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Fabio C. Bagliano

Claudio Morana

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A New Approach to Factor Vector Autoregressive Estimation with an Application to Large-Scale Macroeconometric Modelling ^{*†}

Fabio C. Bagliano

Dipartimento di Scienze Economiche e Finanziarie, Università di Torino (Italy)
and Collegio Carlo Alberto, Moncalieri (Italy)

Claudio Morana

Dipartimento di Scienze Economiche e Metodi Quantitativi,
Università del Piemonte Orientale, Novara (Italy)
and International Centre for Economic Research (ICER, Torino)[‡]

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[‡]Address for correspondence: Claudio Morana, Università del Piemonte Orientale, Facoltà di Economia, Dipartimento di Scienze Economiche e Metodi Quantitativi, Via Perrone 18, 28100, Novara, Italy. E-mail: claudio.morana@eco.unipmn.it

Abstract

In this paper a new approach to factor vector autoregressive estimation, based on Stock and Watson (2005), is introduced. Relative to the Stock-Watson approach, the proposed method has the advantage of allowing for a more clear-cut interpretation of the global factors, as well as for the identification of all idiosyncratic shocks. Moreover, it shares with the Stock-Watson approach the advantage of using an iterated procedure in estimation, recovering, asymptotically, full efficiency, and also allowing the imposition of appropriate restrictions concerning the lack of Granger causality of the variables versus the factors. Finally, relative to other available methods, our modelling approach has the advantage of allowing for the joint modelling of all variables, without resorting to long-run forcing hypotheses. An application to large-scale macroeconomic modelling is also provided.

JEL classification n.: C32, G1, G15.

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1 Introduction

Factor vector autoregressive (FVAR) models are a recent important novelty in the literature, allowing to easily handle the estimation of large scale dynamic econometric models in the convenient vector autoregressive (VAR) framework. Seminal contributions can be traced back to the works of Giannone, Reichlin and Sala (2002), Bernanke and Boivin (2003), Favero and Marcellino (2005), Favero, Marcellino and Neglia (2004), Bernanke, Boivin and Elias (2005), Pesaran, Schuerman and Weiner (2004), and Stock and Watson (2005). A common element in all the above studies is the intuition that the information contained in very large data sets of economic variables can be summarized by a small number of factors, which can then be employed to augment vector autoregressive systems, in order to allow for a better description of economic dynamics in macroeconomic models.

Yet, some differences can also be noted among the various approaches. In fact, while in Bernanke and Boivin (2003), Giannone, Reichlin and Sala (2002), Favero and Marcellino (2005), Favero, Marcellino and Neglia (2004), Bernanke, Boivin and Elias (2005), the framework of the analysis is still a small-scale macroeconomic model, where the additional factors are introduced to alleviate the problem of omitted variables, the contributions of Pesaran, Schuerman and Weiner (2004), and Stock and Watson (2005) aim to large-scale or global macroeconomic modelling. Important differences can also be found in terms of estimation methods. For instance, in all the above papers apart from Stock and Watson (2005), estimation is based on a two-step approach, where first the economic factors are estimated either by the use of principal components analysis (PCA) as in Stock and Watson (1998), or by means of frequency domain principal components as in Forni, Hallin, Lippi and Reichlin (2000), or as cross-sectional averages of observed variables as in Pesaran (2005), and then included as exogenous variables in VAR-X models. Yet, in the above papers also one-step estimation, based on the Kalman filter approach or Bayesian likelihood methods and Gibbs sampling, are discussed. Differently, Stock and Watson (2005) have proposed an iterated two-step approach, still based on the use of PCA, which however allows to recover the efficiency of the single-step estimator. In the light of the recent results of Bai (2002, 2003) and Bai and Ng (2001), which have justified the use of PCA also for the case of strongly persistent processes¹,

¹In particular, Bai (2003) has considered the generalization of PCA to the case in which the series are weakly dependent processes, establishing consistency and asymptotic normality when both the unobserved factors and idiosyncratic components show limited serial correlation, also allowing for heteroschedasticity in both the time and cross section dimension in the idiosyncratic components. In Bai (2002) consistency and asymptotic

the iterated two-step approach appears to be optimal, given its asymptotic properties and simplicity of implementation.

In this paper, a new approach to FVAR modelling and estimation is proposed. The proposed approach is an extension of Stock and Watson (2005), allowing for a more straightforward interpretation of the global factors and for the identification of all shocks, both global and idiosyncratic. Moreover, differently from the FVAR approach of Favero, Marcellino and Neglia (2004), Giannone, Reichlin and Sala (2002), and Bernanke, Boivin and Elias (2005), the proposed method has the advantage of using an iterated procedure in estimation, recovering full efficiency asymptotically, allowing, as in Stock and Watson (2005), the imposition of appropriate restrictions concerning the lack of Granger causality of the variable versus the factors as well. In addition, relative to the approach employed by Pesaran, Schuerman and Weiner (2004), as in Stock and Watson (2005), modelling is not carried out by considering each block of relevant variables at the time, on the basis of long-run forcing, but all the variables are treated as endogenous at the outset. Moreover, in the proposed framework the unknown factors can be interpreted as global factors, while in Pesaran, Schuerman and Weiner (2004) the interpretation is less straightforward.² Finally, while in the proposed approach the optimal weighting is chosen (by means of PCA), in Pesaran, Schuerman and Weiner (2004) the weighting is somewhat arbitrary, albeit a sound economic justification supports the method employed.

After this introduction, the rest of the paper is organized as follows. In section two the econometric methodology is presented, while in sections three an application to the estimation of a large-scale macroeconomic model for the G-7 countries is provided.

normality has been derived for the case of $I(1)$ unobserved factors and $I(0)$ idiosyncratic components, also in the presence of heteroschedasticity in both the time and cross section dimension in the idiosyncratic components. Moreover, Bai and Ng (2001) have established consistency also for the case of $I(1)$ idiosyncratic components. As the authors point out, consistent estimation should also be achieved by PCA in the intermediate case represented by long memory processes.

²In fact, what is denoted as global factor in Pesaran, Schuerman and Weiner (2004) is just a summary feature for all the variables which may have an impact on a given country, but for parsimony reasons are not modelled in details. This is because when the unobserved component is estimated the own country variables are neglected. It is hard, for instance, to justify the exclusion of US data when the global factors for the US are computed.

2 Econometric methodology

Following Stock and Watson (2005), consider the following dynamic factor model

$$X_t = \Lambda F_t + D(L)X_{t\Box 1} + v_t \quad (1)$$

$$F_t = \Phi(L)F_{t\Box 1} + \eta_t, \quad (2)$$

where X_t is a n -variate vector of variables of interest, F_t is a r -variate vector of global factors, v_t is a n -variate vector of idiosyncratic i.i.d. shocks, η_t is a r -variate vector of common or global i.i.d. shocks, $E[\eta_t v_{is}] = 0$ all i, t, s , Λ is a $n \times r$ matrix of loadings, and $D(L)$, $\Phi(L)$ are matrices of polynomials in the lag operator of order p , i.e.

$$D(L) = \begin{bmatrix} \delta_{1,1}(L) & \dots & \delta_{1,n}(L) \\ \vdots & \ddots & \vdots \\ \delta_{n,1}(L) & \dots & \delta_{n,n}(L) \end{bmatrix}, \Phi(L) = \begin{bmatrix} \phi_{r,r}(L) & \dots & \phi_{1,r}(L) \\ \vdots & \ddots & \vdots \\ \phi_{r,1}(L) & \dots & \phi_{r,r}(L) \end{bmatrix}$$

By substituting (2) into (1), the vector autoregressive form (FVAR) of the factor model can be written as

$$\begin{bmatrix} F_t \\ X_t \end{bmatrix} = \begin{bmatrix} \Phi(L) & 0 \\ \Lambda\Phi(L) & D(L) \end{bmatrix} \begin{bmatrix} F_{t\Box 1} \\ X_{t\Box 1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{F_t} \\ \varepsilon_{X_t} \end{bmatrix}, \quad (3)$$

where

$$\begin{bmatrix} \varepsilon_{F_t} \\ \varepsilon_{X_t} \end{bmatrix} = \begin{bmatrix} I \\ \Lambda \end{bmatrix} \eta_t + \begin{bmatrix} 0 \\ v_t \end{bmatrix},$$

with variance covariance matrix

$$E\varepsilon_t\varepsilon_t' = \Sigma_\varepsilon = \begin{bmatrix} \Sigma_\eta' & \Sigma_\eta'\Lambda' \\ \Lambda\Sigma_\eta' & \Lambda\Sigma_\eta'\Lambda' + \Sigma_v \end{bmatrix},$$

where $E\eta_t\eta_t' = \Sigma_\eta$ and $E v_t v_t' = \Sigma_v$. Finally, inversion of the FVAR form yields the vector moving average form (VMA) for the X_t process

$$X_t = B(L)\eta_t + u_t,$$

where $B(L) = [I \Box D(L)L]^{\Box 1} \Lambda [I \Box \Phi(L)L]^{\Box 1}$ and $u_t = [I \Box D(L)L]^{\Box 1} v_t$.

2.1 Estimation

The estimation problem may be written as follows

$$\min_{F_1, \dots, F_T, \Lambda, D(L)} T^{\Box 1} \sum_{t=1}^T [(I \Box D(L)L) X_t \Box \Lambda F_t]' [(I \Box D(L)L) X_t \Box \Lambda F_t],$$

and solved following an iterative process. Differently from Stock and Watson (2005), where the r static factors F_t are estimated as the first r principal components of $(I \square D(L)L) X_t$, with r determined following Bai and Ng (2002), when a priori information concerning the economic interpretation of the factors is available, the estimation of the F_t factors can be carried out considering the relevant sub-set of variables. Therefore, given a preliminary estimates of $D(L)$, the r static factors F_t can be estimated as the first principal component of each of the r -subset of variables $(I_i \square D_i(L)L) X_{i,t}$ $i = 1, \dots, r$; then, conditional to the estimated static factors, an estimate of Λ and $D(L)$ can be obtained by OLS estimation of the block of equations corresponding to X_t in (1) . The procedure is then iterated until convergence. Once the final estimate of F_t is available, the $\Phi(L)$ matrix in (3) can be obtained by OLS estimation of the block of equations corresponding to F_t . Then, by also employing the final estimate of the Λ and $D(L)$ matrices, the restricted VAR coefficients in (3) can be computed. Stock and Watson (2005) provide details about the asymptotic properties, i.e. consistency and asymptotic normality, of the estimation procedure.

The availability of a priori information on the interpretation of the factors is key for the implementation of the proposed approach, which requires the separation of the variables in appropriate blocks. The separation issue is not problematic for econometric modelling, which is always guided by an underlying economic theory. In the application provided in this paper a large scale macroeconometric model for the G-7 countries (USA, Japan, euro area, UK, Canada) has been estimated, considering for each country eight macroeconomic variables, i.e. real output, inflation, short and long term nominal interest rates, nominal money balances, the real oil price, the real effective exchange rate and real stock prices. Hence, eight groups of homogeneous variables have been assessed and global factors bearing the interpretation of global GDP growth rate, real oil price growth, global real stock prices growth, and a global nominal factor, related to monetary policy management, have been extracted.³

2.2 Identification of structural shocks

The identification of the structural shocks in the F-VAR model can be carried out as follows. Denoting by ξ_t the vector of the r structural global shocks,

³On the basis of the results of PCA a global common factor among the real effective exchange rates could not be extracted. Moreover, a single factor (the global nominal factor) could be extracted among all the nominal variables, i.e. inflation, nominal money growth, short and long nominal interest rates for the five countries. See the next section for details.

the relation between reduced form and structural form global shocks can be written as $\xi_t = H\eta_t$, where H is square and invertible. The identification of the structural shocks amounts then to the estimation of the elements of the H matrix. It is assumed that $E[\xi_t \xi_t'] = I_r$, and hence $H\Sigma_\eta H' = I_r$. Moreover, by denoting v_t the n structural idiosyncratic shocks, the relation between reduced form and structural form idiosyncratic shocks can be written as $v_t = \Theta v_t$, where Θ is square and invertible. The identification of the structural idiosyncratic shocks amounts then to the estimation of the elements of the Θ matrix. It is assumed that $E[v_t v_t'] = I_n$, and hence $\Theta \Sigma_v \Theta' = I_n$.

The VMA representation of the factor model in structural form can then be written as

$$X_t = B^*(L)\eta_t + C^*(L)v_t,$$

where $B^*(L) = B(L)H^{\square 1} = [I \square D(L)L]^{\square 1} \Lambda [I \square \Phi(L)L]^{\square 1} H^{\square 1}$, $u_t = C^*(L)v_t$, $C^*(L) = C(L)\Theta^{\square 1}$, $C(L) = [I \square D(L)L]^{\square 1}$, and $E[v_{i,t} v_{j,t}'] = 0$ any i, j . Given r factors, then $r(r \square 1)/2$ restrictions need to be imposed in order to exactly identify the structural global shocks. Moreover, exact identification of the n structural idiosyncratic shocks requires the imposition of additional $n(n \square 1)/2$ zero restrictions.

The imposition of the exactly identifying restrictions is easily achieved by following a double Cholesky strategy, guided by economic theory, carried out as follows. Firstly, the structuralization of the factor or global shocks is achieved by assuming a lower triangular structure for the H matrix, with ordering of the variables set according to economic theory.⁴ The H matrix is then written as

$$H = \begin{bmatrix} H_{11} & & & \\ \vdots & \ddots & & \\ H_{r1} & \cdots & H_{rr} & \end{bmatrix},$$

and estimated by the Choleski decomposition of the matrix $\hat{\Sigma}_\eta$, i.e. from $\xi_t = H\eta_t$ we have $E[\eta_t \eta_t'] = H\Sigma_\eta H' = I$, and hence, $\hat{H} = chol(\hat{\Sigma}_\eta)$.⁵ The identification scheme performed allows for exact identification of the r structural global shocks, imposing $r(r \square 1)/2$ zero restrictions on the contemporaneous impact matrix.

Secondly, the matrix C_0^* is identified by imposing a lower triangular structure, with each non-zero block on the main diagonal showing a lower triangular structure as well, i.e.

⁴For instance, standard economic assumptions concerning the speed of adjustment to shocks, i.e. slow (output inflation), intermediate (interest rates, money growth), and fast (stock prices exchange rates, commodities) variables, could be employed.

⁵See Stock and Watson (2005) for details on alternative identification strategies.

$$C_0^{r*} = \begin{bmatrix} C_{011}^* & & & \\ \vdots & \ddots & & \\ C_{01r}^* & \cdots & C_{0rr}^* & \end{bmatrix},$$

where

$$C_{0jj}^{r*} = \begin{bmatrix} c_{0ij,11}^* & & & \\ \vdots & \ddots & & \\ c_{0ij,1m}^* & \cdots & c_{0ij,mm}^* & \end{bmatrix},$$

and $n = mr$, with m equal to the number of units (for instance, countries, as in the application provided below) in the sample. Again economic theory is called for to guide the ordering of the different units in each block (for instance, the distinction between large and small countries, in terms of GDP, could be employed).

The estimation of the Θ matrix is then carried out as follows:

1. regress $\hat{\varepsilon}_{X,t}$ on $\hat{\xi}_t$ by OLS and obtain \hat{v}_t as the residuals;
2. then from $\begin{bmatrix} \varepsilon_t \\ v_t \end{bmatrix} = \Theta v_t$ we have $E \begin{bmatrix} \varepsilon_t \\ v_t \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ v_t \end{bmatrix}' = \Theta \Sigma_v' \Theta = I$. Hence, $\hat{\Theta} = chol(\hat{\Sigma}_v)$.

The identification scheme performed allows for exact identification of the n structural idiosyncratic shocks, imposing $n(n - 1)/2$ zero restrictions on the contemporaneous impact matrix.

In the proposed application the double triangular structure has been justified on the basis of both the distinction in “slow” (output inflation), “intermediate” (interest rates, money growth), and “fast”-moving variables (stock prices, exchange rates, commodity prices), and the distinction, in terms of GDP size, in large (USA, euro-12 area, Japan) and relatively small (UK, Canada) countries. Hence, for instance, the matrix C_{011}^* contains the GDP growth rate time series for the various countries, in the order the US, Japan, the Euro Area, the UK and Canada.

Alternatively, in order to ensure robustness to variable ordering, estimation may be carried out by following the thick modelling estimation approach of Granger and Jeon (2004), consisting of repeating the analysis considering all the possible ordering of the variables, also simulating the model by Monte Carlo tools in each case, yielding median estimates and 95% confidence levels for the parameters of interests, i.e. impulse response functions and forecast error variance decomposition. Finally, policy analysis can also be carried out by means of generalized impulse response analysis (Pesaran and Shin,

1998), which, by construction, is not affected by variables ordering. Therefore, estimation methods allowing to draw robust conclusions not only to the ordering of the variables, but also to potential misspecification of the econometric model, are available for the proposed approach.

3 Empirical application: the construction of a large scale-macroeconometric model for the G-7

Quarterly time series data for five countries, i.e. the US, Japan, the Euro-12 area, the UK, and Canada, over the period 1980:1-2005:2, have been employed. Eight variables for each country have been considered, i.e. the real GDP growth rate, the real oil price growth rate, real stock market price returns, real effective exchange rate returns, the CPI inflation rate, the nominal money growth rate⁶ and the nominal short and long term interest rates.⁷ On the basis of misspecification tests, the lag length of the F-VAR model has been set to one. On the whole, the econometric model is composed of 39 equations, with the first 35 equations referring to the 35 endogenous variables, i.e. real output growth, inflation, the nominal short term rate, the nominal long term rate, nominal money growth, real exchange rate returns, and real stock returns for the five countries in the system, and the last 4 equations referring to the global factors.⁸

The F-VAR model has been estimated following the iterative procedure described in the methodological section, with 1000 Monte Carlo simulations carried out for the implementation of the thick modelling approach. Selected cumulative median impulse response functions are reported in Figure 1 for illustrative purposes. As is shown in the plot, the response of real output

⁶Nominal money balances are given by M2 for the US, M2+CD for Japan, M3 for the euro area and Canada, and M4 for the UK. The aggregates employed are those usually employed to measure broad money in each of the countries investigated.

⁷The short term rate refers to three-month government bills, while the long term rate to ten-year government bonds.

⁸The proportion of total variance explained by the first principal component for each group of variables is equal to 0.57 for real stock market returns, 0.40 for real output growth, 0.95 for real oil price growth and 0.65 for the nominal variables. In all the cases the first principal component bears the interpretation of global factor since, according to the estimated factor loadings, all the corresponding variables react as expected. On the other hand, all the other principal components tend to capture idiosyncratic dynamics. Hence, the estimated factors have a clear-cut macroeconomic interpretation, being associated with global real out growth, global stock market dynamics, real oil price growth and global nominal/monetary developments, respectively.

to a unitary global output shocks is similar for all countries, with the shock always leading to a permanent increase in real GDP. Similarly, the global nominal shock leads to a permanent impact on the price level, while the global stock market shock leads to a positive permanent impact on real stock prices for all the countries. Yet, the magnitude of the long term impact tends to vary across countries. For instance, for the global output shock the impact ranges between a minimum value of 0.54% (UK) and a maximum value of 3.07% (Japan), while the response for the other three countries is similar, averaging at 1.47%. A similar range of variation can also be noted for the long term impact of the global stock market shock on real stock prices, spanning between a minimum of 0.97% (US) and a maximum of 3.95% (UK), while for the other three countries the impact averages at 1.93%. Finally, for the global nominal shock the range of variation is wider, i.e. 5.99% (Japan) - 14.11% (UK), while for the three remaining countries the average impact is 7.61%. A large number of issues can be explored by means of the estimated macromodel, ranging from the international transmission of global and idiosyncratic shock to international comovements the in real and nominal side of the economy. Applications along the above lines can be found in Bagliano and Morana (2006) and Morana (2006a, 2006b).

4 Conclusions

In this paper a new approach to factor vector autoregressive estimation, based on Stock and Watson (2005), is introduced. With respect to the Stock-Watson approach, the proposed method has the advantage of allowing for a more clear-cut interpretation of the global factors, as well as for the identification of all idiosyncratic shocks. Moreover, it shares with the Stock-Watson approach the advantage of using an iterated procedure in estimation, recovering, asymptotically, full efficiency, also allowing for the imposition of appropriate restrictions concerning the lack of Granger causality of the variable versus the factors. Finally, relative to other available methods, the modelling approach has the advantage of allowing for joint modelling of all the variables, with no need for long-run forcing hypotheses. An application to large scale macroeconomic modelling is also provided.

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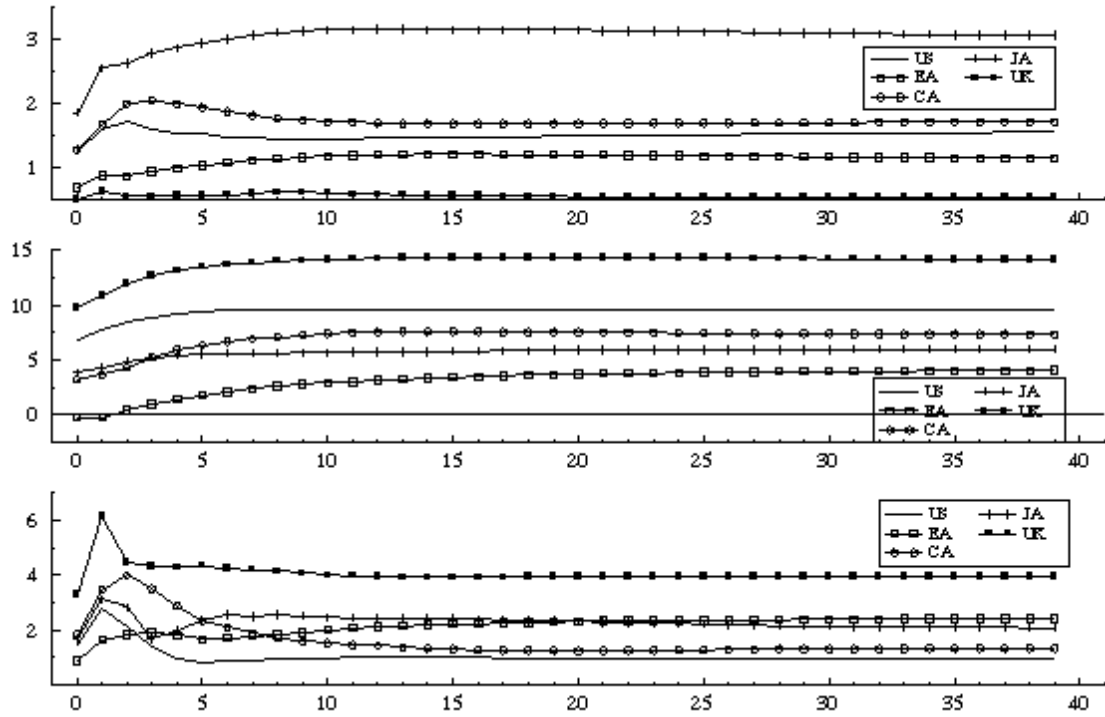


Figure 1: Cumulative median impulse responses of real output (top plot), inflation (center plot), real stock prices (bottom plot) to positive global output, nominal and stock market shocks, respectively (US: United States; JA: Japan; EA: euro area; UK: United Kingdom; CA: Canada).