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Preempting versus Postponing: the Stealing Game

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Abstract

We present an endogenous timing game of action commitment in which players can steal from each other parts of a homogeneous and perfectly divisible pie and the expected effectiveness of a player's theft is proportional to the amount he currently owns. We show how the incentives to preempt or to follow the rivals change with the number of players involved in the game and we investigate the conditions that lead to the occurrence of symmetric or asymmetric equilibria.

Keywords: stealing, endogenous timing games.

JEL Classification: C72, C73.

1 Introduction

Ann owns part of a pie. Bob owns the remaining part, so there is no free pie. The interests of the two players are in conflict as their goal is to hold the majority of the pie at a specific and predetermined point of time in the future. Given that

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trading or bargaining mechanisms are illegal, not enforceable, or simply not in the interest of the parts, the only way Ann and Bob can try to meet their objective is by “stealing” part of the other’s pie. Moreover, the expected effectiveness of such a theft is proportional to the share of the pie that the robber already owns.

This may seem to be the description of a highly specific environment but it broadly fits a number of situations. For example, political parties or candidates struggle to conquer the opponents’ voters with the goal of having the majority at the election date. And given that parties or candidates that already enjoy a large consensus are usually able to raise more funds, there is indeed a positive relationship between a party’s current share and the expected effectiveness of its campaign.¹

To capture a stylized version of situations of this kind we introduce what we call the stealing game. The stealing game is a timing game in which a number of agents can steal from each other parts of a homogeneous and perfectly divisible pie. Agents must decide when and who to rob with the goal to finish the game being the leader, i.e., the player that holds the largest share of the pie.

We model the stealing game as an endogenous timing game of action commitment that follows the structure introduced by Hamilton and Slutsky (1990). Such a structure provides a simple but fruitful framework to study the issue of endogenous timing and the value of action commitment.² Our main goal is to solve for the optimal timing strategies of the agents. We want to find out when is the best moment for a player to behave aggressively and steal part of the pie owned by the rivals. Such a decision is affected by the existence of an intuitive trade-off between preempting or postponing one’s move. A player who moves as soon as possible eliminates the possibility of being preempted but he is then forced to passively suffer the potential

¹A similar analogy holds in business where competing firms aim to steal the opponents’ customers and large firms can afford to launch more expensive advertising campaigns.

²For instance, it has been successfully applied in a series of papers that study the robustness of commitment equilibria (van Damme and Hurkens 1996) and the endogenous emergence of a Stackelberg leader in the presence of cost asymmetries (van Damme and Hurkens 1999) and of price competition (van Damme and Hurkens 2004) in duopolies.

retaliation of those who waited. On the other hand, a player who postpones his move can play the best response but faces the risk of being preempted by a rival. And because of the rules of the game, when a player is robbed his market share goes down and so does the expected effectiveness of his stealing attempt.

We characterize the equilibria of the game under different specifications about the number of players, the duration of the game and the number of stealing attempts players are endowed with. We start by explicitly solving a two periods stealing game in which players have a unique stealing opportunity. Despite its simplicity, this setting highlights the strategic peculiarities of the game and shows how the above mentioned trade-off has different solutions depending on the number of participants. No player postpones his move when the game is played among two agents. The game with three players displays instead different Pareto equivalent equilibria but only those in which all the players postpone their moves survives equilibrium selection based on the concept of \mathbf{p}_u -dominance (a generalization of the concept of risk-dominance to games with more than 2 players, see Harsanyi and Selten, 1988). Finally, when the number of players is larger than three we show that the number of preempting equilibria is strictly larger than the number of postponing equilibria and that asymmetric equilibria may also occur. We then generalize these results to a setting in which n players have K stealing opportunities in a stealing game that lasts for $T > K$ periods.

The stealing game belongs therefore to different categories of timing games commonly studied in economics. The case with two players belongs to the class of preemption games. These are games in which it is better to anticipate the rivals; famous examples are the Stackelberg quantity game (Von Stackelberg 1934) and the centipede game (Rosenthal 1981). The case with three players is instead more similar to a war of attrition (Maynard Smith 1974), a strategic situation in which preempting the others hurts. Finally, the case with $n > 3$ players displays features that are common to both archetypes of timing games. Here lies one of the peculiar-

ities of the stealing game. In fact, a general characteristic of timing games is that optimal timing strategies depend on the payoff structure and not on the number of participants.³ The stealing game provides instead an example of a game in which optimal timing strategies change as a function of the number of players.

The paper is organized as follows: Section 2 formally introduces the stealing game. Section 3 defines the equilibria of the game when players have a unique stealing opportunity and there are only two periods. Section 4 extends some of the results to a more general setting. Section 5 concludes. The appendix collects the proofs of all the propositions that appear in the paper.

2 The stealing game

The stealing game is a game in which $n > 1$ risk-neutral players compete for the possession of a perfectly divisible resource whose size is constant and normalized to 1. Let N denote the set of players such that $|N| = n$. We indicate with $\pi_i^t \in [0, 1]$ the share of the resource that agent $i \in N$ holds at time t . Time is discrete and finite with $t \in \{0, 1, \dots, T\}$, $\pi_i^0 = \frac{1}{n}$ for any i (players are initially symmetric) and $\sum_i \pi_i^t = 1$ at any t .

The goal of the players is to be the largest shareholder at the end of the game. The only way in which a player can increase his holdings is by stealing part of the resource from someone else. More precisely, each player can costlessly steal $K \in \{1, \dots, T - 1\}$ times a certain amount of the resource from another player.⁴ The specific value of K is exogenously given and holds for all the players; we use the

³More recent literature about timing games has focused in generalizing former results (Bulow and Klemperer 1999), in providing a unified framework to study preemption games and wars of attrition (Park and Smith 2008) or in testing experimentally some of the theoretical results (Brunnermeier and Morgan 2010).

⁴As it will become clear in the course of the analysis, all the results presented in the paper would remain unchanged if players could steal *up to* K times as using all the stealing attempts would emerge as a dominant strategy. Similarly, the assumption that there are no explicit monetary costs associated with the decision to rob an opponent is made for simplicity and without loss of generality: all the results would remain valid as far as stealing costs do not trespass a certain threshold.

notation $k_i^t \in \{0, \dots, K\}$ to keep track of the number of thefts perpetrated by player i up to period t included.

The 2-tuple (x_i^t, y_i^t) describes the action taken by agent i at time t : $x_i^t \in \{\{N_{-i}\} \cup \emptyset\}$ indicates the opponent player i decides to rob ($\{N_{-i}\}$ is the set of all players except i and \emptyset is the empty set); y_i^t is the amount that player i steals. Obviously, if $x_i^t = \emptyset$ then $y_i^t = 0$. If on the other hand $x_i^t \in \{N_{-i}\}$ then the actual realization of y_i^t is a random variable defined over the support $[0, y_{\max}]$ with cumulative distribution function $F_{\pi_i^{t-1}}(\cdot)$.^{5,6} The notation $F_{\pi_i^{t-1}}(\cdot)$ indicates that the distribution of y_i^t depends on π_i^{t-1} . In fact, and in line with the reasons mentioned in the introduction, a key assumption of the model is that the expected effectiveness of a player's stealing technology increases with the amount of the resource the player already owns.

Assumption 1: if $\pi_i^{t-1} > \tilde{\pi}_i^{t-1}$ then $F_{\pi_i^{t-1}}(\cdot)$ first-order stochastically dominates $F_{\tilde{\pi}_i^{t-1}}(\cdot)$, i.e., $F_{\pi_i^{t-1}}(y_i^t) \leq F_{\tilde{\pi}_i^{t-1}}(y_i^t)$ for any $y_i^t \in [0, y_{\max}]$ and $F_{\pi_i^{t-1}}(y_i^t) < F_{\tilde{\pi}_i^{t-1}}(y_i^t)$ for some $y_i^t \in [0, y_{\max}]$.

Notice that the distribution $F_{\pi_i^{t-1}}(\cdot)$ depends on the player's current holding of the resource but is independent from the specific identity of player i as well as from the one of his "victim". For example, if $\pi_i^{t-1} = \pi_j^{t-1}$ then y_i^t and y_j^t are i.i.d. random variables with c.d.f. $F_{\pi_i^{t-1}}(\cdot)$. But if $\pi_i^{t-1} > \pi_j^{t-1}$ then the expected amount that player i can steal is larger than the one of player j . In fact, and because of first-order stochastic dominance, the relation $(\bar{y}_i^t > \bar{y}_j^t) \Leftrightarrow (\pi_i^{t-1} > \pi_j^{t-1})$ holds where $\bar{y}_l^t = \int_0^{y_{\max}} y_l^t dF_{\pi_l^{t-1}}(y_l^t)$ is the expected value of the variable y_l^t with $l \in \{i, j\}$.

⁵ y_{\max} , the upper bound of the support of y_i^t , belongs to the interval $(0, \frac{1}{K(n-1)})$ and it is exogenously given. This restriction ensures that there cannot be cases of excess demand, i.e., a player always gets what he steals.

⁶ Notice that the distribution of y_i^t can be discrete or continuous. For instance, $F_{\pi_i^{t-1}}(\cdot)$ could assign probability $p \in (0, 1)$ to the event $y_i^t = 0$ and $(1-p)$ to the event $y_i^t = \lambda$ with $\lambda \in (0, y_{\max}]$. That would model the case in which players' stealing attempts may fail.

We model the stealing game as a T -stages game of action commitment with endogenous timing (see Hamilton and Slutsky 1990). The rules are as follows:

Period 1: each player simultaneously chooses if to remain inactive (i.e., $x_i^t = \emptyset$) or if to rob an opponent of his choice (i.e., $x_i^t \in \{N_{-i}\}$). In the latter case the player steals the amount y_i^1 distributed on $[0, y_{\max}]$ according to $F_{\pi_i^0}(\cdot)$.

Period $t \in \{2, \dots, T\}$: each player is fully informed about the actions previously taken by all the opponents and observes the allocation $\{\pi_1^{t-1}, \dots, \pi_n^{t-1}\}$. A player who run out of stealing possibilities (i.e., $k_i^{t-1} = K$) is forced to remain inactive. A player who still has the possibility to steal (i.e., $k_i^{t-1} < K$) chooses if to remain inactive (i.e., $x_i^t = \emptyset$) or if to rob an opponent of his choice (i.e., $x_i^t \in \{N_{-i}\}$). In the latter case the player steals the amount y_i^t distributed on $[0, y_{\max}]$ according to $F_{\pi_i^{t-1}}(\cdot)$.

Payoffs: the player that at the end of the game holds the largest share of the resource gets a prize of size 1. The others get zero. If there is more than a market leader then the prize is equally shared among the winners.

More formally, payoffs take the following form:

$$u_i = \begin{cases} \frac{1}{1 + \sum_{j \neq i} (\mathbf{1}_{\{\pi_j^T = \pi_i^T\}})} & \text{if } \pi_i^T \geq \pi_j^T \text{ for any } j \neq i \\ 0 & \text{otherwise} \end{cases}$$

where π_i^t is recursively defined as $\pi_i^t = \pi_i^{t-1} + y_i^t - \sum_{j: x_j^t = i} y_j^t$ and $\pi_i^0 = \frac{1}{n}$ for any i . At any period $t \in \{1, \dots, T\}$ a player's holdings are thus the result of three components: the share he had in the previous period plus the amount he (possibly) steals from an opponent minus the amount he is (possibly) stolen from the other players.

In the course of the analysis, we will indicate with $s_i = (s_i^1, \dots, s_i^T)$, where $s_i^t \in \{N_{-i}\} \cup \emptyset$, a player's specific strategy and with $s = (s_i)_i$ a strategy profile. We adopt the formulation introduced in van Damme and Hurkens (1996) such that $s_i^t = j$ indicates the strategy "in period t rob the opponent j if none of the opponents

previously moved. Otherwise play best response”.⁷ We do not consider mixed strategies. Finally, we indicate with $\bar{\pi}_i^t$ and \bar{u}_i agent i 's expected market share at time t and his expected payoff. Notice in fact that players are uncertain about the effectiveness of the thefts they perpetrate as well as of those they suffer; in equilibrium players will thus choose the strategies that maximize their expected payoff, taking into account that their opponents will do the same.

3 The game with $T = 2$ and $K = 1$

We start the actual analysis of the game by focusing on a basic, yet highly informative, case. More precisely, in this section we study a stealing game that lasts two periods (i.e., $T = 2$) and where players have only one stealing opportunity (i.e., $K = 1$). We first analytically solve the game with two and three players and then generalize the results to the case in which $n > 3$.

3.1 The game with two players

If $n = 2$ each player has only one opponent that he can possibly rob. The game is thus essentially a 2x2 game where players must only decide when to be active. For any i and $j \neq i$, players' strategy space is given by $S_i = \{(j, \emptyset), (\emptyset, j)\}$ and expected payoffs are as follows: $\bar{u}_i((j, \emptyset), (i, \emptyset)) = \bar{u}_i((\emptyset, j), (\emptyset, i)) = \frac{1}{2}$, $\bar{u}_i((j, \emptyset), (\emptyset, i)) = 1$ and $\bar{u}_i((\emptyset, j), (i, \emptyset)) = 0$. Players expect to share the prize if they simultaneously rob each other while a player that preempts the opponent expects to result as the unique winner. To see why this is so, consider for instance the profile $((j, \emptyset), (\emptyset, i))$. Final expected market shares are given by $\bar{\pi}_i^2 = \frac{1}{2} + \bar{y}_i^1 - \bar{y}_j^2$ and $\bar{\pi}_j^2 = \frac{1}{2} - \bar{y}_i^1 + \bar{y}_j^2$. Notice that the distribution of y_i^1 (i.e., $F_{\pi_i^0}(\cdot)$) first-order stochastically dominates the distribution of y_j^2 (i.e., $F_{\pi_j^1}(\cdot)$) given that $\pi_i^0 > \pi_j^1$ (in particular $\pi_i^0 = \frac{1}{2}$ while

⁷We are interested in an equilibrium analysis and we thus ignore all those strategies that are not best responses.

$\pi_j^1 = \frac{1}{2} - y_i^1$). Therefore, it follows that $\bar{y}_i^1 > \bar{y}_j^2$ such that $\bar{\pi}_i^2 > \bar{\pi}_j^2$ which in turn implies $\bar{u}_i = 1$ and $\bar{u}_j = 0$.

Proposition 1 *With $n = 2$ the profile $\hat{s} = (j, \emptyset)_i$ is the unique equilibrium of the stealing game.*

3.2 The game with three players

If $n = 3$ the game presents numerous Nash equilibria. The analysis of the game in normal form (see appendix A2) shows that these are actually 16 but only four of them are strict as well as subgame perfect. We restrict our attention to these four equilibria: the two “circles” with all the three players stealing from each other in $t = 1$ and the two “circles” with all the three players stealing from each other in $t = 2$.

Proposition 2 *With $n = 3$ the stealing game has four strict equilibria:*

- the two profiles $\hat{s} = (j, \emptyset)_i$ with $j \in N_{-i}$ and such that $\bar{u}_i = \frac{1}{3}$ for any i ;
- the two profiles $\hat{s} = (\emptyset, j)_i$ with $j \in N_{-i}$ and such that $\bar{u}_i = \frac{1}{3}$ for any i .

With respect to the case with $n = 2$, the interesting feature of the three-player game is that now there exist equilibria in which all the agents postpone their move. The reason is that the possibility of best responding in $t = 2$, even though potentially risky, is now worthwhile. To understand how the trade-off works in this case, consider the hypothetical situation in which player A commits to steal in $t = 1$ while B and C wait. Player A can rob either B or C . Assume $s_A = (B, \emptyset)$ such that $\bar{\pi}_A^1 = \frac{1}{3} + \bar{y}_A^1$, $\bar{\pi}_B^1 = \frac{1}{3} - \bar{y}_A^1$ and $\bar{\pi}_C^1 = \frac{1}{3}$. In $t = 2$ player B is indifferent about who to rob as he has been weakened by the stealing of A and thus in expectations cannot catch up with his initial share: $\bar{\pi}_B^2 = \frac{1}{3} - \bar{y}_A^1 + \bar{y}_B^2$ with $\bar{\pi}_B^2 < \frac{1}{3}$ and thus $u_B = 0$ no matter if $s_B = (\emptyset, A)$ or $s_B = (\emptyset, C)$. Player C is instead confident to win the

game as he can effectively best respond to what happened in $t = 1$. In fact, even assuming that $s_B = (\emptyset, C)$, if C robs A then expected market shares would be: $\bar{\pi}_A^2 = \frac{1}{3} + \bar{y}_A^1 - \bar{y}_C^2 = \frac{1}{3}$ given that y_A^1 and y_C^2 are i.i.d.; $\bar{\pi}_B^2 = \frac{1}{3} - \bar{y}_A^1 + \bar{y}_B^2 < \frac{1}{3}$ given that the distribution of \bar{y}_A^1 first order stochastically dominates the distribution of y_B^2 ; $\pi_C^2 = \frac{1}{3} + \bar{y}_C^2 - \bar{y}_B^2 > \frac{1}{3}$ given that the distribution of \bar{y}_C^2 first order stochastically dominates the distribution of y_B^2 . It follows that $\bar{u}_A = \bar{u}_B = 0$ and $\bar{u}_C = 1$. The outcomes of agents C and B exemplify the advantages/disadvantages of postponing one's move.

3.2.1 The concept of \mathbf{p}_u -dominance

In order to discriminate among the four strict equilibria of the three-player stealing game we introduce the concept of \mathbf{p}_u -dominance. \mathbf{p}_u -dominance is a generalization of risk-dominance (Harsanyi and Selten 1988) to games with more than two players. It is inspired by, and closely related to, the concept of \mathbf{p} -dominance (Morris *et al.* 1995, Kajii and Morris 1997). Indeed all these three criteria tackle the issue of equilibrium selection sharing the same intuition: if agents do not know which equilibrium will arise, they will compute the risk involved in playing each of these equilibria and they will coordinate expectations on the less risky one.

As a preliminary step, we define the vector $\mathbf{p}_u = (p_u^1, \dots, p_u^n)$ that, given n agents and a set $E = \{e_1, \dots, e_k\}$ of k alternative events, indicates a collection of n probabilities distributions such that each agent $i \in N$ believes a certain event $e^* \in E$ will occur with probability p_u^i while each of the remaining events $e_k \neq e^*$ will occur with probability $\frac{(1-p_u^i)}{k-1}$. Using a similar notation, an equilibrium $(\hat{s}_i, \hat{s}_{-i})$ is said to be \mathbf{p}_u -dominant for $\mathbf{p}_u = (p_u^1, \dots, p_u^N)$ if, for any player $i \in N$ and any $j \in N_{-i}$, strategy \hat{s}_i is the unique best response to any probability distribution $\lambda \in \Delta(S_{-i})$ that assigns at least probability p_u^i to the event of j playing his equilibrium strategy \hat{s}_j and lets j uniformly randomize on his alternative strategies with the remaining probability. In other words, \mathbf{p}_u -dominance mimics the process according to which

an agent evaluates the likelihood of an equilibrium by focusing on the probability of the strategies that sustain it while assuming a simplifying uniform distribution for what concerns the alternative strategies.

Definition 1 *Strategy profile* $(\hat{s}_1, \dots, \hat{s}_N)$ *is a* \mathbf{p}_u -*dominant equilibrium with* $\mathbf{p}_u = (p_u^1, \dots, p_u^n)$ *if for all* $i \in N$, $s_i \neq \hat{s}_i$ *and all* $\lambda \in \Delta(S_{-i})$ *with* $\lambda(\hat{s}_j) \geq p_u^i$ *and* $\lambda(s_j) = \frac{(1-\lambda(\hat{s}_j))}{|S_j|-1}$ *for all* $s_j \neq \hat{s}_j$ *and* $j \in N_{-i}$,

$$\sum_{s_{-i} \in S_{-i}} \lambda(s_{-i}) u_i(\hat{s}_i, s_{-i}) \geq \sum_{s_{-i} \in S_{-i}} \lambda(s_{-i}) u_i(s_i, s_{-i}).$$

Some standard concepts of game theory can be formulated in terms of \mathbf{p}_u -dominance. For instance, an equilibrium in dominant strategies is a \mathbf{p}_u -dominant equilibrium with $\mathbf{p}_u = (0, \dots, 0)$ while every Nash equilibrium is a \mathbf{p}_u -dominant equilibrium with $\mathbf{p}_u = (1, \dots, 1)$. Notice also that if the profile $(\hat{s}_1, \dots, \hat{s}_n)$ is a \mathbf{p}_u -dominant equilibrium then it is also a \mathbf{p}'_u -dominant equilibrium for any $\mathbf{p}'_u \geq \mathbf{p}_u$ (using the standard vector ordering). What characterizes an equilibrium is the smallest \mathbf{p}_u for which the equilibrium is \mathbf{p}_u -dominant. This vector, which we indicate with \mathbf{p}_u^* , reports the minimum level of the beliefs $\lambda(\hat{s}_j)$ for which the equilibrium strategy under scrutiny dominates the alternatives. As such, \mathbf{p}_u^* provides a measure of the riskiness of playing a certain equilibrium strategy as well as a tool to identify the equilibrium upon which players' expectations should coordinate. In particular, in the same spirit of what is suggested by Morris *et al.* (1995) for what concerns \mathbf{p} -dominance, the \mathbf{p}_u -dominance criterion selects the equilibrium characterized by the smallest \mathbf{p}_u^* .⁸

⁸Notice that such an equilibrium may not exist as there may be situations in which it is not possible to unambiguously order the \mathbf{p}_u^* vectors associated with the various equilibria.

3.2.2 Equilibrium selection in the 3-players stealing game

We apply the \mathbf{p}_u -dominance criterion to refine the strict equilibria of the stealing game with $n = 3$ (see Proposition 2). Note that all four equilibria are Pareto equivalent such that there is no conflict between payoff realization and risk considerations. Therefore, players should indeed coordinate on the less risky equilibrium.

As before, we call the 3 players A , B and C and we refer to the game in normal form as it appears in appendix A2. Given any strict equilibrium $(\hat{s}_A, \hat{s}_B, \hat{s}_C)$, we compute for each player $E_{p_u}(s_i)$, i.e., the expected payoff of each strategy $s_i \in S_i$ under the conjecture that each opponent plays strategy \hat{s}_j with probability p_u and each of his alternative strategies with probability $\frac{1-p_u}{3}$. Then, by imposing the conditions $E_{p_u}(\hat{s}_i) > E_{p_u}(s_i)$ for any $s_i \neq \hat{s}_i$, we find the components of the vector \mathbf{p}_u^* for which the equilibrium $(\hat{s}_A, \hat{s}_B, \hat{s}_C)$ is \mathbf{p}_u -dominant. Finally, we will select the equilibrium characterized by the smallest \mathbf{p}_u^* .

For instance, starting from the equilibrium $((B, \emptyset), (C, \emptyset), (A, \emptyset))$ and focusing without loss of generality on player A , we have the following: $E_{p_u}(B, \emptyset) = -\frac{5}{9}p_u^2 + \frac{7}{9}p_u + \frac{1}{9}$, $E_{p_u}(C, \emptyset) = -\frac{11}{27}p_u^2 + \frac{4}{27}p_u + \frac{7}{27}$ and $E_{p_u}(\emptyset, B) = E_{p_u}(\emptyset, C) = -\frac{17}{27}p_u^2 + \frac{7}{27}p_u + \frac{10}{27}$. Therefore, the equilibrium strategy (B, \emptyset) dominates strategy (C, \emptyset) for any $p_u \geq 0.25$ and strategies (\emptyset, B) and (\emptyset, C) for any $p_u \geq \frac{3}{2}\sqrt{7} - \frac{7}{2} \cong 0.47$. Given that similar relations also hold for players B and C , the equilibrium $((B, \emptyset), (C, \emptyset), (A, \emptyset))$ is \mathbf{p}_u -dominant with $\mathbf{p}_u^* = (\frac{3}{2}\sqrt{7} - \frac{7}{2}, \frac{3}{2}\sqrt{7} - \frac{7}{2}, \frac{3}{2}\sqrt{7} - \frac{7}{2})$. Not surprisingly, analogous computations show that also the other preempting equilibrium $((C, \emptyset), (A, \emptyset), (B, \emptyset))$ is \mathbf{p}_u -dominant for the same \mathbf{p}_u^* . Now consider one of the postponing equilibria, say $((\emptyset, B), (\emptyset, C), (\emptyset, A))$. Focusing again on player A , we have that $E_{p_u}(B, \emptyset) = -\frac{11}{27}p_u^2 + \frac{4}{27}p_u + \frac{7}{27}$, $E_{p_u}(C, \emptyset) = \frac{1}{27}p_u^2 - \frac{11}{27}p_u + \frac{10}{27}$, $E_{p_u}(\emptyset, B) = -\frac{1}{3}p_u^2 + \frac{1}{3}p_u + \frac{1}{3}$ and $E_{p_u}(\emptyset, C) = -\frac{17}{27}p_u^2 + \frac{7}{27}p_u + \frac{10}{27}$ such that the equilibrium strategy (\emptyset, B) dominates (B, \emptyset) for any $p_u \geq 0$, (C, \emptyset) for any $p_u \geq 1 - \frac{3}{10}\sqrt{10} \cong 0.05$ and (\emptyset, C) for any $p_u = 0.25$. Therefore, the equilibrium $((\emptyset, B), (\emptyset, C), (\emptyset, A))$ is \mathbf{p}_u -dominant with $\mathbf{p}_u^* = (0.25, 0.25, 0.25)$. The same \mathbf{p}_u^* characterizes the equilibrium $((\emptyset, C), (\emptyset, A), (\emptyset, B))$.

Proposition 3 *The postponing equilibria $\hat{s} = (\emptyset, j)_i$ with $j \in N_{-i}$ and such that $\bar{u}_i = \frac{1}{3}$ for any i , are the \mathbf{p}_u -dominant equilibria of the stealing game with $n = 3$.*

In other words, despite the possibility to be preempted, it is less risky to wait until $t = 2$ rather than move in $t = 1$. To have an intuition for this result, consider the case in which player A finds himself in the situation of being the only player who moved in $t = 1$. The payoff matrices in the appendix show that, if this occurs, player A has no chance to win the game. At the opposite end, if A happens to be the only agent who postpones his move then there is still a positive probability, associated with the event of B and C stealing from each other in $t = 1$, that $u_A = 1$. As such, no agent wants to break the initial symmetric situation and all the players postpone their move.⁹

3.3 The game with $n > 3$ players

We now extend the analysis of the stealing game to a situation in which $n > 3$ players compete over two periods (i.e., $T = 2$) and have only one possibility of stealing (i.e., $K = 1$). As before, our main interest lies in investigating the timing of agents' move and their decision if to preempt or to postpone their stealing opportunity. As a first step we formally define the concept of "circle" of players.

Definition 2 *Given any subset $M \subseteq N$ with $|M| = m \geq 2$, a circle of players C_m^t is formed in M at time t if for any $i \in M$ we have that $x_i^t = j$, $j \in M$ and there exists a unique agent $k \in M$ such that $x_k^t = i$.*

In other words a circle C_m^t is such that: all agents that belong to the subset M move simultaneously at time t ; every player steals from a rival that belongs to M ;

⁹This result is reminiscent of the analysis of so-called truels (gun duels among three players) which shows that, under certain conditions, the best strategy that a player can adopt is to postpone his shot or even to shoot in the air rather than against an opponent (see Kilgour 1972 and Kilgour and Brams 1997).

every player is robbed by a single rival who also belongs to M . As such, the random variables y_i^t and y_k^t are i.i.d., they have the same expected value and thus cancel out in expectations. It follows that in every circle $\bar{u}_i = \frac{1}{3}$ for every i .

The notion of circle plays an important role in the characterization of the equilibria of the stealing game. The following proposition defines the preempting equilibria, i.e., the strict Nash equilibria in which all the players move in $t = 1$.

Proposition 4 *All the profiles $\hat{s} = (j, \emptyset)_i$ with $j \in N_{-i}$ and such that every $i \in N$ belongs to a circle are strict Nash equilibria of the Stealing game.*

Notice that the number of preempting equilibria rapidly explodes with the number of players. In fact, for any $n > 3$, equilibrium profiles are not only the $(n - 1)!$ possible circles that involve all the players (i.e., the circles C_n^1) but also those in which the set N is partitioned and smaller circles emerge in every part. This implies that equilibria do not exist in those partitions that include singletons as distinct parts given that the smallest part that allows the creation of a circle contains 2 agents.¹⁰

There also exist equilibria in which all the players postpone their move to $t = 2$. Still the conditions that define them are stricter as shown by the following proposition.

Proposition 5 *All the profiles $\hat{s} = (\emptyset, j)_i$ with $j \in N_{-i}$, and such that every $i \in N$ belongs to a circle that includes at least three agents are strict Nash equilibria of the stealing game.*

¹⁰Consider for instance the case $n = 5$. Strict Nash equilibria in which all the players move in $t = 1$ can only emerge in the partitions (5) and (3, 2). On the contrary, there are no equilibria in the partitions (4, 1), (2, 2, 1), (2, 1, 1, 1) and (1, 1, 1, 1, 1). The number of strict Nash equilibria $\hat{s} = (j, \emptyset)_i$ if $n = 5$ is 44: there are $4! = 24$ possible equilibria in the partition (5) and 20 equilibria in the partition (3, 2) (10 different couples can be drawn from a set of 5 elements; for any of these couples there are two possible circles that can emerge in the part that involves 3 players).

The proposition indicates that there cannot exist postponing equilibria that include circles made of two players (C_2^2). The reason is that within any of these circles players would like to deviate in order to preempt the rival. Proposition 1 showed in fact that the only circle that qualifies as an equilibrium when $n = 2$ is the one in which both players move in $t = 1$ (C_2^1). Three additional results directly follow from the analysis and the comparison of propositions 4 and 5:

- for any $n \neq 3$, the condition $\bar{u}_i = \frac{1}{n}$ for any $i \in N$ is a necessary but not sufficient feature of any Nash equilibrium. In particular not all the profiles in which every player belongs to a circle are Nash equilibria.
- for any $n > 3$, the number of postponing equilibria (i.e., equilibria in which all the players move in $t = 2$) is strictly smaller than the number of preempting equilibria (i.e., equilibria in which all the players move in $t = 1$).
- for any $n > 3$, there exist strict Nash equilibria that are asymmetric with respect to the timing decision (i.e., equilibria in which some players move in $t = 1$ while others move in $t = 2$).¹¹

4 The game with $T > 2$ and $K < T$

As a further generalization we consider a stealing game that lasts for T periods and where players have $K \in \{1, \dots, T - 1\}$ stealing possibilities.¹² Results in this setting maintain the same qualitative features of those presented in the previous section. More precisely: with two players the only equilibrium of the game is such that both agents use all their stealing opportunities as soon as possible. With three

¹¹As an example, consider the case with $n = 5$. The profile $((B, \emptyset), (A, \emptyset), (D, \emptyset), (E, \emptyset), (C, \emptyset))$ in which all the players move in $t = 1$ is a Nash equilibrium. On the contrary the profile $((\emptyset, B), (\emptyset, A), (\emptyset, D), (\emptyset, E), (\emptyset, C))$ in which all the players move in $t = 2$ is not an equilibrium since players A and B want to deviate to $t = 1$. The profile $((B, \emptyset), (A, \emptyset), (\emptyset, D), (\emptyset, E), (\emptyset, C))$ is an equilibrium but notice that this is asymmetric with respect to the timing of players' moves: A and B move in $t = 1$ while C , D and E move in $t = 2$.

¹²If $K \geq T$ the game becomes trivial: stealing opportunities are no more a "scarce resource" and every player would thus steal from a rival in every period.

players there are many strict Nash equilibria but only those in which all the players postpone their moves as late as possible are p_u -dominant. With more than three players asymmetric equilibria in which different circles of agents move in different periods also exist.

Proposition 6 *A stealing game with T periods and $K < T$ stealing opportunities is such that:*

- If $n = 2$ the profile $\hat{s} = \left(\underbrace{j, j, \dots, j}_{K \text{ times}}, \underbrace{\emptyset, \dots, \emptyset}_{T-K \text{ times}} \right)_i$ is the unique Nash equilibrium.

- If $n = 3$ all the profiles that involve the formation of K circles C_3^t over the T periods are strict Nash equilibria but only those such that $\hat{s} = \left(\underbrace{\emptyset, \dots, \emptyset}_{T-K \text{ times}}, \underbrace{j, \dots, j}_{K \text{ times}} \right)_i$ with $j \in N_{-i}$ are p_u -dominant.

- If $n > 3$ equilibria may be symmetric or asymmetric with respect to the periods in which agents move but they necessarily require that every player that steals in generic period $t \in \{1, \dots, T\}$ belongs to a circle C_m^t with $m = |M|$, $M \subseteq N$ and $m \geq 2$.

5 Conclusions

The paper introduced what we called the stealing game. This is an endogenous timing game in which players must decide when to steal from each other parts of a homogeneous good, the expected effectiveness of a player's stealing attempt is proportional to his current holdings and the goal is to finish the game being the agent who owns the largest share. In a stealing game with two agents, players always want to preempt the rival and thus employ their stealing possibilities as soon as possible. At the opposite, with three players, the game is characterized by multiple equilibria but the ones in which agents move as late as possible endogenously

emerge as the focal ones. Finally, when the number of players is larger than three asymmetric equilibria also exist and not all the players necessarily move in the same periods. The stealing game provides thus an example of a timing game in which optimal timing strategies change according to the number of participants. This is an interesting aspect of timing games that has been so far neglected and that possibly requires further and more general research.

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Appendices

A1. Proofs of the propositions

Proof of Proposition 1. Given that $\bar{u}_i((j, \emptyset), (i, \emptyset)) > \bar{u}_i((\emptyset, j), (i, \emptyset))$ and $\bar{u}_i((j, \emptyset), (\emptyset, i)) > \bar{u}_i((\emptyset, j), (\emptyset, i))$ it follows that, for both players, strategy $s_i = (j, \emptyset)$ strictly dominates the alternative strategy $s_i = (\emptyset, j)$. Therefore, the profile $\hat{s} = (j, \emptyset)_i$ is the unique Nash equilibrium of the game.

Proof of Proposition 2. We check for profitable deviations over the two dimensions of the strategy space. First, consider the situation in which, from any of the four equilibrium profiles, player i deviates and robs k instead of j : this would imply that j is not robbed by anyone while k is robbed twice. Therefore, $\bar{\pi}_i^t = \frac{1}{3} + \bar{y}_i^t - \bar{y}_k^t$, $\bar{\pi}_j^t = \frac{1}{3} + \bar{y}_j^t$, and $\bar{\pi}_k^t = \frac{1}{3} + \bar{y}_k^t - \bar{y}_i^t - \bar{y}_j^t$ for $t \in \{1, 2\}$. Given that y_i^t, y_j^t , and y_k^t are i.i.d. random variables with $F_{\frac{1}{3}}(\cdot)$, it follows that $\bar{y}_k^t = \bar{y}_i^t = \bar{y}_j^t$ such that $\bar{\pi}_j^2 > \bar{\pi}_i^2 > \bar{\pi}_k^2$ and therefore $\bar{u}_i = 0$. Then, consider possible deviations over the timing dimension. Start from $\hat{s} = (j, \emptyset)_i$ and let player i postpone his move and deviate to (\emptyset, j) . In this case $\bar{\pi}_i^1 = \frac{1}{3} - \bar{y}_k^1$ and $\bar{\pi}_i^2 = \frac{1}{3} - \bar{y}_k^1 + \bar{y}_i^2$. But notice that $\bar{\pi}_i^2 < \frac{1}{3}$ because $\pi_k^0 > \bar{\pi}_i^1$

and thus the distribution of y_k^1 first order stochastically dominates the distribution of y_i^2 . It follows that $\bar{u}_i = 0$ as the condition $\sum_i \bar{\pi}_i^t = 1$ implies that there exists an agent j with $\bar{\pi}_j^2 > \bar{\pi}_i^2$. Then consider $\hat{s} = (\emptyset, j)_i$ and let i deviate to $s_i = (j, \emptyset)$ such that $\bar{\pi}_i^1 = \frac{1}{3} + \bar{y}_i^1$. In $t = 2$ player k best responds to the new situation and steals from i as this action ensures him the largest expected share. In fact if k steals from i then $\pi_k^2 = \frac{1}{3} + \bar{y}_k^2 - \bar{y}_j^2$ with $\pi_k^2 > \frac{1}{3}$ given that $\pi_k^1 > \bar{\pi}_j^1$, the distribution of y_k^2 first order stochastically dominates the distribution of y_j^2 and thus $\bar{y}_k^2 > \bar{y}_j^2$. On the other hand, $\bar{\pi}_i^2 = \frac{1}{3} + \bar{y}_i^1 - \bar{y}_k^2 = \frac{1}{3}$ given that y_i^1 and y_k^2 are i.i.d. according to $F_{\frac{1}{3}}(\cdot)$. It follows that $\bar{\pi}_k^2 > \bar{\pi}_i^2$ and therefore $\bar{u}_i = 0$ such that also this deviation is unprofitable.

Proof of Proposition 3. The equilibria $\hat{s} = (j, \emptyset)_i$ with $j \in N_{-i}$ and such that $\bar{u}_i = \frac{1}{3}$ for any i are \mathbf{p}_u -dominant with $\mathbf{p}_u^* = (\frac{3}{2}\sqrt{7} - \frac{7}{2}, \frac{3}{2}\sqrt{7} - \frac{7}{2}, \frac{3}{2}\sqrt{7} - \frac{7}{2})$. The equilibria $\hat{s} = (\emptyset, j)_i$ with $j \in N_{-i}$ and such that $\bar{u}_i = \frac{1}{3}$ for any i are \mathbf{p}_u -dominant with $\mathbf{p}_u^* = (0.25, 0.25, 0.25)$. Given that $0.25 < \frac{3}{2}\sqrt{7} - \frac{7}{2}$, the two equilibria in which all the players postpone their move are the equilibria selected by the \mathbf{p}_u -dominance criterion.

Proof of Proposition 4. The proof essentially generalizes part of the proof of Proposition 2 (the case with $n = 3$). Take any profile \hat{s} and let generic player $i \in N$ explore possible deviations. Say that in \hat{s} player i is supposed to play (j, \emptyset) and let him deviate to (l, \emptyset) with $l \neq j$. Then in this modified profile player j is not robbed by anyone such that $\bar{\pi}_j^2 = \frac{1}{n} + \bar{y}_j^1$ and $\bar{\pi}_i^2 = \frac{1}{n}$. Given that $\bar{\pi}_i^2 < \bar{\pi}_j^2$ it follows that $\bar{u}_i = 0$. If player i instead postpones his move and plays (\emptyset, j) then $\bar{\pi}_i^2 = \frac{1}{n} - \bar{y}_l^1 + \bar{y}_i^2 < \frac{1}{n}$ as the distribution of y_l^1 (where l is the agent that in \hat{s} steals from i) first order stochastically dominates the one of y_i^2 . Given the condition $\sum_i \bar{\pi}_i^2 = 1$, it follows that there exists a player j such that $\bar{\pi}_j^2 > \bar{\pi}_i^2$ which in turn implies $\bar{u}_i = 0$.

Proof of Proposition 5. The proof is similar to the proof of Proposition 2. Take any profile \hat{s} : generic agent i 's possible deviations (i steals from a different player in $t = 2$

or i anticipates his move to $t = 1$) are strictly unprofitable. As for the restriction on the size of the smallest part of N , we know that partitions that include singletons as distinct parts cannot match the requirements of \hat{s} as by definition a circle requires the presence of at least two players. Now consider all the possible partitions of N where at least a part contains only two players. The strategic situation within any of these parts replicates the one that characterizes the stealing game when $n = 2$. We thus know (see Proposition 1) that the equilibrium within any part of size 2 is unique and is such that both players move in $t = 1$. It follows that equilibria in which all the players move in $t = 2$ only exist in all those partitions whose smallest parts contain strictly more than 2 agents.

Proof of Proposition 6. For the $n = 2$ case: consider any alternative profile for generic player i , i.e., any profile such that there exists at least a period $\tilde{t} \in \{1, \dots, K\}$ in which $x_i^{\tilde{t}} = \emptyset$. Then, no matter if $x_j^{\tilde{t}} = i$ or $x_j^{\tilde{t}} = \emptyset$, it follows that $\pi_i^{\tilde{t}} < \pi_i^{\tilde{t}}$ where $\pi_i^{\tilde{t}}$ is the share player i would have had if $x_i^{\tilde{t}} = j$. This in turn implies $\bar{y}_i^{\tilde{t}+1} < \bar{y}_i^{\tilde{t}+1}$ and thus $\bar{\pi}_i^{\tilde{t}+1} < \bar{\pi}_i^{\tilde{t}+1}$ and this gap propagates through the game such that $\bar{u}_i < \bar{u}_i'$.

Therefore the profile $\hat{s} = \left(\underbrace{j, j, \dots, j}_{K \text{ times}}, \underbrace{\emptyset, \dots, \emptyset}_{T-K \text{ times}} \right)_i$ is the unique Nash equilibrium of the game.

For the $n = 3$ case: the proof that whenever $x_i^t \neq \emptyset$ then in any equilibrium agent i must belong to a circle C_3^t uses the argument presented in the proof of Proposition 2, i.e., the fact that there are no incentives to deviate from such a circle. As such there cannot exist asymmetric equilibria and all agents must use their K attacks in the same periods. The number of these equilibria is given by $2^K \binom{T}{K}$ where the term $\binom{T}{K}$ is the binomial coefficient that returns the number of K -combinations that can be formed over T periods and the term 2^K acknowledges the fact that in every period in which a circle C_3^t emerges this can go in two directions. The proof that the profiles $\hat{s} = \left(\underbrace{\emptyset, \dots, \emptyset}_{T-K \text{ times}}, \underbrace{j, \dots, j}_{K \text{ times}} \right)_i$ with $j \in N_{-i}$ are the p_u -dominant

equilibria proceeds by induction: imagine first that $T > 2$ and $K = 1$ and start by comparing the equilibria that exist in $t = 1$ and in $t = 2$, i.e., the circles C_3^1 and C_3^2 . By Proposition 3 we know that the equilibria in $t = 2$ are p_u -dominant. Now compare the equilibria in $t = 2$ with those in $t = 3$: the situation is strategically equivalent to a stealing game with $n = 3$, $T = 2$, and $K = 1$ (appendix A2 shows the game in normal form). We thus know, again because of Proposition 3, that the p_u -dominant equilibria are those in which players postpone their move. Therefore, the equilibria in $t = 3$ p_u -dominate those in $t = 2$ and, more in general, the p_u -dominant equilibria of a stealing game with $n = 3$, $T > 2$, and $K = 1$ are the ones in which all the players move in $t = T$ (i.e., the two circles C_3^T). Now imagine to endow players with a second stealing opportunity that they can use in any period $t \in \{1, \dots, T - 1\}$ and replicate the argument as before. The equilibria in which all players move in $t = T - 1$ would then p_u -dominate all the alternatives. By iterating this argument K times, the profiles $\hat{s} = \left(\underbrace{\emptyset, \dots, \emptyset}_{T-K \text{ times}}, \underbrace{j, \dots, j}_{K \text{ times}} \right)_i$ with $j \in N_{-i}$ emerge as the p_u -dominant equilibria.

For the $n > 3$ case: all the profiles in which K circles C_n^t (i.e., circles that involve all the players) form over the T periods are (symmetric) equilibria of the game as profitable deviations do not exist (see the proofs of propositions 4 and 5). But given that $n > 3$ and that the minimum size of a circle is $m = 2$, there also exist equilibria in which all players, whenever they move, always belong to a circle C_m^t (not necessarily the same one over the entire game) with $m < n$, i.e., circles that form within parts of the set N . Now, although the moves of all the players that belong to a specific circle must be simultaneous (and thus symmetric in terms of the timing decision), players that belong to different circles may move in different periods and thus there exist asymmetric equilibrium profiles.

A2. The normal form of the game with $n = 3$

Player A chooses the row, player B chooses the column, player C chooses the matrix. In each cell payoffs appear in the order $\bar{u}_A, \bar{u}_B, \bar{u}_C$. Nash equilibria are those whose payoffs are underlined. Strict Nash equilibria are those whose payoffs also have a “hat”.

$x_C^1 = A$					$x_C^1 = B$				
	$x_B^1 = A$	$x_B^1 = C$	$x_B^2 = A$	$x_B^2 = C$		$x_B^1 = A$	$x_B^1 = C$	$x_B^2 = A$	$x_B^2 = C$
$x_A^1 = B$	0, 0, 1	<u>$\widehat{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}}$</u>	0, 0, 1	0, 0, 1	$x_A^1 = B$	0, 0, 1	1, 0, 0	0, 0, 1	1, 0, 0
$x_A^1 = C$	0, 1, 0	0, 1, 0	<u>0, 1, 0</u>	<u>0, 1, 0</u>	$x_A^1 = C$	<u>$\widehat{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}}$</u>	1, 0, 0	1, 0, 0	1, 0, 0
$x_A^2 = B$	0, 0, 1	0, 1, 0	0, 1, 0	0, 1, 0	$x_A^2 = B$	0, 0, 1	<u>1, 0, 0</u>	1, 0, 0	1, 0, 0
$x_A^2 = C$	0, 1, 0	0, 1, 0	0, 1, 0	<u>0, 1, 0</u>	$x_A^2 = C$	0, 0, 1	<u>1, 0, 0</u>	1, 0, 0	<u>1, 0, 0</u>

$x_C^2 = A$					$x_C^2 = B$				
	$x_B^1 = A$	$x_B^1 = C$	$x_B^2 = A$	$x_B^2 = C$		$x_B^1 = A$	$x_B^1 = C$	$x_B^2 = A$	$x_B^2 = C$
$x_A^1 = B$	<u>0, 0, 1</u>	1, 0, 0	<u>0, 0, 1</u>	0, 0, 1	$x_A^1 = B$	<u>0, 0, 1</u>	1, 0, 0	0, 0, 1	0, 0, 1
$x_A^1 = C$	0, 1, 0	0, 1, 0	<u>0, 1, 0</u>	0, 1, 0	$x_A^1 = C$	0, 1, 0	1, 0, 0	0, 1, 0	0, 1, 0
$x_A^2 = B$	0, 0, 1	1, 0, 0	0, 0, 1	<u>$\widehat{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}}$</u>	$x_A^2 = B$	<u>0, 0, 1</u>	<u>1, 0, 0</u>	0, 0, 1	1, 0, 0
$x_A^2 = C$	0, 0, 1	1, 0, 0	0, 1, 0	0, 1, 0	$x_A^2 = C$	0, 0, 1	1, 0, 0	<u>$\widehat{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}}$</u>	1, 0, 0

References

- [1] Brunnermeier, M.K., Morgan, J., 2010. Clock games: theory and experiments. Games and Economic Behavior 68, 532-550.
- [2] Bulow, J., Klemperer, P., 1999. The generalized war of attrition. American Economic Review 89, 175-189.

- [3] van Damme, E., Hurkens, S., 1996. Commitment robust equilibria and endogenous timing. *Games and Economic Behavior* 15, 290-311.
- [4] van Damme, E., Hurkens, S., 1999. Endogenous Stackelberg leadership. *Games and Economic Behavior* 28, 105-129.
- [5] van Damme, E., Hurkens, S., 2004. Endogenous price leadership. *Games and Economic Behavior* 47, 404-420.
- [6] Hamilton, J.H., Slutsky, S.M., 1990. Endogenous timing in duopoly games: Stackelberg or Cournot equilibria. *Games and Economic Behavior* 2, 29-46.
- [7] Harsanyi, J., Selten, R., 1988. A general theory of equilibrium selection in games. MIT Press, Cambridge.
- [8] Kajii, A., Morris, S., 1997. The robustness of equilibria to incomplete information. *Econometrica* 65, 1283-1309.
- [9] Kilgour, D.M., 1972. The simultaneous truel. *International Journal of Game Theory* 1, 229-242.
- [10] Kilgour, D.M., Brams, S.J., 1997. The truel. *Mathematical Magazine* 70, 315-326.
- [11] Maynard Smith, J., 1974. Theory of games and the evolution of animal contests. *Journal of Theoretical Biology* 47, 209-221.
- [12] Morris, S., Rob, R., Shin, H.S., 1995. p -Dominance and belief potential. *Econometrica* 63, 145-157.
- [13] Park, A., Smith, L., 2008. Caller number five and related timing games. *Theoretical Economics* 3, 231-256.
- [14] Rosenthal, R., 1981. Games of perfect information, predatory pricing, and the chain store paradox. *Journal of Economic Theory* 25, 92-100.