ISSN 2279-9362

Collegio Carlo Alberto

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No. 314
December 2013

Carlo Alberto Notebooks

www.carloalberto.org/research/working-papers

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Abstract

This paper proposes an intuitive definition of status quo that is model-free and given in terms of observable choices only. We do not rationalize status quo-dependent preferences, to the contrary, we show that models of decision under ambiguity already predict behavioral phenomena ascribed to status quo bias. In particular, an ambiguity averse individual keeps the status quo whenever she has a minimal reason to do it. This offers a possible explanation to the experimental evidence that ambiguity averse individuals may prefer their ambiguous status quo to an unambiguous alternative. We provide conditions for the existence of a status quo and we show they are always satisfied by a MaxMin decision maker. Applications to portfolio choice generalizes well known no-trade results proving that, trade can be optimal when the status quo is ambiguous.

KEYWORDS: Ambiguity, Status Quo Bias, Revealed Preferences

1 Introduction and overview of the results

Deviations from rational choice are often ascribed to reference-dependent behavior. Various empirically documented phenomena, as status quo bias, loss aversion, endowment effect and the asymmetric dominance effect, are examples of such violations.[1] The common behavioral trait is the existence of an object (the *reference*) that affects the decision

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I am indebted to Paolo Ghirardato for continuous support and excellent supervision during the writing of this paper. I would like to thank Eric Danan, Edoardo Grillo, Pietro Ortoleva, Todd Sarver and Marciano Siniscalchi. All errors are clearly mine.

[1] See Rabin [35] for a survey of this extensive literature.

maker's (DM) rational choice. Theoretical models rationalizing reference-dependent behaviors have been proposed by a number of authors.[2] However, most of the existing literature assumes exogenously given objects that are taken as references and, either imposes axioms that rationalize reference-dependent behavior (Ok et al. [33], Freeman [13]) or directly assumes a utility functional form that reflects reference-dependence (Kőszegi and Rabin [23]). Consequently, the mechanism beyond the formation of the reference is still obscure. In the last years a growing literature highlighted the importance of reference-point formation in various fields[3], therefore a theory that endogenizes this mechanism would be desirable both from the descriptive and normative viewpoint.

The present work addresses the problem of how to identify the status quo from revealed preferences. The reason why we focus on status quo resides in its intuitive and agreed behavioral content, namely *inertia*. We exploit ambiguity to pin down the status quo, it is an option, or a utility profile, that induces inertia in the Bewley's sense. The main innovation is that we *do not* rationalize status quo bias, but we show that for a class of models including MaxMin EU, a status quo always exists. Moreover, when a DM is ambiguity averse, a phenomenon we deem *justified status quo bias* obtains. If there exists a prior that ranks the expected utility of the status quo above that of the alternative, the DM prefers the status quo. In other words, the DM keeps her status quo whenever she has a minimal "justification" (a prior) to do it. The justified status quo bias offers a possible explanation to the experimental evidence of Roca et al. [36]. They show experimentally, that ambiguity averse subjects tend to keep the status quo even if it is more ambiguous than the alternative. The justified status quo bias predicts that the DM resolves the conflict between status quo bias and ambiguity aversion in favor of the first, when there is a single prior that ranks the expected utility of the status quo above the alternative. Therefore, although ambiguity averse, the DM acts as an ambiguity loving. The rationale behind such result comes from the behavioral definition of status quo, we provide, indeed it posits that a status quo is, in a precise sense, familiar to the DM. Next section intuitively explains how status quo is elicited.

[2] Among the others, the seminal work of Tversky and Kahneman [41] focuses on loss-aversion in the risky setting, while Sugden [40] and Munro and Sugden [32] generalize the savagean expected utility theory to cope with references. Recently Lleras [26] axiomatized a model of loss-aversion in the Anscombe-Aumann setting. Concerning the status quo bias, Sagi [37]'s model is set in a risky setting, while Masatlioglu and Ok [30] and Masatlioglu and Ok [31] and Apesteguia and Ballester [1], use choice correspondences that may or may not depend on a default option. Ortoleva [34] is the first work to explicitly combine status quo bias and ambiguity, in the spirit of Bewley's inertia.

[3] Asset pricing (Barberis et al. [3]), mechanism design (Carbajal and Ely [4], Eisenhuth [11]), voting theory (Grillo [21])

Furthermore, we provides a behavioral condition to ensure that a status quo always exists for general ambiguity averse decision makers. In terms of uniqueness, we identify a class of models for which is meaningful to talk of a "unique" status quo.

Concerning applications, we solve a portfolio optimization problem for an investor with status quo, showing that it is possible to have a price interval for which no-trade is optimal. Moreover the interval is smaller when the investor has no status-quo. This is in line with the no-trade results under ambiguity aversion of Dow and Werlang [10]. However, when the status quo is not constant and its marginal utility is correlated with the asset, trade could still be optimal, confirming the intuition of Masatlioglu and Ok [31] about the distinct nature of the endowment effect and the status quo bias. The message is that the WTA/WTP discrepancy, comes out from a comparison of an uncertain prospect with *certainty* (the endowment), rather than the status quo. On the other side, optimality of trade may explain the Familiarity bias (French and Poterba [14]). If the domestic market portfolio represents the status quo, investing in domestic assets is optimal, since they are more correlated with the status quo than foreign assets.

Concerning the literature, the relation between uncertainty aversion and status quo bias has been already studied by Ortoleva [34], following the lines of Bewley's inertia assumption, however, the novelty of our approach is the *completely endogenous* nature of the status quo.

As anticipated, the endogenous characterization of references is an active area of research. Freeman [13] offered a revealed preference foundation to the "forward-looking" model of Kőszegi and Rabin [23]. In their model, an endogenous reference is defined as a rational expectation about future consumption streams[4]. Along the lines of the forward-looking approach, Sarver [39] models endogenous reference formation as solution of a trade-off problem. A reference is optimally chosen as the equilibrium between two contrasting forces: expected future consumption streams and loss aversion. Higher expectation raises the reference point, but exposes the DM to the possibility of a greater disutility due to loss aversion. Differently, Ok et al. [33] determine references from the observed behavior of a decision maker using choice correspondences. A revealed reference is defined as an object that affects rational choice in a precise way. However, their definition of refer-

[4]They define the reference as the DM's "recent expectations about outcomes (rather than the status quo), and (we) assume that behavior accords to a personal equilibrium. The person maximizes utility given her rational expectations about outcomes, where expectations depend on her own anticipated behavior".

ence is not meant to model status quo bias. More recently, Lleras [26] axiomatized a model similar to Kőszegi and Rabin [23] in the Anscombe-Aumann setting, a unique probability measure is used as "reference" to evaluate loss and gains from the expected value of each act. The present work is technically related to Ortoleva [34] since we exploit ambiguity to identify status quos. However, in Ortoleva [34] status quos are assumed exogenously.

1.1 On the meaning of endogenous status quo

Firstly, we want to point out that a status quo has to be understood as the *utility profiles* generated by an option, rather than the option itself. Doing so, we can compare different prospects with a unique measurement device. Our definition of endogenous status quo is based on two main building blocks. The first is a "local" preference defined observing the optimal choice behavior of a DM. Once "local" preferences are elicited, we use a normatively appealing robustness/no-regret condition to characterize status quos.

To define "local" preferences, we start with a complete preference $\succcurlyeq$ representing actual (observed) choices. Then, we say informally, that an option f is preferred to another option g from a third option h, if a "small" movement from h "toward" f is preferred to a small movement toward g. Consider, for example, h as the current endowment of a DM that have to decide whether to buy or not a prospect. The preference at the endowment $\succcurlyeq_h$ could be interpreted as "constructing a portfolio" containing the current endowment and a small fraction of the alternative prospect and evaluate the increase in utility of adding small quantities of the latter. Then, if the utility of keeping the initial wealth exceeds that of the "portfolio", we think of it as "locally" preferred. Local preferences embody *willingness to optimally leave* a given option[5] and they are clearly *observable*. The use of this kind of local preferences is motivated by Ghirardato and Siniscalchi [16], where it is shown that they exactly represent how local optimization is performed. To characterize status quo, we use a robustness/no-regret condition. To gain intuition, consider the problem of a decision maker who belong to a given pension plan h and she has to choose whether to enter or not a new pension h' plan. Assume first that the modeler *knows* her current pension plan h is the status quo[6]. Since the status quo generates inertia, in the spirit of Bewley and Ortoleva

[5]The intuitive definition is not restrictive, since the idea of "mixing" two prospects should not be thought in "physical" terms, but again, in terms of "utility" profiles.

[6]Pension plan is one of the classical examples of status quo, see Samuelson and Zeckhauser [38] and Madrian and Shea [29]

[34], she will leave the status quo whenever the new pension plan is *unambiguously better* than the old, i.e. according to a set of probabilities[7] C. So she will switch to a new pension plan only if,

$$h' \succsim_h h \implies E_p[u \circ h'] \geq E_p[u \circ h] \quad \forall p \in C$$

In other words, any decision of leaving the status quo must be robust to model uncertainty and therefore cannot be regretted. However, we are building an endogenous definition of status quo and what we actually observe is the decision maker entering or not the new plan, without knowing the true, if any, status quo. So we reverse the previous point of view, we *define* an option to be a status quo, whenever *any optimal departure from it is robust.* If leaving the old plan is a robust decision, we deem the old pension plan as the status quo. What we claim now is that, the previous decision rule is normatively appealing even if the comparison does not involve the status quo. This because the status quo is *not always an available option* (or a replicable utility profile), so there are situations in which the DM is "forced" to leave the status quo. In that case it is natural to suppose that the DM uses the same decision criterion she uses when deciding to leave or not the status quo.

Suppose the DM is forced to abandon her current pension plan. Assume again that the modeler *knows* the DM's status quo is her old plan. It is natural to think that she will select the new plan according to the same procedure she would use if not forced. First, the DM checks if one of the new plans is *unambiguously* better than the old, if it is the case, she chooses it. If no new plan unambiguously dominates the status quo or both dominate the status quo, she decides according to her original preference, since eventually, a choice has to be made.

Again, we reverse this argument to *define* a status quo: whenever a preference for an act f over g at an act h is robust to model uncertainty, we deem h a status quo. In other words, an act h is a status quo if,

$$f \succsim_h g \implies E_p[u \circ f] \geq E_p[u \circ g] \quad \forall p \in C$$

The definition is given in terms of priors, so it might be unsatisfactory from an observational viewpoint. It turns out that an equivalent definition can be given through revealed

[7]A set of priors can always be defined under minimal assumptions through standard technique (see Cerreia-Vioglio et al. [6].)

preferences only. In particular, it is proved in the text that an equivalent definition is: if

$$f \succcurlyeq_h g \implies f \succcurlyeq_{h'} g \quad \forall h'$$

then h is a status quo. When a deviation from an act h toward f rather than g has to survive a robustness checking and this test is the strongest possible, namely it has to be valuable *for all* the alternative utility profiles, h becomes a status quo[8].

The second definition relates the status quo to familiarity. If we consider *counterfactuals as a source of familiarity*, it follows that an act becomes a status quo, when the "familiarity" with it is maximal. The DM tests her preference against all the possible alternatives, in doing this, she gains confidence about the utility profile delivered by the act. When the preference is maximally robust, or equivalently, the DM is maximally familiar with the act, it becomes a status quo.

A further consideration concerns the relation of our definition of status quo with expected utility. In models of reference-dependent or status quo-dependent preferences, expected utility is a benchmark for absence of status quos (or reference), here however, the reverse condition holds. Under expected utility *all* acts are status quos. This is not surprising if we interpret expected utility as a limiting case of Bewley's model.

1.2 Setting and notation

We assume a standard Anscombe-Aumann setting where a set S of states of the world, it becomes a measurable space when endowed with a sigma-algebra Σ that represents events. An act is a function $f : S \to X$, where the set of prizes X is a convex subset of topological vector space. Constant acts $f(s) = x$ for all $s \in S$ are identified with the prizes they deliver. The set of all acts is denoted by $\mathscr{F}$, as usual, the mixture $\gamma f + (1-\gamma) g$ is performed statewise. We say an act f to be *interior*, denoted $f \in \mathscr{F}^{\text{int}}$ if there are two prizes $x, y \in X$ such that $x \succ f(s) \succ y$ for all $s \in S$. We denote $B_0(\Sigma, \Gamma)$ the set of (simple) Σ-measurable functions on S, with values in an interval $\Gamma \subseteq \mathbb{R}$ endowed with the supnorm. If $\Gamma = \mathbb{R}$, we simply write $B_0(\Sigma)$. The Banach space generated by the supnorm closure of $B_0(\Sigma)$ is denoted by $B(\Sigma)$. We denote $\Delta(S) \triangleq ba_1^+(\Sigma)$, the set of probability charges on Σ endowed with the weak*

[8]Another interpretation of such idea of status quo is offered by Gilboa et al. [20]'s "objective rationality", the DM leaves the status quo, if she can convince the others that she is right in doing that, since to leave the status quo she performs (the strongest) unanimity test, she has enough "evidence" to persuade the others.

topology induced by $B(\Sigma)$ (or $B_0(\Sigma)$). A functional $I : B_0(\Sigma,\Gamma) \to \mathbb{R}$ is said to be *monotonic* if $a(s) \geq b(s)$ for all $s \in S$ implies $I(a) \geq I(b)$; *continuous* if it is continuous with respect to the supnorm. It is *normalized* if $I(k1_S) = k$ for all $k \in \Gamma$. It is said to be *positively homogeneous* if $I(ta) = tI(a)$ for all $a \in B_0(\Sigma,\Gamma)$ and $t \geq 0$ such that $ta \in B_0(\Sigma,\Gamma)$ and *translation invariant* if $I(a + k1_S) = I(a) + k$ for all $a \in B_0(\Sigma,\Gamma)$ and $k \in \Gamma$) such that $a + k1_S \in B_0(\Sigma,\Gamma)$. It is *constant linear* if is both positively homogeneous and translation invariant. We indiscriminately use the notation $p(a) = \int a dp$ for $a \in B(\Sigma)$. If a preference relation $\succcurlyeq$ is represented by a positively homogeneous functional I, we say $\succcurlyeq$ is homothetic. If it is represented by a translation invariant functional, we say $\succcurlyeq$ is translation invariant.

2 Endogenous status quos

Following the previous discussion on the meaning of status quo, in this section we introduce the main definition and we study its basic consequences.

A complete binary relation $\succcurlyeq$ is assumed to represent the decision maker's preference over $\mathscr{F}$. Given $\succcurlyeq$, we assume that there exists a non-constant affine function $u : X \to \mathbb{R}$ and a locally Lipschitz, monotone and normalized functional $I : B_0(\Sigma, u(X)) \to \mathbb{R}$ such that

$$f \succcurlyeq g \iff I(u \circ f) \geq I(u \circ g)$$

In this case $\succcurlyeq$ is said to be a MBL (Monotone Bernoullian and Locally Lipschitz) preference.

Most of the existing models in the literature satisfies the above condition, in particular concave (as MaxMin EU, Confidence preference) are locally Lipschitz, whereas translation invariant preference (as Variational, Vector EU and Mean-Dispersion) are globally Lipschitz. For more general preferences as the MBA (Monotone Bernoullian and Continuous) preferences[9] introduced in Cerreia-Vioglio et al. [6], a behavioral condition for locally Lipschitz continuity can be found in Ghirardato and Siniscalchi [16, Online Appendix].

Given $\succcurlyeq$, we derive a preference relation $\succcurlyeq^*$ representing the notion of unambiguous preference. We say f is unambiguously preferred to g, written $f \succcurlyeq^* g$, if and only if, for all $\gamma \in (0,1]$ and $h \in \mathscr{F}$:

$$\gamma f + (1-\gamma)h \succcurlyeq \gamma g(+(1-\gamma)h$$

[9]They generalize the class of uncertainty averse preferences of Cerreia-Vioglio et al. [7] and smooth ambiguity of Klibanoff et al. [24])

It is well known that $\succsim^*$ is a Bewley preference, and that exists a set $\mathscr{C} \subseteq \Delta(S)$, such that,

$$f \succsim^* g \iff \int u \circ f dp \geq \int u \circ g dp, \quad \forall p \in \mathscr{C}$$

Now we introduce a form of convergence for acts. Following Ghirardato and Siniscalchi [16], we say that a sequence $(f^n)_{n\geq 0} \subseteq \mathscr{F}$ converges to an act $f \in \mathscr{F}$, denoted $f^n \to f$, if and only if, for all $x, y \in X$, with $x \succ y$, there exists N such that $n \geq N$ implies

$$\forall s \in S, \quad \frac{1}{2} f(s) + \frac{1}{2} y \prec \frac{1}{2} f^n(s) + \frac{1}{2} x \quad \text{and} \quad \frac{1}{2} f^n(s) + \frac{1}{2} y \prec \frac{1}{2} f(s) + \frac{1}{2} x$$

When a Bernoulli utility u represents $\succsim$ on X as for MBL preferences, it corresponds to uniform convergence on $B_0(\Sigma, u(X))$.

Next definition of local preferences (see Ghirardato and Siniscalchi [16]) is a first step toward a behavioral definition of status quo. It allows us to generate a family of indexed preferences that are often taken as exogenous in other models of decision with status quo. They characterize the informal idea of observing preferences "at an act h" and we use them because they are the behavioral counterpart of local optimization. As Proposition 2 shows, each local preference is associated to a set that exactly contains the priors used to identify solutions to local optimization problems.

Definition 1. *Given a triple of acts $f, g, h \in \mathscr{F}$, we say that f is a weakly better deviation than g near h, written $f \succsim_h g$, if, for every $(\gamma^n)_{n\geq 0} \subset [0,1]$ and $(h^n)_{n\geq 0}$ with $\gamma^n \downarrow 0$ and $h^n \to h$,*

$$\gamma^n f + (1-\gamma^n) h^n \succsim \gamma^n g + (1-\gamma^n) h^n \quad \textit{eventually} \tag{1}$$

In words, $f \succsim_h g$, if moving in the direction of f is always preferred to moving toward g from h, when the movement is small. $\succsim_h$ determines the "directions" that improve utility when a DM "holds" h. This is the formalization of local preference we anticipated in the introduction. A local preference $\succsim_h$ can be thought as evaluating the utility of a "portfolio" containing the reference act h and a small fraction of another act f. $f \succsim_h g$ if *any* portfolio containing a combination of f and h is preferred to a portfolio containing g and h, when most of the weight is given to the latter. $\succsim_h$ is also robust in the sense of being stable for perturbations. When h is the DM's current endowment, $\succsim_h$ is exactly the "local" preference that determines which are the optimal departures from current wealth.

Each element in the family of preferences $\{\succcurlyeq_h\}_{h\in\mathscr{F}}$, is a monotonic and independent preorder (provided it is nontrivial, see Ghirardato and Siniscalchi [16, Online Appendix]). Next proposition gives a quasi-Bewley's style representation for each non-trivial $\succcurlyeq_h$. First we need a bit of notation, given a weak*-closed convex subset D of $\Delta(S)$, we define $\succcurlyeq_D$ as the Bewley's preference generated by D, i.e.

$$f \succcurlyeq_D g \iff \int u\circ f dp \geq \int u\circ g dp, \quad \forall p \in D$$

Then,

Proposition 2. *Given a MBL preference $\succcurlyeq$, for each non-trivial $\succcurlyeq_h$, there exists a weak* compact convex set $C(h) \subset \Delta(S)$ such that, the following holds:*

1. *for all $f, g \in \mathscr{F}$, $f \succcurlyeq_h g \implies \int u\circ f dp \geq \int u\circ g dp$, for all $p \in C(h)$*
2. *$\succcurlyeq_{C(h)}$ is the smallest continuous preference relation that extends $\succcurlyeq_h$ (i.e. the intersection of all Bewley preferences that contains $\succcurlyeq_h$)*

Proof in Appendix B. So, to each non-trivial "local" preference $\succcurlyeq_h$ is assigned a unique set of probabilities $C(h)$ that rationalizes the smallest Bewley's preference that extends $\succcurlyeq_h$.

This result is a generalization of a similar result contained in Ghirardato and Siniscalchi [16]. In that case it is stated only for interior acts h, whereas, the above proposition holds for all acts. In Appendix B (Proposition 24) it is proved that for interior acts, the result coincides, i.e. the set of prior in Proposition 2 is the identical to the set identified by Ghirardato and Siniscalchi [16, Theorem 6].

Now we introduce the main definition. It summarizes the discussion above concerning the meaning of status quo. If any decision to move away from an act h is unambiguously valuable, then h is a status quo.

Definition 3 (Probabilistic Status Quo)**.** *An act h is called status quo, if and only if, $\forall f, g \in \mathscr{F}$,*

$$f \succcurlyeq_h g \implies \int u\circ f dp \geq \int u\circ g dp \quad \forall p \in \mathscr{C}$$

Alternatively, $f \succcurlyeq_h g \implies f \succcurlyeq^* g$. Notice that observing preferences at the status quo, is sufficient to recover the underlying complete preference, since, by definition, $f \succcurlyeq^* g \Rightarrow f \succcurlyeq g$. Moreover, in the limiting case of $\succcurlyeq$ being an expected utility, $\succcurlyeq = \succcurlyeq_h = \succcurlyeq^*$ for all h, hence all acts are status quos.

The first consequence of Definition 3 is that the set of priors $C(h)$, that rationalizes local preferences at a status quo h is *equal* to the set of global priors.

Theorem 4. *h is a status quo, if and only if, $\mathscr{C} = C(h)$.*

This implies that the "local" perceived ambiguity is equivalent to the "global", the consequences of such result will be clearer in the next section, where we discuss local unambiguous acts and events.

We now provide the promised equivalent definition in terms of observable choice only.

Definition 5 (Behavioral Status Quo)**.** *An act h is called status quo, if and only if, $\forall f, g \in \mathscr{F}$,*

$$f \succsim_h g \implies f \succsim_{h'} g \quad \forall h' \in \mathscr{F}$$

The previous definition[10] has some interesting consequences. First, it is given in terms of (in principle) observable preferences only. This is a main innovation with respect to the literature and it provides a behavioral foundation for status quo formation. In light of the introductory discussion we made, a utility profile is a status quo whenever the DM *optimally* deviates form it in the most *robust* possible way. Now the robustness is not with respect to priors, but counterfactuals. The DM tests her local preference against alternative utility profiles, if it is not reversed, then the initial utility profile is a status quo. In a different interpretation, performing counterfactuals increases "familiarity" with the utility delivered by an option, when the familiarity is maximal, the act becomes a status quo.

When we specialize the condition above at the status quo, we recover the Sagi [37] no-regret condition, if $f \succsim_h h$, then $f \succsim_{h'} h$ for all $h' \in \mathscr{F}$. If an act is ranked above the status quo at the status quo, then it is ranked above regardless of the alternative specifications from which it is evaluated, excluding the possibility of regret.

Since, for each $h' \neq h$, $C(h') \subseteq C(h) \subseteq \mathscr{C}$, it follows that $\succsim_{C(h)}$ is the most ambiguity averse (in the sense of Ghirardato and Marinacci [15]) among all the Bewley's preferences generated by $h \in \mathscr{F}$[11].

[10] It is equivalently since, by Ghirardato and Siniscalchi [16], $f \succsim_h g$ for all interior h, implies $f \succsim^* g$. Definition 5 considers all acts then, in particular, the interior ones.

[11] Provided it is non-trivial, otherwise, they do not coincide with $\succsim$ on X and no bernoulli utility exists.

3 The notion(s) of unambiguous act and event

The previous analysis suggest the possibility to define a "local" notion of unambiguous act and unambiguous event. Indeed, suppose there exists a $x \in X$ such that $f \sim_{C(h)} x$, similarly to the global case, we say f is *h-crisp*. The interpretation is clear, an act is h-crisp if, when it is evaluated from h, it is locally indifferent to a constant. It follows that $f \sim_{C(h)} x$ implies $p(u \circ f) = u(x)$ for all $p \in C(h)$. Similarly, we can define an event $E \in \Sigma$ to be *h-unambiguous*, if and only if, for each $x, y \in X$ with $x \not\sim y$, we have $xEy \sim_{C(h)} z$, for some $z \in X$. That is, E is h-unambiguous if xEy is h-crisp. h-unambiguous events are such that, $u(x)p(E) + (1 - p(E))u(y) = u(z)$ for all $p \in C(h)$, then $p(E) = q(E)$ for all $p \in C(h)$. Let the set of locally unambiguous events be

$$\Lambda(h) \triangleq \left\{A \in \Sigma : p(A) = q(A),\ \forall p, q \in C(h)\right\}$$

it is clearly a (finite) λ-system. Now, denote the set of globally unambiguous event $\mathscr{E}$, i.e. $E \in \Sigma : p(E) = q(E),\ \forall p, q \in \mathscr{C}$.

$\mathscr{E}$ is not necessarily equal to the family of events that are considered unambiguous at *any* act h, defined as

$$\Lambda \triangleq \bigcap_{h \in \mathscr{F}} \Lambda(h)$$

Indeed, $A \in \Lambda$ implies that $p(A) = k$ for all $p \in C(h)$ and some h, and $q(A) = k'$ for all $q \in C(h')$ and some h', for possible $k \neq k'$. Then, Λ is generally bigger than the set of globally unambiguous events $\mathscr{E}$. However, when a status quo exists, the two sets coincide.

Proposition 6. *Let $\succcurlyeq$ a MBL preference, if h is a status quo,*

$$\Lambda(h) = \Lambda = \mathscr{E}$$

More generally, we can define the set of h-unambiguous acts as:

$$\mathscr{U}(h) \triangleq \left\{f \in \mathscr{F} : \left\{s \in S : f(s) \succcurlyeq x\right\} \in \Lambda(h),\ \forall x \in X\right\}$$

i.e. the set of acts for which the upper preference set belongs to $\Lambda(h)$. Next proposition from Cerreia-Vioglio et al. [6], characterizes the set of h-unambiguous acts.

Proposition 7 (Cerreia-Vioglio et al. [6])**.** *The following are equivalent:*

(1). $f \in \mathscr{U}(h)$

(2). $p\left(\left\{s \in S : u \circ f(s) = k\right\}\right) = q\left(\left\{s \in S : u \circ f(s) = k\right\}\right)$ *for all* $k \in \mathbb{R}$ *and* $p, q \in C(h)$.

Also in this case if we define the set of all acts that are perceived as unambiguous for every h,

$$\mathscr{U} \triangleq \bigcap_{h \in \mathscr{F}} \mathscr{U}(h)$$

it is not necessarily the case that it is equal to the set of globally unambiguous acts $\mathscr{A}$, defined as

$$\mathscr{A} = \left\{f \in \mathscr{F} : \left\{s \in S : f(s) \succcurlyeq x\right\} \in \mathscr{E},\ \forall x \in X\right\}$$

In general $\mathscr{A} \subseteq \mathscr{U}$, however:

Proposition 8. *Let* $\succcurlyeq$ *a MBL preference, if* h *is a status quo, then* $\mathscr{A} = \mathscr{U}$*. In particular* $\mathscr{U}(h) = \mathscr{A}$ *and* $f \sim_h x$ *implies* $f \in \mathscr{A}$.

Hence with a status quo, locally unambiguous acts are globally unambiguous. The status quo is not only an act that induces inertia but, it also shapes the DM's perception of ambiguity. Next section better explains why local and global preferences are so tightly related in the presence of a status quo.

4 Differential characterization of status quo

In this section we derive an equivalent definition of status quo using a differential approach which enhances its operational tractability.

In Ghirardato and Siniscalchi [16] it is shown that, for interior acts, $C(h)$ is the normalized Clarke subdifferential[12] (see Appendix A for definitions) of I at $u \circ h$,

$$C(h) = \overline{\text{co}}\left(\left\{\frac{q}{q(S)} : q \in \partial^C I(u \circ h),\ q(S) > 0\right\}\right)$$

The local characterization of priors is related to the global set of priors $\mathscr{C}$, by the following relation

$$\mathscr{C} = \overline{\text{co}}\left(\bigcup_{h \in \mathscr{F}^{\text{int}}} C(h)\right)$$

[12]The definition of $C(h)$ given here is slightly different from that of Ghirardato and Siniscalchi [16] although equivalent.

then, up to convex closure, the set of relevant priors is the union of normalized (Clarke) subdifferentials of the representing functional I.

Using the differential characterization of local priors, if h is a status quo and it is an interior act, then

$$\mathscr{C} = C(h) \tag{2}$$

meaning that the normalized elements of the (Clarke) subdifferentials of I at $u \circ h$ are exactly the relevant priors.

Using the differential characterization of status quo, we can elaborate more on the idea of linking local and global preferences. In particular, observing the DM's choice behavior around her status quo, is sufficient to infer her original preferences, since $f \succsim_h g$ implies $f \succsim^* g$, that gives $f \succsim g$. Interestingly, the converse implications holds, although in a weaker sense. It follows from a (nonsmooth) mean value theorem[13] that, if $f \succsim g$ and so $I(u \circ f) \geq I(u \circ g)$, there exists $\gamma \in (0, 1)$ such that

$$f \succsim g \implies \int u \circ f dp \geq \int u \circ g dp \quad \text{for some } p \in C(\gamma f + (1-\gamma)g)$$

However, $C(\gamma f + (1-\gamma)g) \subseteq C(h)$ if h is a status quo. Then

$$f \succsim g \implies \int u \circ f dp \geq \int u \circ g dp \quad \text{for some } p \in C(h)$$

In a sense, every preference can be rationalized by a prior belonging to the set of optimizing priors at the status quo. Hence, $C(h)$ contains all the "information" about DM's preferences $\succsim$, confirming the intuition that preferences with status quo display a local-global similarity.

The differential characterization provides a simple sufficient condition to the existence of a status quo. Given Eq. (2) and the properties of Clarke subdifferential, *a status quo exists when preferences are homothetic, namely I is positively homogeneous.* In that case, $z \in X$, with $u(z) = 0$ is necessarily a status quo[14], since $\mathscr{C} = C(z)$.

We conclude this section with an impossibility result, in the spirit of Sagi [37]. We prove that, if preferences are smooth, i.e. Gateaux differentiable and there is a status quo then, they collapse to expected utility.

[13] Lebourg's mean value theorem (see Proposition 21 in Appendix A).

[14] Provided z is interior.

Theorem 9 (Impossibility of smooth preferences)**.** *There exists a status quo and I is smooth, if and only if, $\succsim$ is expected utility.*

Consequently, non-expected utility preferences with status quo necessarily displays kinked indifference curves. This result rules out some smooth subclasses of well known models, as smooth Variational preferences (see Maccheroni et al. [28]), from being compatible with the presence of a status quo.

5 Uncertainty aversion and the justified status quo bias

In this section we study the interplay of ambiguity aversion and the status quo. We offer a possible explanation for the experimental findings of Roca et al. [36]. We impose an additional axiom, the uncertainty aversion of Gilboa and Schmeidler [19], for all $f, g \in \mathscr{F}$ and $\gamma \in (0,1)$,

$$f \sim g \implies \gamma f + (1-\gamma) g \succsim f$$

Such large class of preferences, corresponds to the uncertainty averse preferences (UAP) axiomatized in Cerreia-Vioglio et al. [7]. Beyond minimal standard requirements, UAP imposes aversion to ambiguity in the sense of preference for hedging. Mathematically, UA preferences are represented by the following quasiconcave functional

$$f \longmapsto \min_{p \in \Delta(S)} G\left(\int u \circ f \, dp, p\right)$$

where G is an index of uncertainty aversion. Several models as Variational, Confidence, concave Smooth Ambiguity and MaxMin, are particular cases of UAP preferences.

Until now, we focused on ambiguity perception (the set $\mathscr{C}$) and the existence of a status quo, showing that a DM with a status quo perceives non-trivial ambiguity. Adding a substantive axiom as uncertainty aversion, we affect the DM's ambiguity attitude. Coupled with the presence of a status quo, an uncertainty averse DM displays a phenomenon we call *justified status quo bias.*

Consider first a MaxMin decision maker with multiple priors D and take $z \in X$ is such that $u(z) = 0$. Suppose there exists a prior $q \in D$, and an act $f \in \mathscr{F}$, such that

$$0 = \int u(z) \, dq \geq \int u \circ f \, dq$$

then clearly, $u(z) = 0 \geq \min_{p\in D} \int u \circ f dp$, i.e. $z \succcurlyeq f$. It follows, that for a MaxMin decision maker, a single prior that ranks z better then f, is sufficient to "justify" a preference for z. This certainty bias clearly holds for any $x \in X$ when $\succcurlyeq$ is MaxMin. By homotheticity of MaxMin, the constant z is a status quo, hence the *justified status quo bias*.

Surprisingly a similar phenomenon turns out to hold for a large class of preferences as those satisfying uncertainty aversion.

Theorem 10. *Suppose $\succcurlyeq$ is a uncertainty averse, MBL preference and h an interior status quo then,*

$$\exists p \in \mathscr{C} \quad \textit{such that} \quad \int u \circ h dp \geq \int u \circ g dp \quad \Longleftrightarrow \quad h \succcurlyeq g$$

The previous theorem clarifies what we mean by *justified status quo bias*. Indeed, for an uncertainty averse DM, it is sufficient to have a prior that ranks the status quo above another act, to prefer the status quo. A single prior "justifies" the preference[15] for the status quo, confirming the DM's willingness to keep "thing as they are".

The result provides a possible explanations for the experimental evidence of Roca et al. [36]. The value attached to the status quo overcomes aversion to ambiguity when there is a minimal reason to do that. Since the DM feels "familiar" with the status quo, she may decide to prefer it over a, possibly more ambiguous, option when a prior justifies such choice. Therefore, our definition of status quo allows, at the same time, uncertainty aversion and "uncertainty seeking" when the choice involves the status quo.

The following proposition characterizes the justified status quo bias using the revealed "local" preferences at $\succcurlyeq_h$:

Proposition 11. *Suppose $\succcurlyeq$ is a uncertainty averse, MBL preference and h an interior status quo. If $g \not\succcurlyeq_h h$, then, $h \succcurlyeq g$.*

Proposition 11 is clear. It says that, although each preference $\succcurlyeq_h$ is incomplete, the status quo works as a completion device. This is another form of status quo bias, if the alternative option is not unambiguously valuable, then the DM acts conservatively and she "keeps" the status quo. Clearly, this can be true also if the alternative is less ambiguous than the status quo.

[15] Lehrer and Teper [25] introduced the class of justifiable preferences. A preference $\succcurlyeq^\sharp$ is a *justifiable*, if and only if, there exists a set $\mathscr{P} \subseteq \Delta(S)$ such that, for all $f, g \in \mathscr{F}$, $f \succcurlyeq^\sharp g \iff \exists p \in \mathscr{P}$ such that $\int u \circ f dp \geq \int u \circ g dp$.

5.1 Characterizing status quo: uncertainty aversion and caution

Proposition 11 can be interpret as a "caution" criterion a DM's uses to evaluate departures from a status quo. It follows from Gilboa et al. [20], that a similar conservative condition is necessary and sufficient to characterize a status quo. In Gilboa et al. [20] and in Cerreia-Vioglio [5], a property called Caution is used to related a complete and an incomplete preference relations[16], representing subjective and objective rationality. It imposes a strong form of ambiguity aversion, if an act is not preferred to a constant according to the incomplete (objective rationality) relation, than the constant is preferred according to the complete (subjective rationality) relation. The link between complete and incomplete preferences is given by a consistency and a caution criterion. In the case of preferences with status quo, the consistency requirement holds by definition, then the caution criterion alone is sufficient to characterize status quo.

We write a caution condition using the preferences $\succcurlyeq_{C(h)}$,

Property (h-Caution)**.** *For all $x \in X$ and $g \in \mathscr{F}$,*

$$g \not\succcurlyeq_{C(h)} x \implies x \succcurlyeq g$$

h-Caution implies strong ambiguity aversion at an act, the DM completes her local preferences with a bias toward certainty. Next Theorem shows that preferences with status quos, are always cautious and h-Caution characterizes status quo.

Theorem 12. *Assume $\succcurlyeq$ is an (unbounded) uncertainty averse MBL preference, h is a status quo, if and only if, h-Caution holds.*

A DM with status quo prefers certainty to possibly ambiguous acts when she is not "objectively" sure that the act beats the certain outcome.

Theorem 12 is an alternative behavioral characterization of status quo holding for UAP preferences. Here the two-steps choice procedure we sketched above is immediate. The DM first performs the unanimity test, if the ambiguous act is not ranked above a constant, then the DM acts conservatively, choosing the safe act.

[16] In Cerreia-Vioglio [5] a slightly weaker condition is imposed to obtain the same result, however, for the economy of this paper the two conditions are essentially equivalent.

6 Ambiguity and status quo

In this section we study under what conditions a status quo is perceived as unambiguous. We will see that imposing some restrictions to the ambiguity attitude of a DM forces the status quo to be constant or unambiguous. For example, in MaxMin EU status quos can only be constant or unambiguous. It gives a deeper understanding of the behavioral implications entailed by models of decision under ambiguity.

A first observation concerns homothetic preference, for which we know that $z \in X$ with $u(z) = 0$ is a status quo. By the properties of Clarke subdifferential[17], if preferences are also translation invariant, any constant act is again a status quo. So, for MaxMin preferences[18] *all* the constants are status quo. However, for more general preferences as uncertainty averse, the characterization given in Theorem 12 does not pose any restriction to the possible ambiguity of a status quo nor to the relations among different status quos.

From now on, we say that a preference $\succcurlyeq$ is *concave* if, $\succcurlyeq$ is represented by I and I is (closed, proper) concave[19].

Intuitively, concave preferences displays a stronger aversion towards ambiguity than Uncertainty Averse preferences and it will be sufficient to restrict the set of acts that a DM could use as status quos. The decision theoretic literature provides different notions of ambiguity or unambiguity for acts and events (For example Epstein and Zhang [12], Ghirardato et al. [18]). In this section we use a general definition as that of Cerreia-Vioglio et al. [6].

Let h be an act, we say that $\succcurlyeq$ satisfies h-independence if, for all $f, g \in \mathscr{F}$ and $\gamma \in (0,1)$

$$f \sim g \implies \gamma f + (1-\gamma)h \sim \gamma g + (1-\gamma)h$$

h-independence holds if preferences are not reversed when mixing with h, or equivalently, h does not provide hedging. Clearly h-independence has some similarities with the definition of local preference $\succcurlyeq_h$. However, they are in general not nested. h-independence holds *for all* $\gamma \in (0,1)$ and a *fixed* h, whereas the local preference $\succcurlyeq_h$ holds for *all* $\gamma \downarrow 0$ and *all* $h^n \to h$, eventually. In models satisfying Certainty Independence, the previous defini-

[17] See Lemma 22.

[18] Actually for all α-MaxMin.

[19] A functional $I : B(\Sigma) \to \mathbb{R}$ is concave if, for $a, b \in B(\Sigma)$, $I(\gamma a + (1-\gamma b)) \geq \gamma I(a) + (1-\gamma)I(b)$ for all $\gamma \in [0,1]$. Notice that concave preferences are everywhere nice, i.e. the zero measure never belongs to the subdifferential of I.

tion is equivalent to *crispness*: h is crisp if there exists $x_h \in X$ such that $h \sim^* x_h$, i.e. h is unambiguously indifferent to a constant. Without certainty independence the two notions are different.

Next results show that, for a general class of preferences, if an act h is a status quo, then, h-independence holds.

Proposition 13. *Let $\succcurlyeq$ be a concave MBL preference, if h is an interior status quo, then h-independence holds.*

It follows that, for the class of concave preferences, status quos necessarily satisfy a form of crispness. It is exactly the crispness definition, when concave preferences satisfy certainty independence, namely with MaxMin. So, under MaxMin status quos are necessarily crisp acts. The extreme ambiguity aversion of MaxMin preferences, excludes that a DM's status quo is a non-constant and ambiguous acts. Consequently, assuming MaxMin in the experimental context of Roca et al. [36] for example, implies that the subjects perceive bets on the ambiguous urns as "crisp".

Next proposition gives the condition to have the reverse inclusion, so to establish a one-to-one relation between status quos and crisp acts.

Theorem 14. *Let $\succcurlyeq$ be a MaxMin preference and h is an interior act, then the following are equivalent:*

(1.) h is a status quo

(2.) h is crisp.

(3.) h-independence holds.

To conclude, unambiguous act and status quos coincides under strong ambiguity aversion and a certain kind of invariance, namely stability for rescaling and translations.

Proposition 13 exclude the possibility that, for concave preferences, a status quo h is ambiguous. Coupled with Theorem 12, it follows that *h-Caution for homothetic concave preferences is equivalent to h being crisp.*

Consequently, it has to be the case that h-Caution, when imposed to MaxMin preferences, is equivalent to crispness of h. Next corollary sum up these considerations.

Corollary 15. *Assume $\succcurlyeq$ is an MaxMin preference, then the following are equivalent:*

1. *h is crisp*

2. *h-Caution: for all $x \in X$ and $g \in \mathscr{F}$,*

$$f \not\succcurlyeq_h x \implies x \succ g$$

3. *h is a status quo*

Consequently, crispness of a status quo is equivalent, under strong ambiguity aversion and invariance, to a completion of the local preference with a caution criterion.

7 Uniqueness and its consequences

Uniqueness properties of a status quo are related to the invariance of the underlying preferences. This follows from the relation between the type of invariance of the preference relation and the Clarke subdifferential. For example, translation invariance preferences (as Variational, Dispersion Aversion and Vector EU) are such that, if $h \in \mathscr{F}^{\text{int}}$ is a status quo, then any "translation" h' is again a status quo, where for translation we mean $h' \in \mathscr{F}^{\text{int}}$ with $u \circ h' = u \circ h + k$ for some $k \in \mathbb{R}$. A similar property holds for homothetic preferences. It follows that a unique status quo is a point where the invariance properties of preferences are somehow particular. As noted, the existence of a status quo relates local and global preferences. Intuitively, the "more invariant" is a preference, the bigger is the set of status quos. Consider the extreme case of invariance holding for expected utility, as a matter of fact, all acts are status quos. Consequently, there is a tension between invariance and uniqueness.

In this section, we offer a model-selection procedure. The selection is not meant as a "selecting" among acts that are status quos, but, conversely, we pick a class of models that are suitable to talk about a certain kind of *uniqueness* of the status quo. Moreover, in that class a complete characterization of *the* status quo is given. In other words, if a modeler commits to study status quo-dependent behavior with a *unique* status quo then, he has to commit to a certain class of preferences.

To formalize the idea of invariance, let $\mathscr{A}_{\succcurlyeq}$, be a group of real line automorphism, i.e. a nonempty collection of bijective functions $A : \mathbb{R} \to \mathbb{R}$ such that, $A \in \mathscr{A}_{\succcurlyeq}$ implies $A^{-1} \in \mathscr{A}_{\succcurlyeq}$, and $A, B \in \mathscr{A}_{\succcurlyeq}$ implies $A \circ B \in \mathscr{A}_{\succcurlyeq}$.

We define a MBL $\succcurlyeq$ to be $\mathscr{A}_{\succcurlyeq}$-*subinvariant* with respect to a group of real line automorphism $\mathscr{A}_{\succcurlyeq}$, if and only if,

$$\partial^C I(u \circ f) = \partial^C I(A \circ u \circ f)$$

for all $f \in \mathscr{F}^{\text{int}}$ and all $A \in \mathscr{A}_{\succcurlyeq}$.

Clearly translation invariant preferences are $\mathscr{A}_{\succcurlyeq}$-subinvariant with respect to the group of translations, $\mathscr{A}_{\succcurlyeq} = \{a \mapsto a + k : k \in \mathbb{R}\}$. Similarly, homothetic preferences are $\mathscr{A}_{\succcurlyeq}$-subinvariant with respect the group of similarities, $\mathscr{A}_{\succcurlyeq} = \{a \mapsto ka : k \geq 0\}$. Finally, α-MEU preferences are $\mathscr{A}_{\succcurlyeq}$-subinvariant w.r.t., $\mathscr{A}_{\succcurlyeq} = \left\{a \mapsto ka + k' : k \geq 0, k' \in \mathbb{R}\right\}$.

A simple observation shows that if there is a "unique" status quo, then it is a fixed point of all the automorphisms under which the preference is invariant. Indeed, if h is a unique status quo then, $\partial^C I(u \circ h)$ is unique, so it is the case that

$$A \circ u \circ h = u \circ h, \quad \text{for all } \ A \in \mathscr{A}_{\succcurlyeq}$$

we call $u \circ h$ a *singular point*. Note that singularity of a status quo does not exclude the possibility that there exists h' with $\partial^C I(u \circ h') = \partial^C I(u \circ h)$. However the next result shows that if such h' exists, it is necessarily a non-singular act, or, equivalently, that a singular status quo, when it exists, is unique. When an act h is said to be "singular", it is clearly meant that $u \circ h$ is singular.

We say that f, g are *essentially equal*, written, $f \hat{=} g$ if $u \circ f(s) = u \circ g(s)$ for all $s \in S$, in particular if $x \neq y$ but $u(x) = u(y)$, then $x \hat{=} y$.

Then we have the following theorem:

Proposition 16. *Let $\succcurlyeq$ a MBL preference, if $h, h' \in \mathscr{F}$ are singular acts with respect to $\mathscr{A}_{\succcurlyeq}$, then h and h' are essentially equal.*

Proposition 16 is crucial, it says that no two status quos can be both singular unless they are essentially equal. A singular status quo is, in some sense, unique.

Next theorem offers the anticipated model selection. It is the main result of this section.

Theorem 17. *Assume $\succcurlyeq$ is a MBL preference then the following holds:*

(1.) if $\succcurlyeq$ is $\mathscr{A}$-subinvariant, w.r.t. $\mathscr{A} = \{a \mapsto ka : k \geq 0\}$, then, $z \in X$ with, $u(z) = 0$, is the unique (up to indifference) singular status quo.

(2.) if $\succcurlyeq$ is $\mathscr{A}$-subinvariant, w.r.t. $\mathscr{A} = \{a \mapsto a + k : k \in \mathbb{R}\}$, then, no $h \in \mathscr{F}^{\text{int}}$ is a unique status quo.

(3.) if $\succcurlyeq$ is $\mathscr{A}$-subinvariant, w.r.t. $\mathscr{A} = \left\{a \mapsto ka + k' : k \geq 0, k' \in \mathbb{R}\right\}$, then, no $h \in \mathscr{F}^{\text{int}}$ is a unique status quo.

Theorem 17 is important in two aspects. First, the unique class of preferences that admit a singular (hence essentially unique) status quo is the class of homothetic preferences. Second, the unique singular status quo is exactly the constant act delivering zero utility. As a modeling implication, any model which assumes the existence of a unique status quo, according to our definition, has to assume *homothetic preferences and take the constant act giving zero utility as the status quo*. For non-homothetic models as translation invariant, the interior status quos are necessarily non-singular.

8 Applications

8.1 Portfolio (quasi-)inertia with status quo and the Familiarity bias

Two of the possible explanation of portfolio inertia are the endowment effect (Kahneman et al. [22]) and ambiguity aversion Dow and Werlang [10]. Status quo bias and the endowment effect are often associated and considered possible explanations to the so called "bid-ask" spread in the price of financial assets, whereas in their seminal paper, Dow and Werlang [10] show that under ambiguity a Choquet Expected utility maximizer can optimally choose portfolio inertia. Similar results are obtained under different specifications by Chateauneuf and Ventura [8] and Asano [2]. However, as pointed out by Masatlioglu and Ok [31], status quo bias and the endowment effect should be regarded as different phenomena. Then it is the case that status quo bias does not necessarily implies a discrepancy between willingness to accept (WTA) and willingness to pay (WTP). Our model allows for such pattern. For example, uncertainty averse preferences with a status quo, displays the *justified status quo bias*, but we will show that they are also consistent with optimality of trade, when the asset is "correlated" with the status quo.

The results of this section confirm the intuition that no-trade results are more likely to happen when the DM's endowment, *as a status quo*, is constant. In other words, no-trade is more willing to result when a DM compares an ambiguous asset with a *safe* status quo (a constant). Since we allowed for non-constant status quos, our preferences are suitable to model at the same time, a bias toward status quos and optimality of trade. For MaxMin preferences, it was already pointed out in Dow and Werlang [10]:

> In terms of empirical implications of the Schmeidler-Gilboa model, broadly similar types of behavior could be caused by transaction costs or asymmetric information, or by the preferences in Bewley's model. The main difference is that in each of those three cases there is a tendency not to trade, whereas in Schmeidler-Gilboa there is a tendency not to hold a position. In other words, the agent's frame of reference here is the safe allocation, rather than the status quo.

Optimality of trade when the asset is correlated with the status quo may also explain the Familiarity bias in international financial markets. It refers to the empirical observation that investors systematically under-diversify their portfolios investing most of their wealth in domestic assets (French and Poterba [14]). If we assume that the investor's status quo is the domestic market portfolio then, domestic assets are clearly "more correlated" to the status quo than foreign assets. Then, our trade theorem predicts optimality of trade and it explains familiarity bias.

When a DM has a constant status quo $h \in X$, the analysis reduces to that of Ghirardato and Siniscalchi [16, Example 3]. Notice that the following results holds similarly if we do not assume preferences with status quo, since the set of local priors of Ghirardato and Siniscalchi [16] is exactly the priors that a DM uses to optimize. The main difference consist in the "size" of this set. When h is not a status quo $C(h)$ is in general smaller than $\mathscr{C}$, implying that the no-trade price interval stemming from optimization at h is smaller than the same interval when h is a status quo. This is again a consequence of the extreme behavioral/probabilistic test that, in this case, an asset has to survive to be optimally chosen at a status quo. As a consequence, *ceteris paribus* a *preference with a constant status quo leads to a bigger interval of prices supporting no-trade than preferences without status quo.*

Let I a locally Lipschitz continuous functional, and assume h is a status quo. Let the bernoulli utility $u : X \to \mathbb{R}$ be differentiable with $u' > 0$ and assume $X \subseteq \mathbb{R}$. Consider an investor with preferences represented by I, that is buying or selling an asset $R : S \to \mathbb{R}$ at a price P. The problem is given by

$$\max_{t\in\mathbb{R}} I(u(h+t(R-P))) \tag{3}$$

A necessary (and sufficient if $t\mapsto I(u(h+t(R-P)))$ is concave) condition for optimality of no-trade at the status quo, is 0 to be an element of the Clarke differential of $t\mapsto I(u(h+t(R-P)))$ at $t=0$. By the chain rule for Clarke derivative (see Clarke [9, Remark 2.3.11]), this is equivalent to

$$E_{q^*}[u'(h)(R-P)] = 0 \quad \text{for some } q^* \in \partial^C I(u\circ h)$$

that after normalization is $\mathscr{C}$, the set of globally relevant priors.

Then no-trade is optimal if the asset's price falls in the implicit interval given by,

$$\min_{p\in\mathscr{C}} E_p\left[u'(h)(R-P)\right] \le 0 \le \max_{p\in\mathscr{C}} E_p\left[u'(h)(R-P)\right] \tag{4}$$

The previous inequalities are the key step for all the next results. First note that, if $h\in X$, then $u'(h)>0$ and we have a condition similar to Ghirardato and Siniscalchi [16] and Dow and Werlang [10], that is:

$$\min_{p\in\mathscr{C}} E_p[R] \le P \le \max_{p\in\mathscr{C}} E_p[R] \tag{5}$$

hence, there is a non-degenerated interval of prices under which no-trade is optimal. As noted above, it is the biggest possible due to the fact that h is a status quo. We can conclude that a DM with status quo, is less willing to trade than one without status quo (when they have the same ambiguity perception). Despite their similarity with the usual no-trade results as in Dow and Werlang [10], inequalities (5) are much more general since no assumptions on the functional specification of I are made, beyond locally Lipschitz continuity.

In the case of a non-constant status quo h, from Eq. (4), it easy to see that trade is still optimal when $u'(h)(R-P)$ tracks the utility of a crisp act even thought R is *not* perceived as crisp. More precisely:

Observation 1. *Trade is optimal if there exists an act g such that $u\circ g = u'(h)(R-P)$ and $p(u\circ g) = k$ for all $p\in\mathscr{C}$.*

Indeed, the condition of Observation 1 implies:

$$\min_{p\in\mathscr{C}} E_p\left[u'(h)(R-P)\right] = 0 = \max_{p\in\mathscr{C}} E_p\left[u'(h)(R-P)\right]$$

and the price is given by,

$$E_p\left[\frac{u'(h)R}{E_p[u'(h)]}\right] = P$$

by simple a covariance formula

$$E_p[R] + \frac{\mathrm{Cov}_p(u'(h),R))}{E_p[u'(h)]} = P \tag{6}$$

So that the bid/ask spread disappears. This is not possible in the Dow and Werlang [10] setting and more generally with a constant status quo. Hence the interplay of a non-constant status quo and the ambiguity of an asset can lead to optimality of trade. Furthermore the price P in Eq. (6) can be higher or lower than the (subjective) expected return of the asset, according to its positive or negative covariance with the status quo's marginal utility.

The above considerations enforce the intuition of Masatlioglu and Ok [31] and the citation of Dow and Werlang [10], regarding the relation between no-trade results and the endowment effect. The endowment effect is stronger when a DM compares an asset to a constant (status quo), whereas portfolio optimization with non-constant status quos may result in a optimality of trade. This remarks the separation between endowment effect (WTA/WTP gap) and status quo bias.

8.2 The case of multiple status quos

The rationale behind no trade price intervals is the discrepancy, under ambiguity, of the highest price at which an agent would buy the asset, given by $I(u(h+R))$ and the lowest price at which she would go short, given by $-I(-u(h+R))$. In the case of a constant status quo $h\in X$ and MEU preferences, it reduces to the familiar Dow and Werlang [10] or Chateauneuf and Ventura [8] condition, $I(u(R)) \le -I(-u(R))$.

In this section we assume that the DM "uses" two status quos, one constant and the other non-constant, to evaluate long and short positions. Buying an asset is taken as a departure from a constant (endowment) status quo, whereas going short, is considered as a departure from a non-constant (portfolio) status quo. In this case the objective function becomes

$$\begin{cases} t \longmapsto I(u(w+t(R-P))) & \text{if } t \geq 0 \\ \\ t \longmapsto -I(-u(h+t(R-P))) & \text{if } t \leq 0 \end{cases} \tag{7}$$

Since we are assuming that w, h are status quos, $C(h) = C(w) = \mathscr{C}$. Optimality of no-trade is equivalent to P solving the following system of equations

$$\begin{cases} P \geq \min_{p \in \mathscr{C}} E_p[R] \\ \\ 0 \leq \max_{p \in \mathscr{C}} E_p\left[u'(h)(R-P)\right] \end{cases} \tag{8}$$

Consider the special case of a DM does not consider the asset R as ambiguous. The striking result is that *no-trade could be optimal even if the DM perceives the asset R as crisp*, if the marginal utility of keeping the non-constant status quo and the asset are "ambiguously" correlated. In that case, we assume the price P to be subjectively fair, $P = E_p[R]$ for all $p \in \mathscr{C}$, and the second inequality becomes

$$\max_{p \in \mathscr{C}} E_p\left[u'(h)(E_p(R) - R)\right] \geq 0$$

and no-trade is optimal if

$$\max_{p \in \mathscr{C}} \mathrm{Cov}_p(u'(h), R) \leq 0$$

Namely, if the covariance of the non-constant (and possibly ambiguous) status quo and the asset is negative for some $p \in \mathscr{C}$, no-trade is possible, even if the asset *per se* is crisp.

Appendix A.

Conic preorders

The first result is a representation of nontrivial, conic and monotonic preorders on $B_0(\Sigma, K)$ for an open $K \subset \mathbb{R}$. First, a preorder $a \trianglerighteq b$ is *monotonic* if $a(s) \geq b(s) \Rightarrow a \trianglerighteq b$. It in *conic* (or affine) if $a \trianglerighteq b \Rightarrow \gamma a + (1-\gamma)c \trianglerighteq \gamma b + (1-\gamma)c$ for all $c \in B_0(\Sigma, K)$ and $\gamma \in (0,1)$. It is *continuous* if $a^n \to a, b^n \to b$ and $a^n \trianglerighteq b^n$ implies $a \trianglerighteq b$. It is *nontrivial* if for some a, b, $a \trianglerighteq b$ and not $b \trianglerighteq a$. Consistently with the previous notation, for a $\mathscr{D} \subseteq \Delta(S)$ we write $a \trianglerighteq_{\mathscr{D}} b$, if and only if, $p(a) \geq p(b)$ for all $p \in \mathscr{D}$.

Next theorem gives a quasi-Bewley representation to nontrivial conic preorders.

Theorem 18. *Assume $\trianglerighteq$ is a nontrivial, conic and monotonic preorder on $B_0(\Sigma, K)$, then:*

(1.) for all $a, b \in B_0(\Sigma, K)$, $a \trianglerighteq b$ implies, $p(a) \geq p(b)$ for all $p \in \mathscr{C}$, for some $\mathscr{C} \subseteq \Delta(S)$

(2.) $\trianglerighteq_{\mathscr{C}}$ is the smallest continuous preference relation that extends $\trianglerighteq$.

Proof. Let $k \in \operatorname{int} K$ and set for all $a, b \in B_0(\Sigma, K-k)$, $a \trianglerighteq' b$ if and only if $a + k \trianglerighteq b + k$. It follows that $\trianglerighteq'$ is a nontrivial, conic and monotonic preorder on $B_0(\Sigma, K-k)$. Now we introduce a Fact due to Gilboa et al. [20]:

Fact 1. Given any $a, b \in B_0(\Sigma, K)$, the following are equivalent:

(i) $a \trianglerighteq' b$.

(ii) There exists $\gamma > 0$ such that $\gamma a, \gamma b \in B_0(\Sigma, K)$ and $\gamma a \trianglerighteq' \gamma b$.

(iii) $\gamma a \trianglerighteq' \gamma b$ for all $\gamma > 0$ such that $\gamma a, \gamma b \in B_0(\Sigma, K)$.

Now for any $a, b \in B_0(\Sigma)$, let define $a \trianglerighteq'' b$ if and only if $\gamma a \trianglerighteq' \gamma b$ for some $\gamma > 0$. Then it easy to see that $\trianglerighteq''$ is a nontrivial, conic and monotonic preorder on $B_0(\Sigma)$. Now let

$$C^\circ \triangleq \{p \in ba(\Sigma) : p(a) \geq 0,\ \forall a \trianglerighteq'' 0\}$$

It is the polar of the cone $C \triangleq \{a \in B_0(\Sigma) : a \trianglerighteq'' 0\}$, hence C° is a weak*-closed convex cone. Obviously, $0 \in C^\circ$ and for all $A \in \Sigma$, $1_A \trianglerighteq'' 0$ by monotonicity of $\trianglerighteq''$, then $p(1_A) \geq 0$, hence C° contains nonnegative elements of $ba(\Sigma)$. Now, $a - b \trianglerighteq'' 0$ implies $p(a-b) \geq 0$ hence, $p(a) \geq p(b)$ for all $p \in C^\circ$. Nontriviality implies $C \neq \{0\}$, then a normalization gives

$$a \trianglerighteq'' b \Leftrightarrow \frac{p(a)}{p(S)} \geq \frac{p(b)}{p(S)},\ \ \forall p \in C^\circ \setminus \{0\}$$

Define C_1° the set of probability measures in C°, it is a weak*-closed and convex set and we have $a \trianglerighteq'' b \Rightarrow p(a) \geq p(b)$ for all $p \in C_1^\circ$. This trivially implies part (1.) defining $\mathscr{C} \triangleq C_1^\circ$. For part (2.), let $\overline{C}$ the closure of C, i.e. the intersection of all closed cones containing C. Hence, $\overline{C}$ is a closed and convex cone, let $\trianglerighteq^\circ$, be the preorder generated by $\overline{C}$ on $B_0(\Sigma)$, i.e. $a \trianglerighteq^\circ b \Leftrightarrow a - b \in \overline{C}$, it is a nontrivial (since contains $\trianglerighteq''$), conic (by definition) and monotonic (since $a \geq b$ implies $a - b \geq 0$ and $0 \in \overline{C}$) preorder. It is also continuous, indeed, let $a_\alpha \to a$ and $b_\beta \to b$, suppose $a_\alpha \trianglerighteq^\circ b_\beta$ for all α, β, then $a_\alpha - b_\beta \in \overline{C}$, since it is closed $a - b \in \overline{C}$, hence $a \trianglerighteq^\circ b$, showing continuity. Then by Ghirardato et al. [18] there exists a weak* closed an convex set $\mathscr{C}^\circ$ such that

$$a \trianglerighteq^\circ b \iff p(a) \geq p(b),\ \forall p \in \mathscr{C}^\circ$$

Now let $a \trianglerighteq_{\mathscr{C}} b$ the Bewley's preference generated by $\mathscr{C}$ of point (1.). Since $\trianglerighteq^\circ$ is the smallest continuous extension of $\trianglerighteq$, $a \trianglerighteq^\circ b$ implies $a \trianglerighteq_{\mathscr{C}} b$, therefore, by Ghirardato et al. [18], $\mathscr{C} \subseteq \mathscr{C}^\circ$. Assume the inclusion is strict, hence there exists $p \in \mathscr{C} \setminus \mathscr{C}^\circ$. By standard separation argument, there is an $c \in B_0(\Sigma)$ such that $p(c) < 0 \leq p'(c)$ for all $p' \in \mathscr{C}^\circ$. This is equivalent to $c \trianglerighteq^\circ 0$ and $c \ntrianglerighteq_{\mathscr{C}} 0$ a contradiction. Therefore $\mathscr{C} = \mathscr{C}^\circ$.

□

A detour on general additivity

In this section we prove a useful additivity result for a general class of preferences including Variational and Confidence preferences. It will enter the subsequent characterization of a status quo for concave preferences, but it is of interest by its own, being a generalization of some results of Ghirardato et al. [17].

We say that preferences $\succcurlyeq$ is *concave* if, $\succcurlyeq$ is represented by I and I is (closed, proper) concave in the space of utilities. By well-known result of convex analysis, monotone, concave preferences are MBL (in the interior of their domain). Examples of concave preferences are MEU preferences, Variational, Confidence preference and Smooth Ambiguity with concave second order utility), Vector Expected utility with concave A. Now, define

$$\pi(u \circ f) \triangleq \operatorname*{argmin}_{p \in \Delta(S)} \left(\int_S u \circ f\, dp - I^*(p) \right)$$

it corresponds to $\Delta(S)\cap\partial I(u\circ f)$, the normalized superdifferential of I at $u\circ f$. Since I is concave, it is everywhere nice, i.e. the zero measure is never an element of the subdifferentials.

Lemma 19. *Suppose $\succcurlyeq$ is a concave MBL preference, then, for all $f,g\in\mathscr{F}^{\text{int}}$, the following are equivalent:*

(1.) $\pi(u\circ f)\cap\pi(u\circ g)\neq\emptyset$

(2.) $I(\gamma u\circ f+(1-\gamma)u\circ g)=\gamma I(u\circ f)+(1-\gamma)I(u\circ g)$, *for all* $\gamma\in[0,1]$.

Proof. Of Lemma 19. (1.) implies (2.). Assume $\pi(f)\cap\pi(g)$, since f,g are interior acts, it follows that $\partial I(u\circ f)\cap\partial I(u\circ g)\neq\emptyset$. Set $a=u\circ f$ and $b=u\circ g$ By definition of superdifferential

$$I(\gamma a+(1-\gamma)b)=\gamma I(a)+(1-\gamma)I(b)$$

for some $\gamma\in(0,1)$. Indeed, if $p\in\partial I(a)\cap\partial I(b)$,

$$I(c)\leq I(a)+p(c-a),\ \text{ and }\ I(c)\leq I(b)+p(c-b) \tag{9}$$

for all $c\in B(\Sigma)$. Then,

$$I(a)-I(b)=p(a-b) \tag{10}$$

If $0<\gamma<1$, by Eq. (9) and (10)

$$I(\gamma a+(1-\gamma)b)\leq I(b)+\gamma p(a-b)=\gamma I(a)+(1-\gamma)I(b)$$

The reverse inequality follows from concavity of I.
(2.) implies (1.) Assume by contradiction $\pi(f)\cap\pi(g)=\emptyset$. Take $p\in\pi(\gamma f+(1-\gamma)g)$, then $p\notin\pi(f)$ or $p\notin\pi(g)$, so $I(u\circ f)\leq p(u\circ f)-I^*(p)$ and $I(u\circ g)\leq p(u\circ g)-I^*(p)$ and one of these inequalities must be strict. Then:

$$\begin{aligned}I(\gamma u\circ f+(1-\gamma)u\circ g)=&p\left(\gamma u\circ f+(1-\gamma)u\circ g\right)-I^*(p)\\>&\gamma\left(p(u\circ f)-I^*(p)\right)+(1-\gamma)\left(p\left(u\circ g\right)-I^*(p)\right)\\\geq&\gamma I(u\circ f)+(1-\gamma)I(u\circ g)\end{aligned}$$

a contradiction. □

Lemma 19 gives necessary and sufficient conditions for additivity with general preferences[20]. A first consequence for our theory of status quo, is that if an interior h is a status quo, then a concave preference $\succcurlyeq$ satisfies independence with respect to h, more precisely, from point (2.) it follows that for $f\sim g\in\mathscr{F}^{\text{int}}$ and all $\gamma\in(0,1)$

$$f\sim g\implies\gamma f+(1-\gamma)h\sim\gamma g+(1-\gamma)h$$

However, the reverse implication does not holds. Assuming such independence does not imply h to be a status quo, unless $\succcurlyeq$ is MaxMin.

Clarke subdifferential and beyond

Definition 20. *Let $U\subset B$ where B is either $B_0(\Sigma,\Gamma)$ or $B(\Sigma,\Gamma)$, with* $\operatorname{int}\Gamma\neq\emptyset$, *an open subset a metric space. Let I a locally Lipschitz functional $I:U\to\mathbb{R}$. For every $a\in U$ and $b\in B$, the*

[20] It can be proved that Lemma 19 is valid for all $f,g\in\mathscr{F}$, rather then only for interior acts. However, this version is sufficient for our results.

Clarke (upper) derivative of I in a in the direction b is

$$I^{\circ}(a;b) = \limsup_{\substack{c\to a\\ t\downarrow 0}} \frac{I(c+tb)-I(c)}{t}$$

The Clarke (sub)differential of I at a is the set

$$\partial^C I(a) = \left\{\mu \in ba(\Sigma) : \int b d\mu \leq I^{\circ}(a;b), \forall b \in B\right\}$$

Next proposition collects some useful properties of Clarke differential (see Ghirardato et al. [18]).

Proposition 21. *Let $I : B \to \mathbb{R}$ be locally Lipschitz. Then:*

1. *$\partial^C I(a)$ is a nonempty, convex, weak* compact subset of $ba(\Sigma)$.*
2. *For every $a \in \operatorname{int} B$, $I^{\circ}(a;b) = \max_{\mu\in\partial^C I(a)} \int a d\mu$*
3. *If I is convex, $\partial^C I(a) = \partial I(a)$ (the usual subdifferential).*
4. *(Lebourg mean value theorem) For any $a, b \in B$, there exists $\gamma \in (0,1)$, such that*

$$I(a) - I(b) \in \langle \partial I(\gamma a + (1-\gamma)b), a-b\rangle$$

The Clarke (upper) derivative can also be rewritten as:

$$I^{\circ}(a;b) = \lim_{\epsilon\to 0}\left(\sup_{\substack{\|c-a\|\leq\gamma\epsilon\\ 0<t<\gamma\epsilon}} \frac{I(c+tb)-I(c)}{t}\right)$$

then:

Lemma 22. *If I is a translation invariant functional and $a = b + k$ for some $k \in \mathbb{R}$, then $\partial^C I(a) = \partial^C I(b)$*

Proof. If I is translation invariant it (globally) Lipschitz and, for all $k \in \mathbb{R}$, and $b, c \in B(\Sigma)$, $I(c+tb) - I(c) = I(c-k+tb) - I(c-k)$. Then

$$\begin{aligned}
I^{\circ}(a+k;b) &= \lim_{\epsilon\to 0}\left(\sup_{\substack{\|c-a-k\|\leq\gamma\epsilon\\ 0<t<\gamma\epsilon}} \frac{I(c+tb)-I(c)}{t}\right)\\
&= \lim_{\epsilon\to 0}\left(\sup_{\substack{\|c'-a\|\leq\gamma\epsilon\\ 0<t<\gamma\epsilon}} \frac{I(c+tb)-I(c)}{t}\right)\\
&= \lim_{\epsilon\to 0}\left(\sup_{\substack{\|c'-a\|\leq\gamma\epsilon\\ 0<t<\gamma\epsilon}} \frac{I(c'+tb)-I(c')}{t}\right) = I^{\circ}(a;b)
\end{aligned}$$

Assume $a = b + k$, then

$$\max_{\mu\in\partial^C I(a)} \int c d\mu = I^{\circ}(a;c) = I^{\circ}(a+k;c) = I(b;c) = \max_{\mu\in\partial^C I(b)} \int c d\mu$$

then $\partial^C I(a) = \partial^C I(b)$. $\square$

From the previous lemma, it follows that $\partial^C I(0) = \partial^C I(k)$ for all $k \in \mathbb{R}$

Now, we introduce some useful results about the relation between Clarke (sub)differential and the Greenberg-Pierskalla (super)differential, which is commonly used in quasi-convex analysis.

Definition 23. *Let $I : B \to \mathbb{R}$ a monotone sup-norm continuous quasi-convex function, the Greenberg-Pierskalla (super)differential of I at a, written $\partial^{GP} I(a)$ is defined as:*

$$\partial^{GP} I(a) \triangleq \left\{ p \in ba^+(\Sigma) : \int a dp \geq \int b dp \implies I(a) \geq I(b) \right\}$$

By Cerreia-Vioglio et al. [7, Proposition 11], if $\Gamma = \mathbb{R}$, the normalized (i.e. $\partial^{GP} I(a) \cap \Delta(S)$) Greenberg-Pierskalla (super)differential of I at a can be written as

$$\pi(a) = \operatorname*{arginf}_{p \in \Delta(S)} G\left(\int a dp, p \right) \tag{11}$$

Appendix B. Proofs.

Proof. Of Proposition 2. Given a MBL preference with representation (I, u) and $h \in \mathscr{F}$, define an order relation $\trianglerighteq_h$ on $B_0(\Sigma, u(X))$ by:

$$a \trianglerighteq_h b \Leftrightarrow f \succcurlyeq_h g$$

and $a = u \circ f, b = u \circ g$. Since $\succcurlyeq_h$ is non-trivial [21], monotonic and independent pre-order, so is $\trianglerighteq_h$, then the conclusion follows from Theorem 18. □

Proof. Of Theorem 4. By definition $f \succcurlyeq^* g \Rightarrow f \succcurlyeq_h g$ for all $h \in \mathscr{F}$. Now, the definition of status quo, implies the reverse implication, hence, $\succcurlyeq_h$ and $\succcurlyeq^*$ are equivalent. Since $\succcurlyeq^*$ is non-trivial, $\succcurlyeq_h$ is non-trivial as well, hence they coincides on X, so a bernoulli utility function v representing $\succcurlyeq_h$ exists. By a normalization choose $v = u$. Then, $f \succcurlyeq_h g$ if and only if $p(u \circ f) \geq p(u \circ g)$ for all $p \in \mathscr{C}$. By Proposition 2, $f \succcurlyeq_h g \Rightarrow f \succcurlyeq_{C(h)} g$, by Ghirardato et al. [18, Prop. A1], $C(h) \subseteq \mathscr{C}$. Let's prove the "if" part. For the converse implication ($f \succcurlyeq_{C(h)} g \Rightarrow f \succcurlyeq^* g$), consider $f \succcurlyeq_{C(h)} g$. Assume first that f, g are interior. Then for an arbitrary $\epsilon > 0$, $p(u \circ f + \epsilon) > p(u \circ g - \epsilon)$ for all $p \in C(h)$, by definition $f' \succcurlyeq_h g'$ where $u \circ f' = u \circ f + \epsilon$ and $u \circ g' = u \circ g - \epsilon$. By definition of status quo, we have $f' \succcurlyeq^* g'$ and then $p(u \circ f') \geq p(u \circ g')$ for all $p \in C$, or equivalently, $p(u \circ f) + \epsilon \geq p(u \circ g) - \epsilon$ for all $p \in \mathscr{C}$, since ϵ was arbitrarily chosen, by continuity of $\succcurlyeq^*$, it follows that $p(u \circ f) \geq p(u \circ g)$, $\forall p \in \mathscr{C}$, hence $f \succcurlyeq^* g$. If f, g are not interior, let x be an interior constant, then $p(\gamma f + (1-\gamma)x) \geq p(\gamma g + (1-\gamma)x)$ for all $p \in \mathscr{C}$ and $\gamma \in (0,1)$, by the previous point $\gamma f + (1-\gamma)x \succcurlyeq^* \gamma g + (1-\gamma)x$. By continuity, $f \succcurlyeq^* g$. Then, by Ghirardato et al. [18, Prop. A1], $\mathscr{C} \subset \overline{\text{co}}(C(h))$, and the result follows since $C(h)$ is weak* compact and convex.

For the "only if". Suppose $\mathscr{C} = C(h)$. Then, $\succcurlyeq^* = \succcurlyeq_{C(h)}$. Suppose $f \succcurlyeq_h g$, then, by Proposition 2, $f \succcurlyeq_{C(h)} g$, so $f \succcurlyeq^* g$. □

Proposition 6 is a particular case of Proposition 8.

Proof. Of Proposition 8. If h is a status quo, then $C(h) = \mathscr{C}$ and so $\mathscr{A} = \mathscr{U}(h)$. Moreover $C(h') \subseteq C(h)$ for all h'. It follows that $\mathscr{U}(h) \subseteq \mathscr{U}(h')$, so $\mathscr{U}(h) = \bigcap_{h' \in \mathscr{F}} \mathscr{U}(h') = \mathscr{U}$. If $f \sim_h x$, then $p(u \circ f) = u(x)$ for all $p \in C(h)$, then by point (2) of Proposition 7 it belongs to $\mathscr{U}(h) = \mathscr{A}$. □

[21] $\trianglerighteq_h$ can be equivalently defined as: $a \trianglerighteq_h b \Leftrightarrow I(\gamma^n a + (1-\gamma^n u \circ h^n) \geq I(\gamma^n b + (1-\gamma^n) u \circ h^n)$, eventually for all $h^n \to h$, and $\gamma^n \to 0$.

Next proposition shows that, when $h \in \mathscr{F}^{\text{int}}$, the local priors obtained in the differential characterization is equal to that obtained in the quasi-Bewley characterization of Proposition 2.

Proposition 24. *For all* $h \in \mathscr{F}^{\text{int}}$, $C(h) = \overline{\text{co}}\left(\left\{\frac{p}{p(S)} : p \in \partial^C I(u \circ h), p(S) > 0\right\}\right)$

Proof. Of Proposition 24. Let $D(h)$ the set in the right-hand side of the equation above. By Ghirardato and Siniscalchi [16, Theorem 6], for any $h \in \mathscr{F}^{\text{int}}$, $\succcurlyeq_{D(h)}$ is the smallest Bewley's extension of $\succcurlyeq_h$. By Theorem 2, $\succcurlyeq_{C(h)}$ is also the smallest continuous extension of $\succcurlyeq_h$ for all $h \in \mathscr{F}$, then they coincide on $h \in \mathscr{F}^{\text{int}}$. This implies $C(h) = D(h)$. □

Proof. Of Theorem 9. Suppose $C(h) = \mathscr{C}$ for some $h \in \mathscr{F}^{\text{int}}$. Smoothness of I means that for all $h' \in \mathscr{F}^{\text{int}}$, $\partial^C I(u \circ h') = \nabla I(h') = \{p_{h'}\}$ for some $p_{h'} \in ba^+(\Sigma)$. If $p_h = 0$ (the zero measure), then $C(h) = \emptyset$, then $C(h') = \emptyset$ for all $h' \in \mathscr{F}^{\text{int}}$. It follows that $\partial^C I(u \circ h') = \{0\}$ for all $h' \in \mathscr{F}^{\text{int}}$, then $I = k$, contradicting nontrivality. Then, $p_h \neq 0$, for a status quo $C(h) = \frac{p_h}{p_h(S)}$, and $C(h') \subseteq C(h)$ implies $p_h = p_{h'}$ for all $h' \in \mathscr{F}^{\text{int}}$. It follows that, $\partial^C I(u \circ h) = \partial^C I(u \circ h') = \{p_h\}$ and this implies I to be linear (expected utility) on $\text{int}\, B_0(\Sigma, u(X))$. By continuity it is linear in the whole space. The converse implication is trivial. □

Proof. Of Theorem 10. Assume $h \succcurlyeq g$, then, $I(u \circ h) - I(u \circ g) \geq 0$, by Lebourg mean value theorem, for some $\gamma \in (0,1)$, there exists $p \in \partial^C I(\gamma u \circ h + (1-\gamma) u \circ g)$ such that, $p(u \circ h) \geq p(u \circ g)$, since h is an status quo, $\partial^C I(\gamma u \circ h + (1-\gamma) u \circ g) \subseteq \partial^C I(u \circ h)$, then $C(\gamma f + (1-\gamma) g) \subseteq C(h)$ for all $f, g \in \mathscr{F}^{\text{int}}$, then $h \succcurlyeq^\circ g$. For the converse, we need first to the following fact:

Fact 2. When $\succcurlyeq$ admits an interior status quo h then, $\pi(u \circ h) = C(h)$.

Proof. Of Fact 2. By Cerreia-Vioglio et al. [7, Proposition 11] and Theorem 4, if h is a status quo, then $\pi(u \circ h) = \mathscr{C} = C(h)$, where $\pi(u \circ h)$ is defined as Eq. (11). □

Now assume $h \succcurlyeq^\circ g$, and $\succcurlyeq$ is UAP. By definition, for some $p \in \mathscr{C} = C(h)$, $p(u \circ h) \geq p(u \circ g)$, since $C(h)$ correspond to the the normalized Greenberg-Pierskalla subdifferential of I at $u \circ h$, $p \in C(h)$ is such that $p(u \circ h) \geq p(u \circ g)$ implies $h \succcurlyeq g$. □

Proof. Of Proposition 11. If $g \not\succcurlyeq_h h$, then for some $p \in C(h)$, $p(u \circ h) > p(u \circ g)$, since $C(h)$ coincides with the normalized Greenberg-Pierskalla subdifferential of I at $u \circ h$ by Fact 2, we have that $p \in \mathscr{C}$ is such that $p(u \circ h) \geq p(u \circ g)$ implies $h \succcurlyeq g$. Hence the result. □

Proof. Of Theorem 12. If h is a status quo, the $C(h) = \mathscr{C} = \text{cl}(\text{dom}_\Delta G)$, by definition. Then, by Cerreia-Vioglio [5, Theorem 4] h-Caution holds. For the reverse implication, it is sufficient to prove that h-Caution implies $C(h) = \mathscr{C}$. The inclusion $C(h) \subseteq \mathscr{C}$ follows from definition. For the reverse inclusion. Define

$$\hat{G}(t,p) = \begin{cases} G(t,p) & \text{if } p \in C(h) \\ \infty & \text{otherwise} \end{cases}$$

then $\min_{p \in \Delta(S)} \hat{G}\left(\int a\, dp, p\right) = \min_{p \in C(h)} G\left(\int u \circ g\, dp, p\right)$, and $\text{cl}(\text{dom}_\Delta \hat{G}) = C(h)$. Now suppose that $\mathscr{C} \neq C(h)$, then for some $g \in \mathscr{F}$, we have $I(u \circ g) > \min_{p \in C(h)} G\left(\int u \circ g\, dp, p\right)$, so that there is an $x \in X$ such that

$$I(u \circ g) > u(x) > \min_{p \in C(h)} G\left(\int u \circ g\, dp, p\right) \tag{12}$$

The first inequality implies $g \succ x$. The second implies that $g \not\succcurlyeq_{C(h)} x$. Indeed, if $g \succcurlyeq_{C(h)} x$, then $p(u \circ g) \geq u(x)$ for all $p \in C(h)$. Since $G(\cdot, p)$ is increasing for all $p \in \Delta(S)$, $G(p(u \circ g), p) \geq u(x)$ for all $p \in C(h)$, or equivalently, $\min_{p \in C(h)} G\left(\int u \circ g\, dp, p\right) \geq u(x)$, so Eq. (12) implies $g \not\succcurlyeq_{C(h)} x$. By h-Caution, $x \succcurlyeq g$ a contradiction. □

Proof. Of. Proposition 13. Assume h is an interior status quo, then $\mathscr{C} = C(h) = \pi(u \circ h)$. For any $f \in \mathscr{F}^{\text{int}}$, $\pi(u \circ f) \cap \pi(u \circ h) \neq \emptyset$. By Lemma 19, for all $\gamma \in [0,1]$, $I(\gamma u \circ f + (1-\gamma) u \circ h) = \gamma I(u \circ f) + (1-\gamma) I(u \circ h)$. Take $g \in \mathscr{F}^{\text{int}}$, with $g \sim f$, then

$$\begin{aligned} I(\gamma u \circ f + (1-\gamma) u \circ h) =& \gamma I(u \circ f) + (1-\gamma) I(u \circ h) \\ =& \gamma I(u \circ g) + (1-\gamma) I(u \circ h) \\ =& I(\gamma u \circ g + (1-\gamma) u \circ h) \end{aligned}$$

Then $\gamma f + (1-\gamma) h \sim \gamma g + (1-\gamma) h$, so h-independence holds. □

Proof. Of Theorem 14. (1.) implies (2.). Assume h is a status quo but not crisp. Then there exists $p, p' \in \mathscr{C}$ such that $p(u \circ h) \neq p'(u \circ h)$. W.l.o.g. let $p(u \circ h) > p'(u \circ h)$, then $p \notin \pi(u \circ h)$. Since h is a status quo, $\pi(u \circ h) = \mathscr{C}$, then $p \notin \mathscr{C}$. A contradiction.
(2) implies (1). If h is crisp, then it is a status quo. Assume $\succcurlyeq$ is a MaxMin preference, then take a crisp h, then $I(u \circ h) = p(u \circ h)$ for all $p \in \mathscr{C}$. The normalized superdifferential of I at $u \circ h$ (i.e. $C(h)$) is given by[22]

$$\Delta(S) \cap \partial I(u \circ h) = \left\{ p \in \Delta(S) : p(u \circ f) - p(u \circ h) \geq I(u \circ f) - I(u \circ h), \text{ for all } u \circ f \in B_0(\Sigma) \right\}$$

which is equal to

$$\Delta(S) \cap \partial I(u \circ h) = \left\{ p \in \Delta(S) : p(u \circ f) \geq I(u \circ f), \text{ for all } u \circ f \in B_0(\Sigma) \right\} = \Delta(S) \cap \partial I(0)$$

but for homothetic preferences $\Delta(S) \cap \partial I(u \circ h) = \Delta(S) \cap \partial I(0) = \mathscr{C}$, then h is a status quo by Theorem 4.
The equivalence of (3), follows from Ghirardato et al. [18, Prop. 10].
□

Before proving Proposition 16, we need some definitions and results.

Define a structure (see Luce [27] for a similar definition) $(u(X), \mathscr{A}, \succcurlyeq)$, to be *finitely unique*, if and only if, there exists an integer N such that any automorphism in $\mathscr{A}$ with N or more fixed points is the identity. If N is the least integer for which this is true, then the structure is called *N-point unique*. Clearly preferences represented by a positively homogeneous functional are 1-point unique, whereas preferences satisfying constant linearity are 2-point unique.

Lemma 25. *Let $(u(X), \mathscr{A}, \succcurlyeq)$ be a N-finite structure and $h, h' \in \mathscr{F}$ are singular acts with respect to $\mathscr{A}$, then h and h' are essentially equal*

Proof. Of Lemma 25. Assume the existence of h, h' and both are singular. Assume $h \neq h'$, then take any non-null event $E \in \Sigma$ and define the act hEh', it follows that $u(hEh') = A \circ u(hEh')$ for all $A \in \mathscr{A}$, then hEh' is also singular, proceeding inductively we can define countably many singular acts, but this contradict N-finiteness of $(\mathscr{F}, \succcurlyeq)$. □

Proof. Of Proposition 16. It is a trivial application of the previous Lemma. □

Proof. Of Theorem 17. (1.) Assume $\succcurlyeq$ is invariant w.r.t. the positive similarities, then $A \circ u \circ h = u \circ h$ is equivalent to $k u \circ h = u \circ h$ for all $k \geq 0$. But this is true only if $u \circ h(s) = 0$ for all $s \in S$, hence $h \in X$ and $u(h) = 0$. (2.) Assume $\succcurlyeq$ is invariant w.r.t. the translations, then $A \circ u \circ h = u \circ h$ is equivalent to $u \circ h + k = u \circ h$ for all $k \in \mathbb{R}$, but this is true only if $u \circ h(s) = \infty$ for all $s \in S$, hence $h \notin \mathscr{F}^{\text{int}}$. (3.) Follows form point (2.). □

[22] Since monotone concave functionals are everywhere nice, then $C(h) = \left\{ \frac{p}{p(S)} : p \in \partial I(u \circ h), p(S) > 0 \right\} = \Delta(S) \cap \partial I(u \circ h)$.

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