{2} \tfrac{\partial_{\mu,\mu} f_C}{f_C} \sigma_\mu^2 - \tfrac{\partial_z f_C}{f_C} \lambda_z (\bar{z} - z) - \tfrac{1}{2} \tfrac{\partial_{z,z} f_C}{f_C} \hat{\sigma}_z^2 \hat{z} - f_J, \\
\theta_x(t) &= -\tfrac{\partial_X f_C}{f_C} \sigma_x X, \\
\theta_\mu(t) &= -\tfrac{\partial_\mu f_C}{f_C} \sigma_\mu, \\
\theta_z(t) &= -\tfrac{\partial_z f_C}{f_C} \hat{\sigma}_z \sqrt{\hat{z}},
\end{aligned}$$

An exact solution for $g(\mu, \hat{z})$ satisfying Eq. (OA.16) does not exist for $\psi \neq 1$. Therefore, I look for a solution of $g(\mu, \hat{z})$ around the unconditional mean of the consumption-wealth ratio. Aggregate wealth is given by

$$Q_{s,t} = \mathbb{E}_t \left[\int_t^\infty \xi_{t,u} C_{s,u} du \right],$$

and, applying Fubini's Theorem and taking standard limits, the consumption-wealth ratio satisfies

$$\tfrac{C_{s,t}}{Q_{s,t}} = r(t) - \tfrac{1}{dt} \mathbb{E}_t \left[\tfrac{dQ}{Q} \right] - \tfrac{1}{dt} \mathbb{E}_t \left[\tfrac{d\xi}{\xi} \tfrac{dQ}{Q} \right]. \tag{OA.18}$$

Guess

$$Q_{s,t} = C_{s,t} \beta^{-1} \left(g(\mu_t, \hat{z}_t) e^{(\gamma_s - 1)(d_0 + d_z \hat{z}_t)} \right)^{1/\chi}$$

and apply Itô's Lemma to get $\frac{dQ}{Q}$. Then, plug in the wealth dynamics, the risk-free rate and the pricing kernel into Eq. (OA.18): after tedious calculus you can recognize that the guess solution is correct. Notice that the consumption-wealth ratio approaches to β when $\psi \to 1$ as usual.

Denote $cq = \mathbb{E}[\log C_{s,t} - \log Q_{s,t}]$, hence, a first-order approximation of the consumption-wealth ratio around cq produces

$$\tfrac{C_{s,t}}{Q_{s,t}} = \beta g(\mu_t, \hat{z}_t)^{-1/\chi} e^{(1-1/\psi)(d_0 + d_z \hat{z}_t)} \approx e^{cx} \left(1 - cx + \log \beta - \tfrac{1}{\chi} \left(\log g(\mu_t, \hat{z}_t) + (\gamma_s - 1)(d_0 + d_z \hat{z}_t) \right) \right).$$

Using such approximation in the Bellman equation (OA.16) leads to

$$\begin{aligned}
0 = {} & \mu - \tfrac{1}{2}\gamma_s \sigma_x^2 + \tfrac{g_\mu}{g} \tfrac{\lambda_\mu(\bar{\mu}-\mu)}{1-\gamma_s} + \tfrac{1}{2} \tfrac{g_{\mu,\mu}}{g} \tfrac{\sigma_\mu^2}{1-\gamma_s} + \tfrac{g_z}{g} \tfrac{\lambda_z(\bar{z}-\hat{z})}{1-\gamma_s} + \tfrac{1}{2} \tfrac{g_{z,z}}{g} \tfrac{\hat{\sigma}_z^2 \hat{z}}{1-\gamma_s} \\
& + \tfrac{1}{1-1/\psi} \left(e^{cx} \left(1 - cx + \log \beta - \tfrac{1}{\chi} \log g(\mu, \hat{z}) + (1 - 1/\psi)(d_0 + d_z \hat{z}) \right) - \beta \right),
\end{aligned}$$

which has exponentially affine solution $g(\mu,\hat{z}) = e^{u_0+(1-\gamma_s)d_0+u_\mu\mu+(u_z+(1-\gamma_s)d_z)\hat{z}}$ where

$$u_0 = \tfrac{1}{2}e^{-cq}(1-\gamma_s)\left(\frac{2\beta\psi}{1-\psi} + \frac{2e^{cq}\psi(cq-1-\log\beta)}{1-\psi} - \sigma_x^2\gamma_s + \frac{2\bar{\mu}\lambda_\mu}{e^{cq}+\lambda_\mu} - \frac{2\bar{z}\lambda_z\left(e^{cq}+\lambda_z-\sqrt{e^{2cq}+\lambda_z^2+2e^{cq}\left(\lambda_z+d_z(\gamma_s-1)\hat{\sigma}_z^2\right)}\right)}{(\gamma_s-1)\hat{\sigma}_z^2} + \frac{(1-\gamma_s)\sigma_\mu^2}{\left(e^{cq}+\lambda_\mu\right)^2}\right),$$
$$u_\mu = \frac{1-\gamma_s}{e^{cq}+\lambda_\mu},$$
$$u_z = \frac{e^{cq}+\lambda_z+d_z(\gamma_s-1)\hat{\sigma}_z^2-\sqrt{e^{2cq}+\lambda_z^2+2e^{cq}\left(\lambda_z+d_z(\gamma_s-1)\hat{\sigma}_z^2\right)}}{\hat{\sigma}_z^2},$$

and the endogenous constant cq satisfies $cq = \log\beta - \chi^{-1}(u_0+u_\mu\bar{\mu}+u_z\bar{z})$. The risk-free rate and the prices of risk take the form:

$$r_0 = \frac{2\beta\psi+2d_z\bar{z}\lambda_z-(1+\psi)\gamma_s\sigma_x^2}{2\psi} + \frac{(1-\psi)(\psi\gamma_s-1)\sigma_\mu^2}{2\psi^2(e^{cq}+\lambda_\mu)^2},$$
$$r_\mu = \frac{1}{\psi},$$
$$r_z = \frac{1}{2\psi^2(1-\gamma_s)^2\hat{\sigma}_z^2}\left(2e^{2cq}(\psi-1)(\psi\gamma_s-1)+2(\psi-1)(\psi\gamma_s-1)\lambda_z^2-2(\psi-1)\lambda_z(\Theta(\psi\gamma_s-1)+d_z(\gamma_s-1)\hat{\sigma}_z^2)\right.$$
$$\left.+d_z(\gamma_s-1)\hat{\sigma}_z^2\left(2\Theta(\psi\gamma_s-1)+d_z(\gamma_s-1)\hat{\sigma}_z^2\right)+2e^{cq}(\psi\gamma_s-1)\left(\Theta-\Theta\psi+2(\psi-1)\lambda_z+(\psi-2)d_z(\gamma_s-1)\hat{\sigma}_z^2\right)\right),$$
$$\theta_x(t) = \gamma_s\sigma_x,$$
$$\theta_\mu(t) = \frac{(\gamma_s-1/\psi)\sigma_\mu}{e^{cq}+\lambda_\mu},$$
$$\theta_z(t) = \sqrt{\hat{z}_t}\left(d_z\gamma_s\hat{\sigma}_z - \frac{(\psi\gamma_s-1)\left(e^{cq}-\Theta+\lambda_z+d_z(\gamma_s-1)\hat{\sigma}_z^2\right)}{\hat{\sigma}_z\psi(\gamma_s-1)}\right),$$

where $\Theta = \sqrt{e^{2cq}+\lambda_z^2+2e^{cq}\left(\lambda_z+d_z(\gamma_s-1)\hat{\sigma}_z^2\right)}$. *q.e.d.*

Proposition C *The following conditional expectation has exponential affine solution:*

$$\mathcal{M}_{t,\tau}(\bar{c}) = \mathbb{E}_t[e^{c_0+c_1\log\xi_{0,t+\tau}+c_2x_{t+\tau}+c_3\mu_{t+\tau}+c_4\hat{z}_{t+\tau}}] = \xi_{0,t}^{c_1}X_t^{c_2}e^{\ell_0(\tau,\bar{c})+\ell_\mu(\tau,\bar{c})\mu_t+\ell_z(\tau,\bar{c})\hat{z}_t}, \tag{OA.19}$$

where $\bar{c} = (c_0,c_1,c_2,c_3,c_4)$, model parameters are such that the expectation exists finite and ℓ_0,ℓ_μ and ℓ_z are deterministic functions of time.

Proposition D *The moment generating function of the logarithm of dividends has the following approximation:*

$$\mathbb{D}_t(\tau,n) = \mathbb{E}_t[D_{t+\tau}^n] \approx e^{nx_t+B_0(n,\tau)+B_\mu(n,\tau)\mu_t+B_z(n,\tau)\hat{z}_t}, \tag{OA.20}$$

where n and model parameters are such that the expectation exists finite, the approximation makes use of Eq. (OA.6) and B_0,B_μ and B_z are deterministic functions of time.

Proof of Proposition C: Consider the following conditional expectation:

$$\mathcal{M}_{t,\tau}(\bar{c}) = \mathbb{E}_t[e^{c_0+c_1\log\xi_{0,t+\tau}+c_2\log X_{t+\tau}+c_3\mu_{t+\tau}+c_4\hat{z}_{t+\tau}}] \tag{OA.21}$$

where $\bar{c} = (c_0,c_1,c_2,c_3,c_4)$ is a coefficient vector such that the expectation exists. Guess an exponential affine solution of the kind:

$$\mathcal{M}_{t,\tau}(\bar{c}) = e^{c_1\log\xi_{0,t}+c_2\log X_t+\ell_0(\tau,\bar{c})+\ell_\mu(\tau,\bar{c})\mu_t+\ell_z(\tau,\bar{c})\hat{z}_t}, \tag{OA.22}$$

where $\ell_0(\tau,\bar{c}), \ell_\mu(\tau,\bar{c})$ and $\ell_z(\tau,\bar{c})$ are deterministic functions of time. Feynman-Kac gives that $\mathcal{M}$ has to meet the following partial differential equation

$$\begin{aligned}0 = &\mathcal{M}_t - \mathcal{M}_\xi(r_0 + r_\mu\mu + r_z\hat{z}) + \tfrac{1}{2}\mathcal{M}_{\xi,\xi}(\theta_x(t)^2 + \theta_\mu(t)^2 + \theta_z(t)^2) + \mathcal{M}_X(\mu X) + \tfrac{1}{2}\mathcal{M}_{X,X}\sigma_x^2 X^2 \\ &+ \mathcal{M}_\mu\lambda_\mu(\bar{\mu}-\mu) + \tfrac{1}{2}\mathcal{M}_{\mu,\mu}\sigma_\mu^2 + \mathcal{M}_z\lambda_z(\bar{z}-\hat{z}) + \tfrac{1}{2}\mathcal{M}_{z,z}\hat{\sigma}_z^2\hat{z} - \mathcal{M}_{\xi,X}\theta_x(t)\sigma_x X \\ &- \mathcal{M}_{\xi,\mu}\theta_\mu(t)\sigma_\mu - \mathcal{M}_{\xi,z}\theta_z(t)\hat{\sigma}_z\sqrt{\hat{z}},\end{aligned}$$

where the arguments have been omitted for ease of notation. Plugging the resulting partial derivatives from the guess solution into the pde and simplifying gives a linear function of the states μ and $\hat{z}$:

$$\begin{aligned}0 = &\ell_0'(\tau,\bar{c}) + \ell_\mu'(\tau,\bar{c})\mu + \ell_z'(\tau,\bar{c})\hat{z} - c_1(r_0 + r_\mu\mu + r_z\hat{z}) + \tfrac{1}{2}c_1(c_1-1)(\theta_x(t)^2 + \theta_\mu(t)^2 + \theta_z(t)^2) + c_2\mu + \tfrac{1}{2}c_2(c_2-1)\sigma_x^2 \\ &+ \ell_\mu(\tau,\bar{c})\lambda_\mu(\bar{\mu}-\mu) + \tfrac{1}{2}\ell_\mu(\tau,\bar{c})^2\sigma_\mu^2 + \ell_z(\tau,\bar{c})\lambda_z(\bar{z}-\hat{z}) + \tfrac{1}{2}\ell_z(\tau,\bar{c})^2\hat{\sigma}_z^2\hat{z} - c_1c_2\theta_x(t)\sigma_x \\ &- c_1\ell_\mu(\tau,\bar{c})\theta_\mu(t)\sigma_\mu - c_1\ell_z(\tau,\bar{c})\theta_z(t)\hat{\sigma}_z\sqrt{\hat{z}}.\end{aligned}$$

Hence, we get three ordinary differential equations for $\ell_0(\tau,\bar{c}), \ell_\mu(\tau,\bar{c})$ and $\ell_z(\tau,\bar{c})$:

$$\begin{aligned}0 = &\ell_0'(\tau,\bar{c}) - c_1r_0 + \tfrac{1}{2}c_1(c_1-1)(\theta_x(t)^2 + \theta_\mu(t)^2 + \theta_z(t)^2) + \tfrac{1}{2}c_2(c_2-1)\sigma_x^2 + \ell_\mu(\tau,\bar{c})\lambda_\mu\bar{\mu} \\ &+ \tfrac{1}{2}\ell_\mu(\tau,\bar{c})^2\sigma_\mu^2 + \ell_z(\tau,\bar{c})\lambda_z\bar{z} - c_1c_2\theta_x(t)\sigma_x - c_1\ell_\mu(\tau,\bar{c})\theta_\mu(t)\sigma_\mu, \\ 0 = &\ell_\mu'(\tau,\bar{c}) - c_1r_\mu + c_2 - \ell_\mu(\tau,\bar{c})\lambda_\mu \\ 0 = &\ell_z'(\tau,\bar{c}) - c_1r_z - \ell_z(\tau,\bar{c})\lambda_z + \tfrac{1}{2}\ell_z(\tau,\bar{c})^2\hat{\sigma}_z^2 - c_1\ell_z(\tau,\bar{c})\theta_z^0\hat{\sigma}_z\end{aligned}$$

with $\theta_z^0 = \theta_z(t)/\sqrt{\hat{z}}$ and initial conditions: $\ell_0(0,\bar{c}) = c_0, \ell_\mu(0,\bar{c}) = c_3$ and $\ell_z(0,\bar{c}) = c_4$. Explicit solutions are available. *q.e.d.*

Proof of Proposition D: The conditional moment generating function $\mathbb{D}_t(\tau,n)$ of Eq. (OA.20) obtains as a special case of $\mathcal{M}_{t,\tau}(\bar{c})$ with $\bar{c} = (nd_0, 0, n, 0, nd_z)$. Therefore, it is given by

$$\mathbb{D}_t(\tau,n) = X_t^n e^{\ell_0(\tau,\bar{c}) + \ell_\mu(\tau,\bar{c})\mu_t + \ell_z(\tau,\bar{c})\hat{z}_t}$$

with $\ell_0(0,\bar{c}) = nd_0, \ell_\mu(0,\bar{c}) = 0$ and $\ell_z(0,\bar{c}) = nd_z$ and $B_0(n,\tau), B_\mu(n,\tau)$ and $B_z(n,\tau)$ are implicitly defined. *q.e.d.*

Proposition E *The equilibrium price of the market dividend strip with maturity τ is given by*

$$P_{t,\tau} = \mathbb{E}_t\left[\xi_{t,t+\tau}D_{t+\tau}\right] \approx X_t e^{A_0(\tau) + A_\mu(\tau)\mu_t + A_z(\tau)\hat{z}_t}, \tag{OA.23}$$

where model parameters are such that the expectation exists finite, the approximation makes use of Eq. (OA.6) and A_0, A_μ and A_z are deterministic functions of time. The instantaneous volatility and premium on the dividend strip

with maturity τ *are given by*

$$\sigma_P(t,\tau) = \sqrt{\sigma_x^2 + \frac{(1-e^{-\lambda_\mu \tau})^2 (r_\mu - 1)^2}{\lambda_\mu^2}\sigma_\mu^2 + \left(\frac{\lambda_z}{\hat{\sigma}_z} + \theta_z^0 + \frac{\mho}{\hat{\sigma}_z}\tan\left(\tau\mho/2 - \arctan((\theta_z^0\hat{\sigma}_z + \lambda_z - d_z\hat{\sigma}_z^2)/\mho)\right)\right)^2 \hat{z}_t}, \tag{OA.24}$$

$$(\mu_P - r)(t,\tau) = \theta_x(t)\sigma_x + \frac{(1-e^{-\lambda_\mu \tau})(1-r_\mu)\theta_\mu(t)\sigma_\mu}{\lambda_\mu} + \theta_z^0\left(\frac{\lambda_z}{\hat{\sigma}_z} + \theta_z^0 + \frac{\mho}{\hat{\sigma}_z}\tan\left(\tau\mho/2 - \arctan((\theta_z^0\hat{\sigma}_z + \lambda_z - d_z\hat{\sigma}_z^2)/\mho)\right)\right)\hat{z}_t. \tag{OA.25}$$

with $\theta_z^0 = \theta_z(t)/\sqrt{\hat{z}_t}$ *and* $\mho = \sqrt{-\lambda_z^2 - 2\theta_z^0\lambda_z\hat{\sigma}_z - (\theta_z^0 + 2r_z)\hat{\sigma}_z^2}$.

Proof of Proposition E: The equilibrium price of the market dividend strip with maturity τ of Eq. (OA.25) obtains as a special case of $\mathcal{M}_{t,\tau}(\bar{c})$ with $\bar{c} = (d_0, 1, 1, 0, d_z)$. Therefore, it is given by

$$P_{t,\tau} = \xi_{0,t}^{-1}\mathcal{M}_{t,\tau}(\bar{c}) = X_t e^{\ell_0(\tau,\bar{c}) + \ell_\mu(\tau,\bar{c})\mu_t + \ell_z(\tau,\bar{c})\hat{z}_t}$$

with $\ell_0(0,\bar{c}) = d_0, \ell_\mu(0,\bar{c}) = 0$ and $\ell_z(0,\bar{c}) = d_z$ and $A_0(\tau), A_\mu(\tau)$ and $A_z(\tau)$ are implicitly defined. The dynamics of the market dividend strip price obtains by applying Itô's Lemma to $P_{t,\tau}$:

$$dP_{t,\tau} = [\cdot]dt + \partial_x P_{t,\tau}\sigma_x dB_{x,t} + \partial_\mu P_{t,\tau}\sigma_\mu dB_{\mu,t} + \partial_z P_{t,\tau}\hat{\sigma}_z\sqrt{\hat{z}_t}dB_{z,t}.$$

Therefore the return volatility is given by

$$\sigma_P(t,\tau) = P_{t,\tau}^{-1}\sqrt{(\partial_x P_{t,\tau}\sigma_x)^2 + (\partial_\mu P_{t,\tau}\sigma_\mu)^2 + (\partial_z P_{t,\tau}\hat{\sigma}_z\sqrt{\hat{z}_t})^2} = \sqrt{\sigma_x^2 + (A_\mu(\tau)\sigma_\mu)^2 + (A_z(\tau)\hat{\sigma}_z\sqrt{\hat{z}_t})^2},$$

and the premium is given by

$$(\mu_P - r)(t,\tau) = -\frac{1}{dt}\left\langle \frac{d\xi_{0,t}}{\xi_{0,t}}, \frac{dP_{t,\tau}}{P_{t,\tau}}\right\rangle = \theta_x(t)\sigma_x + \theta_\mu(t)A_\mu(\tau)\sigma_\mu + \theta_z(t)A_z(\tau)\hat{\sigma}_z\sqrt{\hat{z}_t}.$$

q.e.d.

Proposition F *The equilibrium price of the market asset is given by*

$$P_t = \mathbb{E}_t\left[\int_t^\infty \xi_{t,u}D_u du\right] \approx X_t\beta^{-1}e^{u_0\chi^{-1} + d_0 + u_\mu\chi^{-1}\mu_t + \left(u_z\chi^{-1} + d_z\right)\hat{z}_t} \tag{OA.26}$$

where model parameters are such that the expectation exists finite, the approximation makes use of Eq. (OA.6), and u_0, u_μ *and* u_z *are endogenous constants depending on the primitive parameters. The instantaneous volatility and premium on the market asset are given by*

$$\sigma_P(t) = \sqrt{\sigma_x^2 + \frac{(1-1/\psi)^2 u_\mu^2\sigma_\mu^2}{(1-\gamma_s)^2} + \left(d_z\hat{\sigma}_z + \frac{(1-1/\psi)u_z\hat{\sigma}_z}{1-\gamma_s}\right)^2\hat{z}_t}, \tag{OA.27}$$

$$(\mu_P - r)(t) = \theta_x(t)\sigma_x + \frac{(1-1/\psi)u_\mu\theta_\mu(t)\sigma_\mu}{1-\gamma_s} + \left(\theta_z^0 d_z\hat{\sigma}_z + \frac{\theta_z^0(1-1/\psi)u_z\hat{\sigma}_z}{1-\gamma_s}\right)\hat{z}_t, \tag{OA.28}$$

where $\theta_z^0 = \theta_z(t)/\sqrt{\hat{z}_t}$.

Proof of Proposition F: Under Assumption 2 (limited market participation), the shareholders act as a representative agent on the financial markets and, hence, the equilibrium price of the market asset is equal to the shareholders' wealth. Therefore, using the results of Proposition A, the market asset price can be written as

$$P_t = Q_{s,t} = C_{s,t} e^{-cq_t} = X_t e^{-\log\beta + u_0\chi^{-1} + d_0 + u_\mu \chi^{-1}\mu_t + (u_z\chi^{-1}+d_z)\hat{z}_t}.$$

The dynamics of the market asset price obtains by applying Itô's Lemma to P_t:

$$dP_t = [\cdot]dt + \partial_x P_t \sigma_x dB_{x,t} + \partial_\mu P_t \sigma_\mu dB_{\mu,t} + \partial_z P_t \hat{\sigma}_z \sqrt{\hat{z}_t} dB_{z,t}.$$

Therefore the return volatility is given by

$$\sigma_P(t) = P_t^{-1}\sqrt{(\partial_x P_t \sigma_x)^2 + (\partial_\mu P_t \sigma_\mu)^2 + (\partial_z P_t \hat{\sigma}_z \sqrt{\hat{z}_t})^2} = \sqrt{\sigma_x^2 + (u_\mu \chi^{-1}\sigma_\mu)^2 + ((u_z\chi^{-1}+d_z)\hat{\sigma}_z\sqrt{\hat{z}_t})^2},$$

and the premium is given by

$$(\mu_P - r)(t) = -\tfrac{1}{dt}\left\langle \tfrac{d\xi_{0,t}}{\xi_{0,t}}, \tfrac{dP_t}{P_t}\right\rangle = \theta_x(t)\sigma_x + \theta_\mu(t) u_\mu \chi^{-1}\sigma_\mu + \theta_z(t)\big(u_z\chi^{-1}+d_z\big)\hat{\sigma}_z\sqrt{\hat{z}_t}.$$

q.e.d.

Online Appendix D. Results under Power Utility

Proposition A *The equilibrium state price density satisfies*

$$\xi_{0,t} = e^{-\beta t} Y_t^{-\gamma_s}\big(1 + \tfrac{\alpha}{\alpha^{\phi^*} e^{\phi^* z_t} - \alpha}\big)^{\gamma_s} = e^{-\beta t} Y_t^{-\gamma_s}(1-\omega(z_t))^{-\gamma_s}, \tag{OA.29}$$

and has dynamics: $d\xi_{0,t} = \xi_{0,t}\big(-r(t)dt - \theta_x(t)dB_{x,t} - \theta_z(t)dB_{z,t}\big)$, *where the instantaneous risk-free rate satisfies*

$$r(t) = r_0 + r_\mu(\mu_t) + r_z(z_t), \tag{OA.30}$$

with

$$r_0 = \beta - \tfrac{\gamma_s(1+\gamma_s)\sigma_x^2}{2}, \quad r_\mu(\mu_t) = \gamma_s\mu_t, \quad r_z(z_t) = \lambda_z\gamma_s\big(1 + \tfrac{\alpha\phi^*}{\alpha^{\phi^*}e^{\phi^* z_t} - \alpha}\big)(\bar{z} - z_t) - \frac{\gamma_s^2\big(e^{z_t\phi^*}\alpha^{\phi^*} + \alpha(\phi^*-1)\big)^2 + \gamma_s e^{z_t\phi^*}\alpha^{1+\phi^*}(\phi^*)^2}{2\big(\alpha - e^{z_t\phi^*}\alpha^{\phi^*}\big)^2}\sigma_z^2,$$

and the instantaneous prices of risk are given by

$$\theta_x(t) = \gamma_s\sigma_x, \qquad \theta_z(t) = \gamma_s\sigma_z\big(1 + \tfrac{\alpha\phi^*}{\alpha^{\phi^*}e^{\phi^* z_t} - \alpha}\big).$$

Proposition B *The equilibrium price of the market dividend strip with maturity* τ *is given by*

$$P_{t,\tau} = \mathbb{E}_t\left[\xi_{t,t+\tau}D_{t+\tau}\right] = X_t e^{-\beta\tau}\mathcal{P}_0(z_t)\mathcal{P}_1(\tau,\mu_t,z_t), \tag{OA.31}$$

where model parameters are such that the expectation exists finite,

$$\mathcal{P}_0(z_t) = \big(e^{z_t} - \alpha^{1-\phi^*}e^{(1-\phi^*)z_t}\big)^{\gamma_s}, \tag{OA.32}$$

$$\mathcal{P}_1(\tau,\mu_t,z_t) = \sum_{j=0}^{\infty}\mathcal{B}(1-\gamma_s,j)(-\alpha^{1-\phi^*})^j e^{G_0(\tau,j)+G_z(\tau,j)\mu_t+G_z(\tau,j)z_t}, \tag{OA.33}$$

with $\mathcal{B}(r,k)=\frac{r\Gamma(r)}{(r-k)k!\Gamma(r-k)}$, *and* G_0, G_μ *and* G_z *are deterministic functions of time. The instantaneous volatility and premium are given by*

$$\sigma_P(t,\tau)=\sqrt{\sigma_x^2+\left(\partial_\mu \log \mathcal{P}_1(\tau,\mu_t,z_t)\right)^2\sigma_\mu^2+\left(\partial_z \log \mathcal{P}_0(z_t)+\partial_z \log \mathcal{P}_1(\tau,\mu_t,z_t)\right)^2\sigma_z^2}, \tag{OA.34}$$

$$(\mu_P-r)(t,\tau)=\gamma_s\sigma_x^2+\gamma_s\left(\partial_z \log \mathcal{P}_0(z_t)+\partial_z \log \mathcal{P}_1(\tau,\mu_t,z_t)\right)\left(1+\frac{\alpha\phi^*}{\alpha^{\phi^*}e^{\phi^* z_t}-\alpha}\right)\sigma_z^2. \tag{OA.35}$$

Proof of Proposition A: Under Assumption 2 (limited market participation), the marginal rate of substitution of market participants equals the marginal utility of shareholders evaluated at their optimal consumption: $C_{s,t}=D_t\ \forall t$. Therefore, we have that the state price density is given by

$$\xi_{0,t}=V'(D_t)=c_0e^{-\beta t}D_t^{-\gamma_s},$$

where c_0 is a positive constant which can be standardized to one. Using the resource constraint $Y_t=W_t+D_t$, the dividend process can be written as $D_t=Y_t\left(1-\alpha^{1-\phi^*}e^{-\phi^* z_t}\right)$ and Eq. (OA.29) follows. Consequently, the state price density has dynamics given by

$$\frac{d\xi_{0,t}}{\xi_{0,t}}=\frac{\partial_t e^{-\beta t}D_t^{-\gamma_s}}{e^{-\beta t}D_t^{-\gamma_s}}dt+\frac{\partial_X e^{-\beta t}D_t^{-\gamma_s}}{e^{-\beta t}D_t^{-\gamma_s}}dX_t+\frac{\partial_z e^{-\beta t}D_t^{-\gamma_s}}{e^{-\beta t}D_t^{-\gamma_s}}dz_t+\frac{1}{2}\frac{\partial_{X,X} e^{-\beta t}D_t^{-\gamma_s}}{e^{-\beta t}D_t^{-\gamma_s}}d\langle X\rangle_t+\frac{1}{2}\frac{\partial_{z,z} e^{-\beta t}D_t^{-\gamma_s}}{e^{-\beta t}D_t^{-\gamma_s}}d\langle z\rangle_t.$$

The results about $r(t),\theta_x(t)$ and $\theta_z(t)$ automatically follow. *q.e.d.*

Proof of Proposition B: The equilibrium price of the market dividend strip with maturity τ of Eq. (OA.31) is defined as

$$P_{t,\tau}=\mathbb{E}_t\left[e^{-\beta\tau}\frac{D_{t+\tau}^{1-\gamma_s}}{D_t^{-\gamma_s}}\right]=e^{-\beta\tau}X_t^{\gamma_s}\left(e^{z_t}-\alpha^{1-\phi^*}e^{(1-\phi^*)z_t}\right)^{\gamma_s}\mathbb{E}_t\left[X_{t+\tau}^{1-\gamma_s}\left(e^{z_{t+\tau}}-\alpha^{1-\phi^*}e^{(1-\phi^*)z_{t+\tau}}\right)^{1-\gamma_s}\right].$$

Provided $z_t>\frac{1-\phi^*}{\phi^*}\log\alpha\ \forall t$, using the generalized binomial formula we have

$$\begin{aligned}P_{t,\tau}=&e^{-\beta\tau}X_t^{\gamma_s}\left(e^{z_t}-\alpha^{1-\phi^*}e^{(1-\phi^*)z_t}\right)^{\gamma_s}\mathbb{E}_t\left[X_{t+\tau}^{1-\gamma_s}\sum_{j=0}^{\infty}\mathcal{B}(1-\gamma_s,j)(-\alpha^{1-\phi^*})^je^{(1-\gamma_s-j\phi^*)z_{t+\tau}}\right]\\=&e^{-\beta\tau}X_t^{\gamma_s}\left(e^{z_t}-\alpha^{1-\phi^*}e^{(1-\phi^*)z_t}\right)^{\gamma_s}\sum_{j=0}^{\infty}\mathcal{B}(1-\gamma_s,j)(-\alpha^{1-\phi^*})^j\mathbb{E}_t\left[X_{t+\tau}^{1-\gamma_s}e^{(1-\gamma_s-j\phi^*)z_{t+\tau}}\right],\end{aligned}$$

where $\mathcal{B}(r,k)=\frac{r\Gamma(r)}{(r-k)k!\Gamma(r-k)}$. For each $j=0,1,\ldots$, the conditional expectation $\mathbb{E}_t\left[X_{t+\tau}^{1-\gamma_s}e^{(1-\gamma_s-j\phi^*)z_{t+\tau}}\right]$, is a special case of $\mathcal{M}_{t,\tau}(\bar{c})$ with $\bar{c}=(0,0,1-\gamma_s,0,1-\gamma_s-j\phi^*)$. Therefore, it is given by

$$\mathbb{E}_t\left[X_{t+\tau}^{1-\gamma_s}e^{(1-\gamma_s-j\phi^*)z_{t+\tau}}\right]=X_t^{1-\gamma_s}e^{G_0(\tau,j)+G_\mu(\tau,j)\mu_t+G_z(\tau,j)z_t},$$

where

$$G_0(\tau,j) = -\frac{1}{4\lambda_z\lambda_\mu^3}\left((\gamma_s-1)^2\lambda_z\left(\left(3+e^{-2\tau\lambda_\mu}-4e^{-\tau\lambda_\mu}-2\tau\lambda_\mu\right)\sigma_\mu^2-2\tau\lambda_\mu^3\sigma_x^2\right)+\left(e^{-2\tau\lambda_z}-1\right)\lambda_\mu^3\sigma_z^2\left(\gamma_s-1+j\phi^*\right)^2\right.$$
$$\left.+2\tau(1-\gamma_s)\lambda_z\lambda_\mu^3\sigma_x^2+4\lambda_z\lambda_\mu^2\left(\bar{\mu}(1-\gamma_s)\left(1-e^{-\tau\lambda_\mu}-\tau\lambda_\mu\right)+\left(e^{-\tau\lambda_z}-1\right)\bar{z}\lambda_\mu(1-\gamma_s-j\phi^*)\right)\right),$$
$$G_\mu(\tau,j) = \frac{1-e^{-\tau\lambda_\mu}(1-\gamma_s)-\gamma_s}{\lambda_\mu},$$
$$G_z(\tau,j) = e^{-\tau\lambda_z}(1-\gamma_s-j\phi^*).$$

The result of Eq. (OA.31) automatically follows. Eq. (OA.34) and (OA.35) obtain by an application of Itô's Lemma. *q.e.d.*